Vector Fields

Representation Series

Instructor's Guide

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Developed by the Teaching and Learning Laboratory at MIT
for the Singapore University of Technology and Design

Introduction

When to Use this Video

- In Math 201, in class or recitation, during or after Unit 4: Vector Calculus on the Plane, Lecture 3: Vector Fields
- Prior knowledge: representing vector fields with vectors attached to points, and representing vector fields with flow lines

Key Information

Duration: 9:52
Narrator: Prof. Dave Darmofal
Materials Needed:
• Paper
• Pencil

Learning Objectives

After watching this video students will be able to:

- Understand that flow quantities around bodies are often analyzed in Eulerian frame.
- Recognize that flow velocity is a vector field, which can be a function of space or a function of space and time.

Motivation

- In multivariable calculus, students become very adept at computing quantities involving vector fields. However, there is difficulty in connecting the abstract concept of a vector field to the representation of an observable quantities.
- Understanding what variables a physical quantity depends on is important in modeling and problem solving. This video explores when a vector field may or may not depend on time, and how conditions may effect this dependence.

Student Experience

During the video, students will:

- Observe the flow field around an airplane wing in a wind tunnel, using a smoke probe visualization.
- Connect the definition of the vector field and their understanding of flow lines to a visual example.

Video Highlights

This table outlines a collection of activities and important ideas from the video.

Time	Feature	Comments
1:27	Wright Brothers Wind Tunnel	The features, dimensions, and capabilities of the wind tunnel are described.
2:35	Prof. Dave Darmofal describes the airplane model.	A wooden model of an F16-like aircraft is used in this flow visualization. Prof. Dave Darmofal describes the parts of this model and why it was created.
4:36	Low angle of attack flow condition	This video segment contains visualizations of flow over a wing, and vortex flows imaged at a low angle of attack (about 10 degrees off of horizontal). In this flow regime, the flow is shown to be steady, or time independent.
6:25	High angle of attack flow condition	This video segment shows what happens to flow over the wing when the airplane is at a high angle of attack. Note the vortices created by the leading edge strake.
6:21	Unsteady flow	At this higher angle, the flow is seen to be unsteady, which causes the smoke to "mix-out", losing the steady flow line observed in the low angle of attack case. This shows that the flow is time dependent.
8:35	Use of flow data in engineering	Prof. Dave Darmofal discusses the ways that flow visualizations in combination with other measurements are used in engineering. He discusses an example of a US Navy plane that had to be removed from service with very few flight hours due to the wear on the tail wing caused by the instability of the strake vortex.

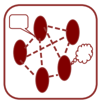
Video Summary

This video takes a look at a smoke probe visualization of airflow over a model of an F16 aircraft. The visualization is created in the Wright Brothers Wind Tunnel. Two flow regimes are shown. The first is a low angle of attack visualization, where the flow field appears steady, or independent of time. The second flow regime has the model at a high angle of attack. In this case, the flow appears unsteady, which shows that the flow field has a time dependence. Prof. Dave Darmofal ends the demonstration by discussing how these visualizations are used in engineering.

Math 201 Materials

Pre-Video Materials

When appropriate, this guide is accompanied by additional materials to aid in the delivery of some of the following activities and discussions.



1. Break students into small groups of 3 to 4 students. Have them brainstorm lists of vector field quantities. Have them explain why the quantities on their list are aptly represented by vector fields, and how they might try to visualize the behavior of this field.

2. How does your interpretation of flow lines for a velocity field differ from your interpretation of flow lines for an acceleration field?

The Physics Education Technology group (PhET) from the University of Colorado created a simulation that students can play around with that shows how objects behave in force fields, which are proportional to acceleration fields.

This simulation is available from:
http://phet.colorado.edu/en/simulations/category/physics

Post-Video Materials

1. Return students to small groups of 3 to 4 students. In these groups, have students find videos from YouTube® and other sources of their choosing which contain flow fields that depict stable and/or unstable flows. Have students discuss why a particular video depicts a vector field quantity, and how they recognize the vector field's dependence on different variables.

Note that it may be very difficult for students to find video of flow fields that are stable. Stable flows have no time dependence. It would be worth pursuing this line of discussion. Ask students why it is so difficult to find time independent vector fields. Student discussion may lead to the notion of a steady state, or approach to an equilibrium state in trying to describe situations where the vector field would be steady.

