Analysis of a Forecasting-Production-Inventory System with Stationary Demand

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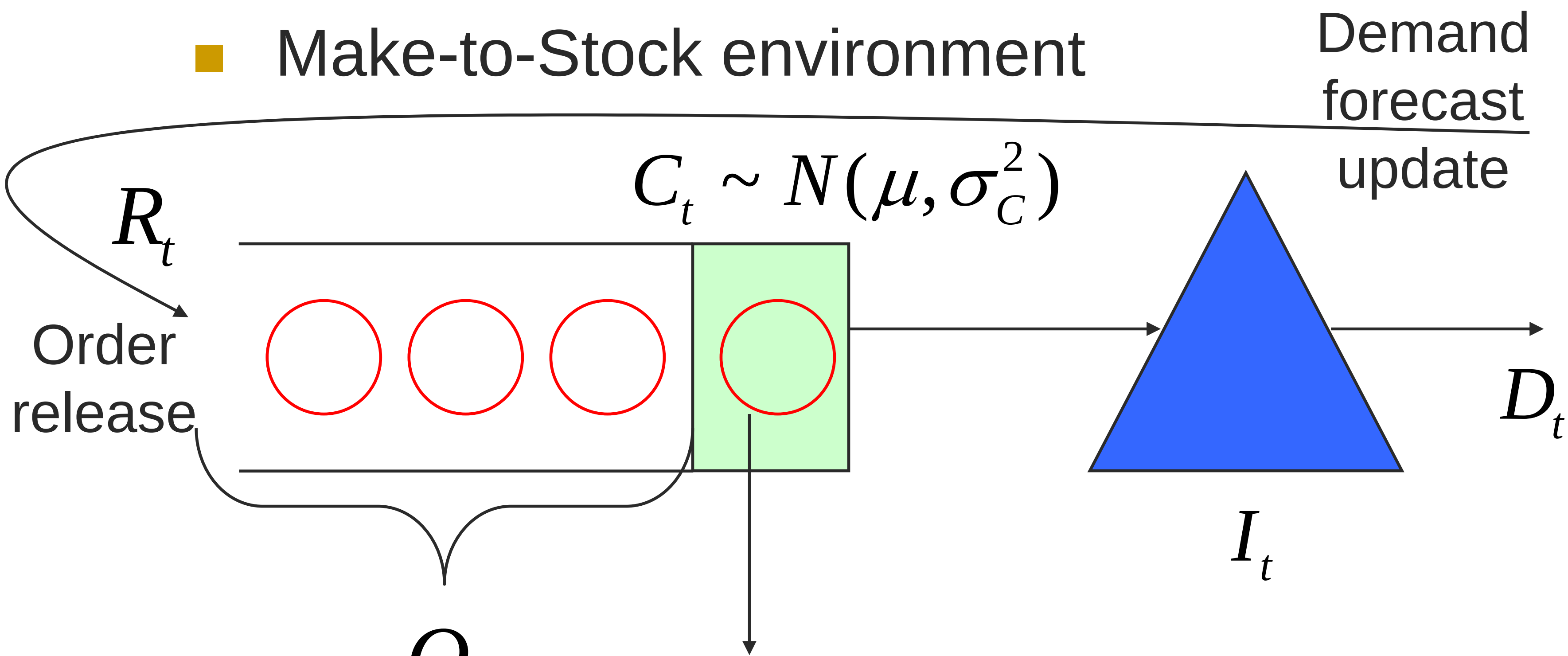
Forecasting-Production-Inventory

- Make-to-Stock environment



$$P_t = \min\{Q_{t-1}, C_t\}$$

Objective

- Minimize steady-state
  - Inventory holding costs *h*
  - Shortage penalty costs *b*

Recap: MMFE Model

- Rolling horizon H
- Forecast $D_{t,t+i} \quad i = 0, \ldots, H$
- Forecast Update

$$\varepsilon_{t,t+i} = D_{t,t+i} - D_{t-1,t+i}$$

- Assumptions
  - Stationary Demand with rate λ
  - Unbiased Forecasts
  - Uncorrelated Forecast Updates

Production-Inventory Model

- MRP-Type Release Policy:

$$R_t = \sum_{i=0}^{H-1} \varepsilon_{t,t+i} + D_{t,t+H} = e^T \varepsilon_t + \lambda$$

- Inventory Policy

$$\underbrace{Q_t + I_t - \sum_{i=1}^{H} D_{t,t+i}}_{\tilde{I}_t} = s_H$$

Production Policy

- Forecast-corrected base-stock policy

$$P^*(\tilde{I}_{t-1}) = \begin{cases} C_t & \text{if } s_H > \tilde{I}_{t-1} + C_t \\ s_H - \tilde{I}_{t-1} & \text{if } s_H \le \tilde{I}_{t-1} + C_t \end{cases}$$

