Suppose you get a text message. Your friend tells you to go to Lobby 7 at MIT to find the gift they left you 7 meters from the center of the lobby. Is that enough information to find the gift right away? As you can see, there are many locations 7 meters from the center of the room. Don't forget that we live in 3 dimensions, so there are actually even more points 7 meters away from the center of the room. Fortunately, in this problem, you can ignore most of them since we don't expect our gift to be hanging in mid air. Distance alone wasn't enough information. It would have been helpful to have both the distance and the direction. This video is part of the Representations video series. Information can be represented in words, through mathematical symbols, graphically, or in 3-D models. Representations are used to develop a deeper and more flexible understanding of objects, systems, and processes. Hi, my name is Dan Hastings and I am Dean of Undergraduate Education and a Professor of Engineering Systems, and Aeronautics and Astronautics here at MIT. Today, I'd like to talk to you about the utility of thinking about displacements as vectors when trying to recall vector properties, and how you determine if a physical quantity can be represented using vectors. Before watching this video, you should know how to add and scale vectors. You should also understand how to decompose vectors, and how to find perpendicular basis vectors. After you watch this video, you will be able to understand the properties of vectors by using displacement as an example, and you will be able to determine whether a physical quantity can be represented using vectors. Meet the vector. The vector is an object that has both magnitude and direction. One way to represent a vector is with an arrow. You have seen other algebraic representations of vectors as well. There are many physical quantities that have both magnitude and direction. Can you think of some? Make a list of quantities that can be described by a magnitude and direction. Feel free to discuss your list with other people. We'll come back to this list at the end of the video. Pause the video here. In engineering, there are many physical quantities of interest that have both magnitude and direction. Consider the following example: Here you see a video of airflow over the wing of an F16 fighter jet model in the Wright Brothers wind tunnel at MIT. The air that flows over the wing has both speed and direction. The direction is always tangent to the path of the airflow. We can represent the air velocity with an arrow at each point around the wing. The length of the arrow represents speed, and the direction represents the direction of motion. Such a collection of vectors is called a vector field. The vector field of airflow over the wing creates a lift force via the Bernoulli effect. This effect suggests that because the horizontal component of the airflow velocity is the same throughout the flow field, the air flowing over the wing is moving faster than the air flowing beneath the wing. This creates a difference in air pressure, which provides the lift force, another physical quantity that we can represent with a vector. Depending on the angle of the wing, the magnitude and direction of the lift force changes. Lift is just one example of a vector quantity that is very important in designing aircraft. We are quite used to thinking of forces as vectors, but do forces exhibit the properties necessary to be aptly represented by vectors? Let's review the properties of the vector. To add vector b to vector a, we connect the tail of b to the tip of a and the sum is the vector that connects the tail of a to the tip of b. An important property of vector addition is that it is commutative. That is a + b = b + a. You can see this visually from the parallelogram whose diagonal represents both sums simultaneously. Another important property is that vectors can be multiplied by real numbers, which are called scalars, because they have the effect of scaling the length of the vector. Multiplying by positive scalars increases the length for large scalars, and shrinks the vector for scalars less than one. Multiplying a vector by -1 has the effect of making the vector point in the opposite direction. Another important property of vectors is that the initial point doesn't matter. Any vector pointing in the same direction with the same magnitude represents the same vector. To make this seem less abstract, we can think of vector properties in terms of displacement. Suppose you walk from a point P to a point Q. The displacement, or change is position from P to Q, is aptly represented by an arrow that starts at the point P and ends at the point Q. Let's see how displacement motivates the correct form of vector addition. Consider the following example: you start at home, which is represented by a star on the map. You walk 300 meters east to get a cup of tea before you walk southeast 500 meters to school. After class you walk 400 meters southwest of your school to play tennis. Your friend, who lives in your apartment complex, is going to meet you there. What vector would represent the displacement vector for your friend who leaves home directly and meets you to play tennis? Pause the video here and discuss your answer with someone. Answer: The vector that starts at your home and moves down to the tennis court. This is interesting because the arrow that connects your starting location to your ending location represents the total displacement from your starting point. In other words, this vector is the sum of the other 3 displacement vectors. Displacement also helps you understand vector decomposition. Suppose you have walked a few blocks away, represented by the following displacement. To get there, you probably didn't walk through other people's houses and yards. Your path more likely looked something like this. This process of breaking a vector down into component parts pointing along particular directions is completely analogous to decomposing a vector into components that point along perpendicular basis vectors. When in doubt about the mathematics of the vector, take a moment to rephrase your problem in terms of displacements, and see if your intuition can guide the mathematics. Now, let's go back to forces -- do they have the vector properties that we expect them to? When representing physical quantities with vectors, the quantity must have both magnitude and direction. But it must also scale and add commutatively. Let's see if force has these properties. Force