After this discussion, you may wish to point students to the National Committee for Fluid Mechanics Film: Flow Instabilities (link available in the references section).

2. After the previous activity, go back to the low angle of attack video. Is the smoke visualization really steady?

Students can discuss reasons for this apparent unsteadiness to be present. In truth, the flow is steady. The apparent unsteadiness is due to the smoke probe itself, as well as some unsteadiness due to the fan creating the wind.

3. In the video, we considered the flow around a body in an Eulerian frame. However, we could instead consider the flow field from the reference frame of the moving air particles. What are the pros and cons of doing this? In small groups of 3 to 4 students, have students brainstorm different vector field quantities that are better represented in a different reference frame.

You can have students watch the National Committee for Fluid Mechanics Film: Eulerian Lagrangian Description (link available in references section).

Additional Resources

Going Further

Vector field quantities are pervasive in electromagnetism, which is taught in Physics 201. While vector fields themselves are not taught, students have various difficulties in understanding the meaning of various representations of vector fields. Students having difficulty understanding what a force field really is may benefit from the Simulation "Electric Field Hockey" created by the Physics Education Technology (PhET) at the University of Colorado, Boulder, CO. This simulation is available at: http://phet.colorado.edu/en/simulations/category/physics

The example in this video is entirely concerned with the velocity field of air around an airplane wing and body. But there are other velocity fields at play even in this example that are difficult to perceive. It is important that students understand the difference in how velocity and force fields behave, and the response created by bodies placed into the field.

Coming back to this notion of vector fields and reference frame may be useful when starting to do numerical methods, solving differential equations and the like. It may be helpful for students to understand the connection between the algorithm for numerically solving a differential equation, and the reference frame.

References

The following videos and associated film notes were created by Prof. Ascher Shapiro and the National Committee for Fluid Mechanics Films at MIT as part of a grant from the National Science Foundation. They describe in detail ways of visualizing flows, and how flow instabilities arise and can be recognized.

- Kline, S.J., (1969). *Film Notes for Flow Visualization*. Available from http://web.mit.edu/hml/ncfmf.html
- Lumley, J.L., (1969). *Film Notes for Eulerian and Lagrangian Descriptions in Fluid Mechanics*. Available from http://web.mit.edu/hml/ncfmf.html
- Mollo-Christensen, E.L. (Unknown Date). *Film Notes for Flow Instabilities.* Available from http://web.mit.edu/hml/ncfmf.html
- Pesetsky, A. H., Smith, K. and Educational Services Incorporated (Producers), (unknown). *Flow Instabilities* [Film]. Available from http://web.mit.edu/hml/ncfmf.html
- Pesetsky, A. H. (Producer), (1968). *Eulerian Lagrangian Description* [Film]. Available from http://web.mit.edu/hml/ncfmf.html
- Smith, K. and Educational Services Incorporated (Producers), (1969). *Flow Visualization* [Film]. Available from http://web.mit.edu/hml/ncfmf.html

Vector fields are introduced in of the MIT Open CourseWare course listed below.

- Auroux, Denis. 18.02 Multivariable Calculus, Fall 2007. (Massachusetts Institute of Technology: MIT OpenCourseWare), http://ocw.mit.edu (Accessed 24 Jan. 2012). License: Creative Commons BY-NC-SA
  - Lecture 19: Vector Fields

The following are references on aerodynamics and flight. The first is suitable for first year undergraduate students interested in pursuing these ideas. The second was developed from an advanced course at MIT, and is intended as a resource to the instructor.

- Anderson, D.F., Eberhardt, S. (2001). *Understanding Flight.* New York: McGraw-Hill.
- Ashley, H., Landahl, M. (1985). *Aerodynamics of Wings and Bodies.* New York: Dover Publications, Inc.

The following articles discuss education research into student difficulties with understand vector fields, and in particular the electric field.

- Curjel, C.R. (1990). Understanding Vector Fields. *The American Mathematical Monthly, 97*(6), 524–527.
- Furió, C., Guiasola, J. (1998). Difficulties in Learning the Concept of the Electric Field. *Science Education*, *82*(4), 511–526.
- Törnkvist, S., Pettersson, K.-A., Tranströmer, G. (1993). Confusion by Representation: on student's comprehension of the electric field concept. *Am. J. Phys., 61*(4), 335–338.