- State-dependent Optimal Policy

$$(D_{t-1,t}, D_{t-1,t+1}, \ldots, D_{t-1,t+H-1}, \lambda)$$

Benchmark: Myopic Policy

- Do not use available forecast information

$$R_t = D_t$$

- Constant Inventory

$$Q_t + I_t = s_m$$

Outline

- Model
- Steady-State Distribution of WIP
- Base-Stock Levels
- Discussion
- Conclusion

In Heavy Traffic, the WIP has an exponential distribution

Average Excess Capacity

$$v = \frac{2(\mu - \lambda)}{e^T \Sigma e + \sigma_C^2}$$

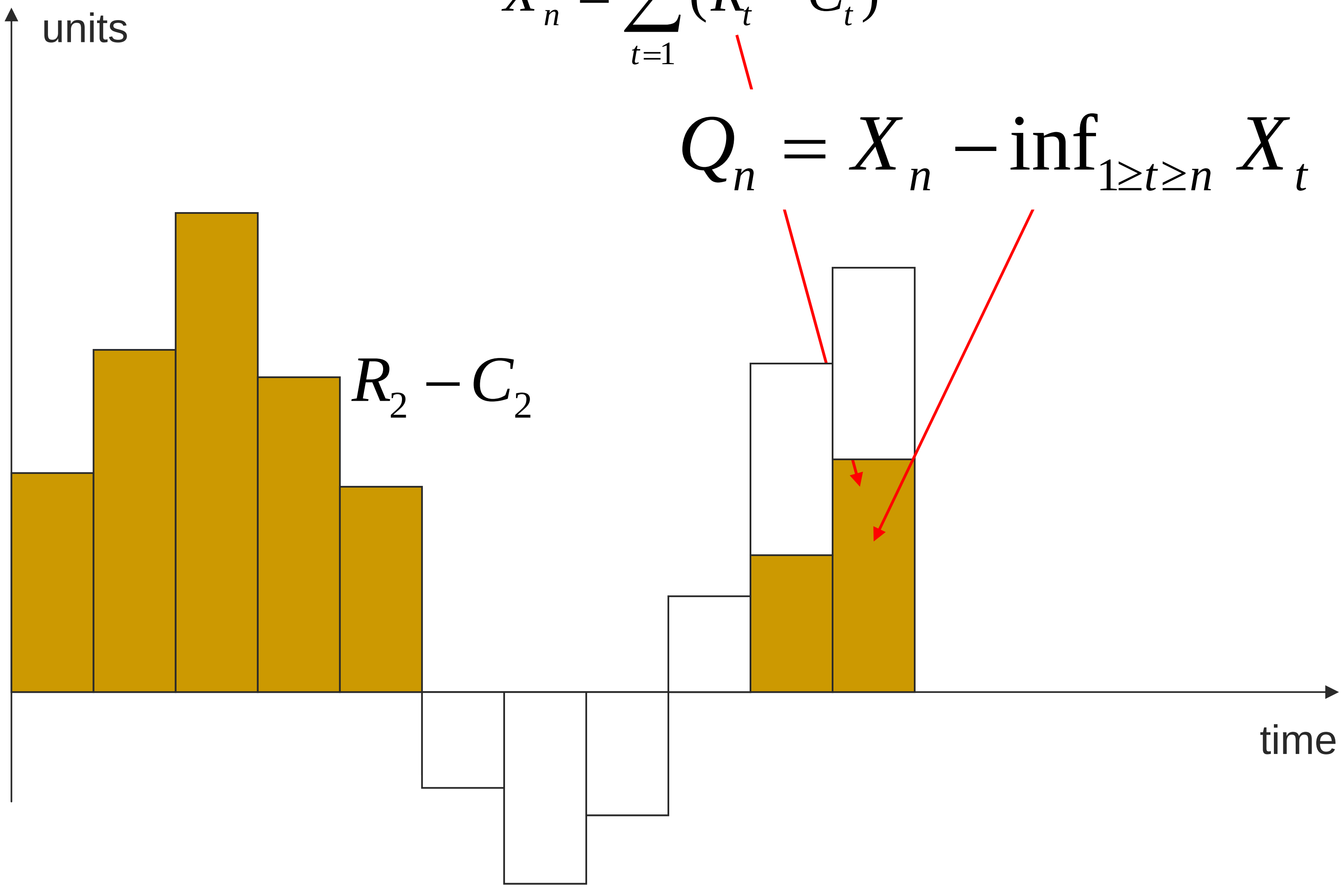
Variance of the forecasts

Variance of the production

WIP at time *n*

$$X_n = \sum_{t=1}^{n} (R_t - C_t)$$

$$Q_n = X_n - \inf_{1 \geq t \geq n} X_t$$



Heavy traffic analysis 1

Consider a sequence of systems k s.t.:

