15.066 Systems Optimization and Analysis
Optimization Project

Labor Planning for a Manufacturing Line

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Introduction

The objective of this project was to create an LP model which could allocate labor on a manufacturing line. The line manufactures precision thermostats for the aerospace industry (see Exhibits 1 and 2). Current monthly demand for the product ranges from 25 thousand and 45 thousand units with an average of 35 thousand units per month. Parts are processed continuously through the line so there is no need to complete all operations on all parts during the month (or day) and parts are shipped on a daily basis. There are three main manufacturing areas: an open area and two clean rooms.

Machine operators are paid on an hourly basis depending on their job grade and the area they are working. Job grades range from one to five – grade fives perform the highest skilled work and are paid the highest. A grade five worker is able to perform all jobs from one to five, but a grade one worker can only perform grade one jobs. There are 2nd and 3rd shift hourly premiums as well as clean room hourly premiums paid to workers. For safety reasons, there is also a minimum requirement of at least five people per shift if the shift is to be run. A single worker can only work a single machine at any given time. The hourly pay rates for each job grade can be seen in Exhibit 3 and the minimum grade required to perform each job can be seen in Exhibit 4.

The LP model minimizes the total labor cost per month and allows supervision to schedule daily work. Analysis of the output of the model shows the effect of demand fluctuations on labor cost and machine utilization. This optimization tool can be used to determine the size and skill level of the workforce required to meet product demand as well as aid the supervisor to direct each shift to the operations needed to be done. Note that the LP model is not intended for the scheduling of individual workers to a specific task per hour.

Description of the LP Model:

The LP model is built to analyze the number of work hours needed from each grade level worker for different operations on different shifts. The model gives the plant information for deciding how many workers are needed to meet the demand in a given month.

Decision Variables

Work hours required: This is the number of work hours required for each operation at each grade level by shift.

Shifts: This decision variable is a binary variable. It indicates which shift has to run depending on the demand. It is equal to 1 if shift has to be run, 0 otherwise.

Constants and Inputs

Number of days per month: This is an input of the work days in a given month. This number can be changed based upon holidays and the plant schedule for each month. This is used later in the constraints section of the LP model.

Demand: Demand is the number of units that the plant needs to produce in a given month. It can be changed by changing the demand cell in the LP model.

Fraction Fraction is a constant that is percentage of the products that goes through a given operation. Thus, there is a fraction for every operation. If all of the products have to go through an operation the fraction is equal to one. If 50% of the products have to go through an operation, the fraction is equal to 0.5.

Current work force: In some of the LP runs the current work forces is entered as an input to the model. All of the hours of the current workforce must be used before additional workers can be added.

Number of work stations: The number of available machines for each operation was entered as an input to the model. (See Exhibit 4)

Number of units produced per hour: This is the number of parts that can be processed per hour by each machine for a given operation. (See Exhibit 4)

Second and third shift penalty: For 2^{nd} and 3^{rd} shifts, there is a fixed cost of starting up the additional shift. This represents the additional electricity consumption, additional security and so on. This fixed cost is the same for both shifts and it is estimated at $5000 per month.

Constraints

Labor demand per day: We assume that there is level demand for the month and that production is continuous. We calculate the number of required production hours for an operation by using the demand per month, number of work days per month, the fraction of the demand that goes through the operation and the number of products produced per hour for the operation. Therefore, for a given operation:

The required production hours per day= Monthly demand*fraction / (number of work days per month*number products produced per hour)

For each operation, the actual hours have to be at least the required production hours.

Machine capacity per shift: There are a limited number of machines for each operation and these machines can be run only for 8 hours per shift with the exception of laser weld which can be run for 7 hours (this machine is shared part-time with another production line). This capacity is the number of hours that the machine can be worked in a shift. The number of labor work hours spent on each operation must be less than the machine capacity. The machine capacities are calculated by multiplying the number of machines with 8 since every shift lasts for 8 hours, except for laser weld.

Number of workers for each shift: The number of workers for each shift has to be more than 5. Therefore for each shift the total number of labor hours is forced to be greater than 40 hrs, which is the total number of labor hours that can be contributed by 5 workers on each shift.

