In this video for 3D geometry, we will be talking about two topics.

One is the angle bisector of two planes and one is line of intersection of two planes.

Both of the topics are important especially the line of intersection of two planes, and I will be spending some time on that.

Angle bisector of two planes is a very easy topic, something you should remember while preparing for JEE.

I will just give you the formula and I wonâ€™t be doing a problem but I will spending sometime on this (Line of Intersection of two planes), because this is a relatively important topic.

So let us say we have a pair of planes.

And you have to find the angle bisector.

This is the angle bisector â€” plane which is the angle bisector of two planes.

This angle is the same as this angle.

You have been given equations for P1 and P2.

You have find equation of angle bisector.

If you think about this â€” easiest way to think about planes is using a notebook.

I always recommend you do that if you are a little confused.

So let us say I have two planes like this.

Again, they are infinite.

One plane will be the plane bisecting this angle.

This plane can also be extended to this direction.

Thus there can also be an angle bisector of this angle between the two planes.

There are always two angles â€” one is acute and one is obtuse.

You can find equations for both of the planes.

The way you do it is very simple.

I will describe the equations of both kind of planes.

Again, there are two angles â€” acute and obtuse.

How will you find the angles?

Just take dot product between n_1 and n_2.

You will get an angle â€” either obtuse or acute.

If there is one angle, you can always subtract 180 - that to get the other angle.

Let us say we have equations â€” this is just how you should remember to do this.

I am currently writing d_1 here so please take note of this.

Generally I write d on the right hand side but this time it is here.

Whenever you get something like this, ensure d_1 and d_2 are greater than 0.

$d_1>0$, $d_2>0$.

Whatever equation you get, take constant to the left hand side, multiply by negative if there is a need to make this greater than 0.

Once you have ensured that, check if $a_1a_2 + b_1b_2 + c_1c_2 > 0$.

If it is greater than 0, then $(a_1x + b_1y + c_1z - d_1)/(a_1^2+b_1^2+c_1^2)^{1/2} = (a_2x + b_2y + c_2z - d_2)/(a_2^2+b_2^2+c_2^2)^{1/2}$.

This would be the angle bisector of the obtuse side.

Similarly, let me just write it down so there is no confusion.

This would be the angle for the acute side with negative sign in front.

This is only when $a_1a_2 + b_1b_2 + c_1c_2 > 0$.

If it is not greater than 0, vice-verse otherwise.

The signs would reverse if this is less than 0.

Just say opposite if a1a2 + b1b2 + c1c2

Think like this that whatever will be the sign of a1a2 + b1b2 + c1c2, that will be the sign of obtuse side.

Thats how I remember.

Again, you have to ensure that d1>0, d2>0.

If is not a very conceptual thing and I can explain to you how this equation was derived, it is not hard to see.

Thats not the important point here.

If the question comes, just remember this thoroughly.

It is literally like being careful with signs and calculation.

So I am not doing a problem for this, you can do very easily yourself.

You can construct two planes, and then find the angle bisector.

I think its pretty straightforward.

You just have to remember the formulas.

Now let us go to the next part which is a very interesting topic â€” line of intersection of two planes.

Let us say you have two planes P1 and P2.

What is the line of intersection of two planes?

That is the line which is â€” this line i.e. the line of intersection of two planes.

If you have a notebook.

The way you think about it â€” this line is the line of intersection of two planes.

So just take a notebook and look at this line and that is the line of intersection.

How would you find the equation of line?

If you know P1 and P2, the question would be how to find this line L here.

How would you define this.

What all do you need to define a line?

You need a parallel vector.

You will have n1_vector and n2_vector.

You need a point here, a_vector.

And you need a parallel vector.

What is the property of this parallel vector?

Property of this parallel veto is that it is parallel to both P1 and P2 planes.

Because it parallel to the line and the line is in both the planes, so it parallel to both planes P1 and P2.

In other words, it is perpendicular to both n1 and n2.

And as soon as I say this, it should flash into your mind that b_vector = n1 \cross n2.

This is something straight out of the cross-product definition that b_vector = n1 \cross n2.

Please recall the chapter of vectors if you are forgetting cross-product.

So the first step is that you will fine n1 \cross n2 for getting b_vector.

And second thing you would do is find a common point by putting one coordinate to be zero.

I will solve a problem so that it will hopefully make it easier for you to understand.

The equation for P1 that has been give to you is $x+y+z=1$.

And the other plane has been give to you is $x-y+z=1$.

First thing we have to do is that we have to find b_vector. What is n1_vector? n1 = i_cap + j_cap + k_cap. And n2_vector = i_cap - j_cap + k_cap.

Now b_vector would be = |i j k; 1 1 1; 1 -1 1|. If you open this up, it will come 2i_cap - 2k_cap. You can check by taking a dot with both of them. This 2-2=0 and 2-2=0. This is a vector which is perpendicular to both n1 and n2.

So this is the vector parallel to the line.

Then you have to find the point. This was Step-1. Step-2 would be â€” we can put any coordinate to be zero. Let us put z=0 in both the planes. What we are doing is that we have to find a point. So we have to get the x coordinate, y coordinate and z coordinate. But you have two-equations and three variables. So we are putting one coordinate to be zero. If we put z=0, then you have two equations and two variables, and then you know the point.

So we have x+y=1 and x-y=1. So that will give you x=1 and y=0. This will become x+y=1 and x-y=1. This becomes x=1, y=0. So the point A is (1,0,0).

So equation of line L is i_cap + \lambda (2i_cap - 2k_cap).

This line would lie in both plane 1 and plane 2.

And you can also write it in the coordinate form. (x-1)/2 = (y-0)/0 = z/(-2).

I hope that this made sense and gave you the idea of what it means. Line of intersection like a notebook is a line common to both the planes. It is perpendicular to both n1 and n2. So the parallel vector has to be n1 \cross n2. And then you can find the point by putting one coordinate to be zero.

So this was the topic of angle bisector of two planes and line of intersection of two planes. The next video we will start mixing up line and planes, and points and planes, and things like that. I hope to see you in the next video. I hope you enjoyed this video.

Thank you.