seems to have magnitude and direction. Force also scales appropriately. We think of forces as being small or large, we can increase them and decrease them. When we draw a free body diagram, we are implicitly assuming that forces are vectors, and that they add like vectors. But how do we know this? We do an experiment. In this next segment we'll see a demonstration of how forces, do indeed, add like vectors. [Pause] Here you see 3 Newton Scales connected by strings. We'll call the two strings on top String A and String B. String A is 135 degrees off of horizontal. String B is 45 degrees off of horizontal. The scale reads out the magnitude of the tension force on each string. We first want to get a reading of the scales while there is no mass added to the system. The scales do not have very precise measurement; we can only guarantee the measurement to within .5 Newtons. When looking at the bottom scale, we see that the reading is approximately -.3 Newtons. The tension on string A is approximately .5 Newtons, and the tension on string B is .3 Newtons. These tension forces are due to the weight of the bottom scale and the strings. So we will need to subtract these amounts off of any reading when mass is added into the system to get the tension force of the mass alone. Let's add a 1kg mass to the hook below the bottom scale. The bottom scale now reads about 9.6 Newtons. Now we look at the top two scales. We see that the tension force on string A is 7.4 Newtons, and the tension on string B is 7.5 Newtons. We want to decompose these forces to see if they do in fact add like vectors. Note that Newton's second law says that the sum of these forces must be zero, since our system of strings and scales is stationary. Let's start by subtracting off the readings we got from our Newton scale system with no added mass to find the net tension force due to the mass. The tension in the string A is 7.4-.5 = 6.9 N. And the tension on string B is 7.5-.3=7.2 N. We find that the net force down is 9.6-(-.3) = 9.9 Newtons. Using F=mg, we would predict that the force due to a 1kg mass would be 9.8Newtons. So the fact that we are measuring 9.9Newtons indicates that we have some experimental error in our measurements. How would you use this setup to show whether or not forces add like vectors? We want to see that the forces sum to zero. To do this, let's decompose the forces into horizontal and vertical components. We use the fact that the string A is at a 135 degree angle. Because the magnitude of sin(135 )and cos(135) are both one over square root of 2, we simply need to divide by the square root of two. We find that the horizontal and vertical components of this tension force are approximately 4.9 Newtons. Because sin(45) and cos(45) are both one over the square root of two, we divide by the square root of two and find that each component is approximately 5.1 N. Thus the horizontal forces subtract to give a net force of .2N in the positive x direction. The three vertical components add to 10-9.9 = .1 in the positive y direction. Because .1 and .2 are small, and because we know that there are errors associated with the limits of accuracy of out measurements, we can be confident that these forces do, in fact, sum to 0. So this demo does in fact suggest that forces add like vectors. But we want to make sure that this wasn't an artifact of having so much symmetry in the system. To do this, we move string B 60 degrees off of horizontal. As you can see, the tension force on the bottom string did not change. It still reads 9.6N. But the force on each upper Newton scale has changed. The tension force on String A is 5.5N, and the tension force on the string B is 7.5N We leave it as an exercise to you to decompose the tension forces into horizontal and vertical components and verify that within the expected measurement error the forces sum to zero. Forces really do add like vectors!! Okay, so forces can indeed be represented with vectors. Let's look back at the list you generated of physical quantities with both magnitude and direction. If force was on your list, we now know that force is indeed a vector quantity. Maybe you also listed rotation. Let's see if rotations have vector properties. Rotation seems like a physical quantity that has magnitude and direction. The direction could be determined by the axis of rotation. We choose which way the arrow points based on the right hand rule. The magnitude determines how many radians through which the object rotates. Consider the following two rotations: Rz rotates an object by 90 degrees about the z-axis: which rotates an object as such. Ry rotates an object by 90 degrees or Ï€/2 radians about the y-axis, which rotates the same object in this manner. Do rotations scale like vectors? Let's see what happens if we take the vector that represents a rotation of Ï€/2 radians about the z-axis and add it to itself -- it seems that it should be a rotation of Ï€ radians, or 180 degrees. And this agrees with what we get by rotating 90 degrees about the z-axis twice. So scaling rotations makes sense. Question: Do rotations add like vectors? If we rotate the object a quarter turn about the z-axis, followed by a quarter turn about the y-axis, the object ends up in the following position. If instead we rotate a quarter of a turn about the y-axis, followed by a quarter turn about the z-axis, the object ends up in this position. Are the ending positions the same for the two different permutations of rotations? [Pause] No, they are not. This means that rotations don't add commutatively, but vector addition must be commutative. So this tells us that we CANNOT use vectors to represent rotations. You'll learn that it is better to use matrices and matrix multiplication to represent combinations of rotations. The tricky thing is that a vector can be used to represent the rotation rate, the time derivative of rotation, quite well. During this video, you came up with several physical quantities that you theorized behave like vectors. Consider the following list of quantities. Compare our list to your own list, and determine whether each one is best represented by a vector, a scalar, or neither. You may need to design an experiment or thought experiment in order to verify your hypothesis. To review, you have learned that: Displacements help guide our intuition for vector algebra. Physical quantities can be represented with vectors only when they have magnitude, have direction, scale, and add commutatively. Forces can be represented by vectors, while rotations cannot.