Work hours by each grade vs. the available number of work hours at each grade level: This constraint is applied by making the works hours at each grade level to be greater than or equal to the available work hours. This constraint causes each current worker to work to his/her maximum capacity, before additional workers are hired.

Non-negativity constraints for decision variables: applied when running the solver by using the check box toggle.

Objective Function

The objective function of the model is to minimize the overall daily cost of production. The cost is calculated by adding up the cost of labor on each operation per shift. See Exhibit 3 for a listing of all the labor costs. There is a $0.25 per hour premium paid on a second shift and a $0.75 per hour shift premium paid on third shift. There is also a premium of $0.50 per hour for the work hours spent in the clean areas.

Our Experiment with an Integer Model

The LP model provides the work hours required by each operation for each shift. These numbers are allowed to be fractional and assume a perfect allocation of a worker. This, however, does not take into account the integrality of a worker since in reality there are a certain number of workers in a shift and not just a number of work hours to be completed. To try to solve this problem we created an Integer model. This model added the number of workers for each grade and shift as decision variables to the LP model. The number of workers was forced to be an integer and a constraint was added which required the number of hours worked for each grade level and for each shift to be equal to eight times the number of workers.

This model was able to run for small demand quantities and returned an exact number of workers. However as demand increased and multiple shifts were required, the model ran for too long to be practical. At a high demand level the solver ran for more than 10 hours without returning a solution. Since there was only marginal benefit in this integer model we decided to continue all of our analysis with the simpler LP that optimized on an hourly basis. We assume that a manager can take the required number of work hours and translate this into the number of workers that he needs to schedule.

Use of the LP Model:

Monthly Use:

Exhibit 7 shows the factory manager interface of our optimization model. Each month the manager simply inputs the demand, number of days in the month, and current workforce and clicks solve. The model then produces the following outputs;

- *Daily Pay code Specific Staffing Hours*: Indicates daily by-shift hours required by each pay code and where (which operation) each pay code hour must be utilized in the factory in order to achieve lowest costs. This information is critical to monitor factory resources, insuring the level loading of the factory.

- *Daily Operation Specific Staffing Hours*: Indicates the by-shift total hours of each shift on each individual operation. Very useful at identifying shift-constrained operations.

- *Daily Overall Pay code Staffing Hours*: Summary of total staffing hours for each pay code used to adjust pay code levels (training).

- *Daily Overall Shift Staffing Hours*: Summary of total staffing hours on each shift used by management/facilities to gauge level of operation for each shift.

- *Daily and Monthly Staffing Costs*: Indicates the lowest cost per day/month for staffing given current factory constraints.

Long Range Planning:

Target Workforce

There are other important uses for the model in addition to daily and monthly scheduling. A senior manager could use it to help determine exactly how many permanent workers to hold on staff at each grade. When you run the model without any current workers, it shows the total number of hours required at each grade for a day. This can then be interpreted to give a rough idea of the total number of people required to do those jobs. For example, if demand is expected to grow then the model can determine how many workers of each type you might want to hire or train in advance of the increase to be well prepared. For a given demand, the ideal number of hours is shown by the model.

There are also monthly variations in customer demand, which means that the ideal number of workers to meet that demand will vary as well. Having a permanent staff of workers that is greater than the ideal number required is an excess cost. However, there is also a tradeoff in the cost to hire a new worker to meet demand (or lost revenue in not being able to produce enough). By running the model with different sets of permanent staff and then comparing the total overall labor costs, the manager can begin to understand this tradeoff (the cost of hiring or lost revenue is not known in this analysis, however).

Exhibit 6 shows this type of analysis with 4 different types of workforce: No permanent workers, the number of workers required to meet demand of 25,000 units, workers required for 35,000 and workers required for 45,000. In each case, demand was simulated in a range from 10,000 to 75,000 units and the daily cost of labor was recorded. In this case we assumed that

Grade 1 and Grade 2 workers could be easily hired, and that there is a permanent staff of Grade 3 to Grade 5 workers. Exhibit 6 shows the required workforce for each case.