$$\lambda^{(k)} \to \lambda$$

$$\mu^{(k)} \to \mu$$

$$\sqrt{k}(\lambda^{(k)} - \mu^{(k)}) \to c < 0$$

Heavy traffic analysis 2

$$\frac{Q_n^{(k)}}{\sqrt{k}} = \frac{X_n^{(k)}}{\sqrt{k}} + \frac{-\inf_{1 \geq t \geq n} X_t^{(k)}}{\sqrt{k}}$$

$$\frac{X_{\lfloor kt \rfloor}^{(k)}}{\sqrt{k}} = \frac{X_{\lfloor kt \rfloor}^{(k)} - m^{(k)} \lfloor kt \rfloor}{\sqrt{k}} + \frac{m^{(k)} \lfloor kt \rfloor}{\sqrt{k}}$$

where $m^{(k)} = \lambda^{(k)} - \mu^{(k)}$

Heavy traffic analysis 2

$$\sigma B(t) \qquad\qquad ct$$

$$\frac{X^{(k)}_{\lfloor kt \rfloor}}{\sqrt{k}} = \frac{X^{(k)}_{\lfloor kt \rfloor} - m^{(k)}\lfloor kt \rfloor}{\sqrt{k}} + \frac{m^{(k)}\lfloor kt \rfloor}{\sqrt{k}}$$

$$BM(c, \sigma^2)$$

Heavy traffic analysis 3

$$\frac{Q_n^{(k)}}{\sqrt{k}} = \frac{X_n^{(k)}}{\sqrt{k}} + \frac{-\inf_{1\geq t\geq n} X_t^{(k)}}{\sqrt{k}}$$

$$RBM(c,\sigma^2)$$

Reflected Brownian Motion on the nonnegative halfline

Estimated by an exponential random variable

Steady-state WIP distribution

$$P(Q_\infty = 0) = 1 - e^{-v\beta}$$ impulse

$$P(Q_\infty > x) = e^{-vx}(x + \beta)$$

- β is a correction term, coming from Random Walks

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Determination of the Base-Stock levels 1

- Myopic Policy

$$s_m = F_{Q_\infty}^{-1}\left(\frac{b}{b+h}\right)$$

- Newsboy quantity…
- … considering the distr. of the WIP!

$$s_m = \frac{1}{v}\ln\left(1+\frac{b}{h}\right)-\beta$$

Determination of the Base-Stock levels 2

- MRP-type Policy

$$s_H = F_W^{-1}\left(\frac{b}{b+h}\right)$$

$$W = \max\{Q_\infty + Y_0, \max_{1 \le k \le H} Y_k\}$$

- Y_0 is the difference between:
  - Total Forecast Error over the horizon *H* and
  - Total Capacity

Determination of the Base-Stock levels 3

- MRP-type policy: asymptotic

$$b \gg h$$

$$s_H^a = s_m^* + \mu_{Y_0} + \left(\frac{1}{2}\right)\sigma_{Y_0}^2 \, v$$

Proportional to the variance!

- Good approximation when
  - b/h large
  - High utilization rate

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Capacity – Stock Trade-Off

- No advance information

$$s_H^a = s_m^* - H\lambda$$

- Full advance information

$$s_H^a = s_m^* - H\lambda - H(\mu - \lambda)\left(\frac{\sigma_D^2}{\sigma_D^2 + \sigma_C^2}\right)$$

- Interchangeability Capacity/Safety Stock
- Demand variability $\longrightarrow$ Capacity

Discussion

- Correlation $\uparrow \rightarrow v \downarrow \rightarrow s_m^* \uparrow$
- $\sigma_{Y_0}^2$ is the system variability over *H* not resolved at the beginning of the horizon
  - Preference for accurate early forecasts
- Optimize over all planning horizons

$$R_t = \sum_{i=0}^{H-1} \varepsilon_{t,t+1} + D_{t,t+H}$$

- Greater costs due to
  - misspecification of the forecast model than
  - misuse of the information in production

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Conclusion

- Integrated view
  - Forecast
  - Production
  - Inventory
- Lots of improvement for current MRP systems
- Is heavy traffic of practical value?

Thank you

Questions?