The graph shows that there is a fixed cost for the permanent staff at low levels of demand, and then it becomes variable at higher levels of demand. For example, with a large permanent workforce, the plant is using Grade 3 to 5 workers (the permanent staff) for all jobs until demand is above about 25,000 units. This is not as cost efficient as having Grade 1 and 2 workers to do the lower skill jobs. We see that at higher levels of demand that all of the costs converge. This, however, does not take into account the fixed cost of hiring new workers beyond their hourly wage. Above 45,000 units, the large workforce would still need to be augmented with new workers. This graph would be very useful for a manager deciding at what level to keep a permanent number of workers. It shows that excess cost of holding too large a workforce when demand does not fully utilize these workers to their highest skills.

While the true number of workers in the plant would vary, this would help the manager to set a target workforce. The model is an excellent planning tool for hiring and setting training policies.

Plant and Machine Capacity
The model also provides information about machine capacities in the plant. By looking at allocation of people to machine resources we can quickly see the bottlenecks. With increasing demand, the most constraining resource is the one that causes a second shift (and then 3rd) to start up. In our model the constraint is the Vacuum Bake/Tig Weld operation. In addition to other analytical tools available, the model can help the manager to see the constraining resources in the plant.

This information also can show the tradeoff in making new capital investment versus operating more shifts. There is a significant cost to having a second or third shift and depending upon the cost of making a capital investment it may or may not be wise to increase capacity. For example, it may be relatively inexpensive to invest in a new Vacuum Bake machine and the cost of this machine would be quick made up by the fact that the plant would not have to operate a second shift. The model helps the manager to better understand these tradeoffs.

LP Model Limitations

While the model is very effective at producing the above outputs given a monthly demand, there are several limitations to the model and we must remember the model is a forecasting tool. The model does not account for factory variability such as down machines, training, vacations, sick-leave, hiring limitations, or part defects. Statistical data must be analyzed in conjunction with the LP model to reduce the risk of factory variation. Also, you will notice the model works in hours per pay code, not employees per pay code. As discussed earlier, we attempted to add this feature into model, but were unsuccessful given our modeling software and computer power. So when the LP model, as in Exhibit 7, indicates you need to bring in a pay code 4 on 2nd shift to do only 1.86 hours of work, obviously the manager should use the remaining 6.14 hours of the workers time doing other pay code 3, 2, or 1 work. Knowledge of the above limitations simply means to use the model together with past data and common sense to create and effective tool for forecasting staffing levels.

Conclusion

Given the monthly variable demand in precision thermostat units and the complex pay scheme for the various pay codes as they relate to shifts, work environment, and training, the constructed LP model is an effective tool for planning optimal staffing levels. Along with optimizing monthly staffing levels the LP model can be used for long range planning to develop a target permanent workforce as well as plant and machine optimal capacities. Direct results of this analysis are workforce training/hiring requirements and future capital investments. Overall, the LP Model accomplished our objective, to create a forecasting tool which could allocate labor on a manufacturing line.

Exhibit 1: Sample Device Styles

(Images of six Klixon thermostats.)

Exhibit 2

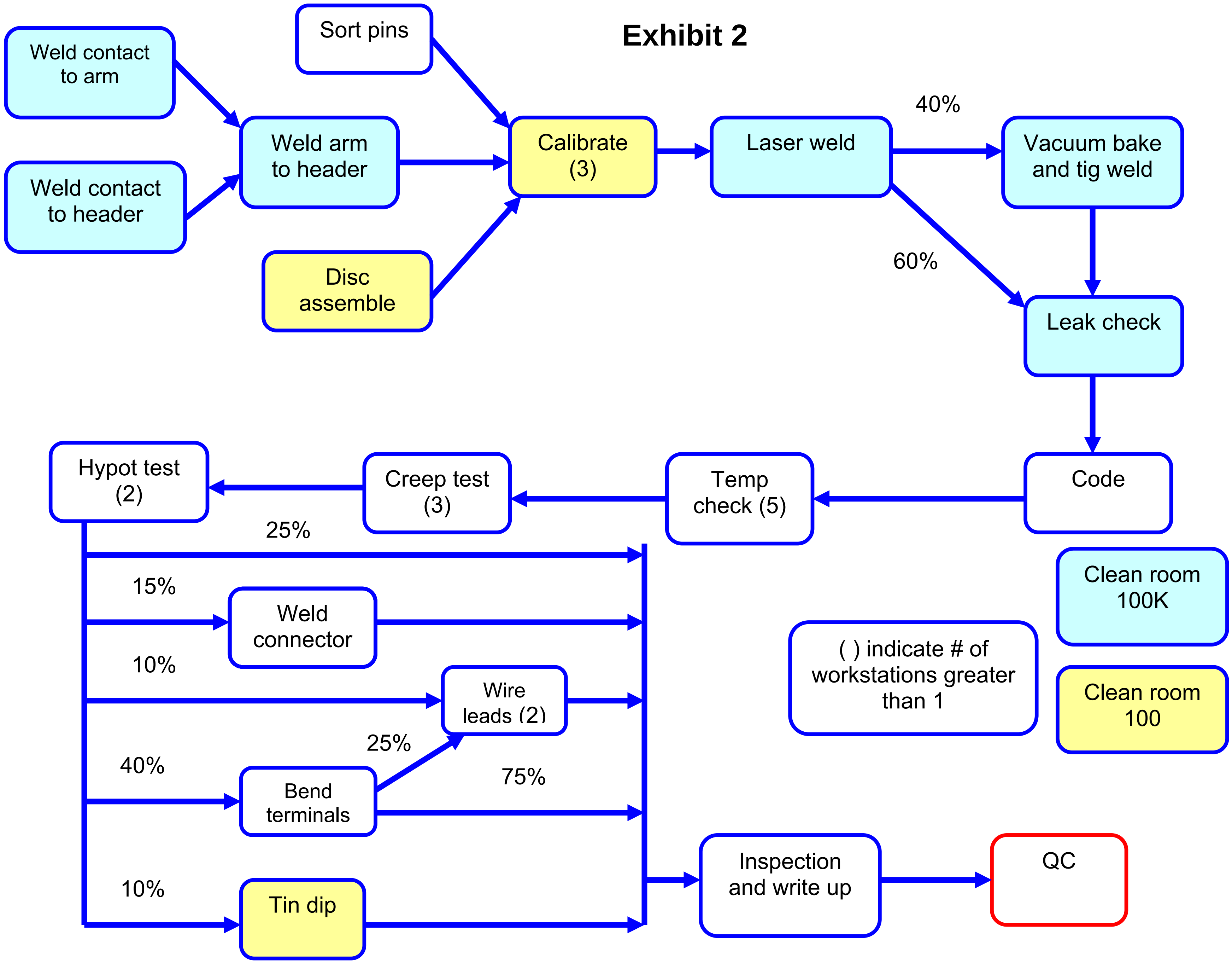
Weld contact to arm
Sort pins
Weld contact to header
Weld arm to header
Calibrate (3)
Disc assemble
Laser weld
40%
60%
Vacuum bake and tig weld
Leak check
Code
Temp check (5)
Creep test (3)
Hypot test (2)
25%
15%
Weld connector
10%
Wire leads (2)
25%
40%
Bend terminals
75%
10%
Tin dip
Inspection and write up
QC
Clean room 100K
Clean room 100
( ) indicate # of workstations greater than 1

Exhibit 3

Pay Rates

Job Grade	Base pay rate/hour	First Shift base rate	First Shift clean room rate	Second Shift base rate	Second shift clean room rate	Third Shift base rate	Third Shift clean room rate
1	8.00	8.00	8.50	8.25	8.75	8.75	9.25
2	9.00	9.00	9.50	9.25	9.75	9.75	10.25
3	14.00	14.00	14.50	14.25	14.75	14.75	15.25
4	18.00	18.00	18.50	18.25	18.75	18.75	19.25
5	20.00	20.00	20.50	20.25	20.75	20.75	21.25

Exhibit 4

Capacity and Job grade by operation

Operation	pcs/hour	# stations	Grade
Sort pins	500	N/A	1
Weld contact to header	180	1	2
Weld contact to arm	180	1	2
Weld arm to header	180	1	2
Disc assemble	720	N/A	1
Calibrate	60	3	5
Laser weld	180	1	3
Vac bake/tig weld	60	1	2
Leak check	300	1	2

Operation	pcs/hour	# stations	Grade
Code	120	1	2
Temp test	60	5	3
Creep test	180	3	1
Hypot test	180	2	1
Bend terminals	180	1	1
Weld wire leads	30	2	2
Tin dip	60	1	3
Weld connector	30	1	2
Inspection/write up	60	N/A	4

Exhibit 5

Indexes
i = index for employee job grade level. In this problem there are 5 job grades. Workers who can do job grade 5 can do all grade levels from 1 to 5.
j = index for shift. There are up to 3 shifts in a day.
k = index for job function (operation).

Constants and Inputs
A_k = Number of stations for each job function.
B_k = Pieces per hour produced per operation
C_{ijk} = Per hour cost of job grade i for shift j, doing operation k.
D = Demand for the final product per month. This will vary from month to month.
F_k = Ratio of final product that must go through the operation. Not all products must go through every operation.
W = Number of work days in the month
E_i = Number of workers for grade level i

Decision Variables
H_{ijk} = number of hours allocated to each job function during each shift and by grade. For example, during shift 1 a welding job can be done by anyone of grade 3, 4 or 5. Since welding cannot be done by someone of grade 1 or 2, then these are not valid decision variables. See Exhibit 2 for a sample array of the decision variables. This is a daily schedule that could be repeated for a period of time such as weekly, monthly or more.

S_j = Binary variable determining if the plant is running during a given shift. 1 if the plant is running, 0 if it is not.

Objective Function
Minimize total Labor cost: $_j\sum\ _k\sum\ _i\sum\ C_{ijk}H_{ijk}$

Constraints
Labor Demand per day
For each k $_i\sum\ _j\sum H_{ijk}$ >= (D * F_k) / (W * B_k)
Machine Capacity per shift
For each k and j $_j\sum H_{ijk}$ <= 8 * A_k
Minimum number of people per shift
For each j (10000)S_i >= $_i\sum\ _k\sum H_{ijk}$ >= 40S_j
Available work hours vs. needed work hours per grade level
For each i $_j\sum\ _k\sum H_{ijk}$ >= E_i *8
Non negativity: H_{ijk} >= 0

Exhibit 6

Number of Grades 3, 4 and 5 Required to Meet Demand				
Demand Level	Demand	G5	G4	G3
Low	25000	3	3	4
Mean	35000	4	4	5
High	45000	5	5	7

Daily Cost to Meet Demand Given Predetermined Staffs at Three Levels					
	No current staff	# Shifts	Small staff	Medium Staff	Large Staff
10000	$ 689.41	1	$ 1,371.43	$ 1,787.43	$ 2,315.43
15000	$ 1,034.11	1	$ 1,377.14	$ 1,793.14	$ 2,321.14
20000	$ 1,378.81	1	$ 1,607.20	$ 1,819.78	$ 2,326.85
25000	$ 1,723.52	1	$ 1,849.00	$ 2,049.00	$ 2,332.57
27500	$ 2,133.14	2	$ 2,207.17	$ 2,407.17	$ 2,657.47
30000	$ 2,305.49	2	$ 2,328.07	$ 2,528.07	$ 2,768.07
32500	$ 2,477.84	2	$ 2,477.84	$ 2,648.97	$ 2,888.97
35000	$ 2,650.20	2	$ 2,650.20	$ 2,769.87	$ 3,009.87
37500	$ 2,822.55	2	$ 2,822.55	$ 2,890.77	$ 3,130.77
40000	$ 2,994.90	2	$ 2,994.90	$ 3,011.87	$ 3,251.67
42500	$ 3,167.25	2	$ 3,167.25	$ 3,167.25	$ 3,372.57
45000	$ 3,339.60	2	$ 3,339.60	$ 3,339.60	$ 3,493.47
50000	$ 3,686.02	2	$ 3,686.02	$ 3,686.02	$ 3,736.97
55000	$ 4,286.28	3	$ 4,286.28	$ 4,286.28	$ 4,286.28
60000	$ 4,632.39	3	$ 4,632.39	$ 4,632.39	$ 4,632.39
65000	$ 4,982.62	3	$ 4,982.62	$ 4,982.62	$ 4,982.62
70000	$ 5,333.07	3	$ 5,333.07	$ 5,333.07	$ 5,333.07
75000	$ 6,138.06	3	$ 6,138.06	$ 6,138.06	$ 6,138.06

Note: Yellow shading indicates need to hire new workers

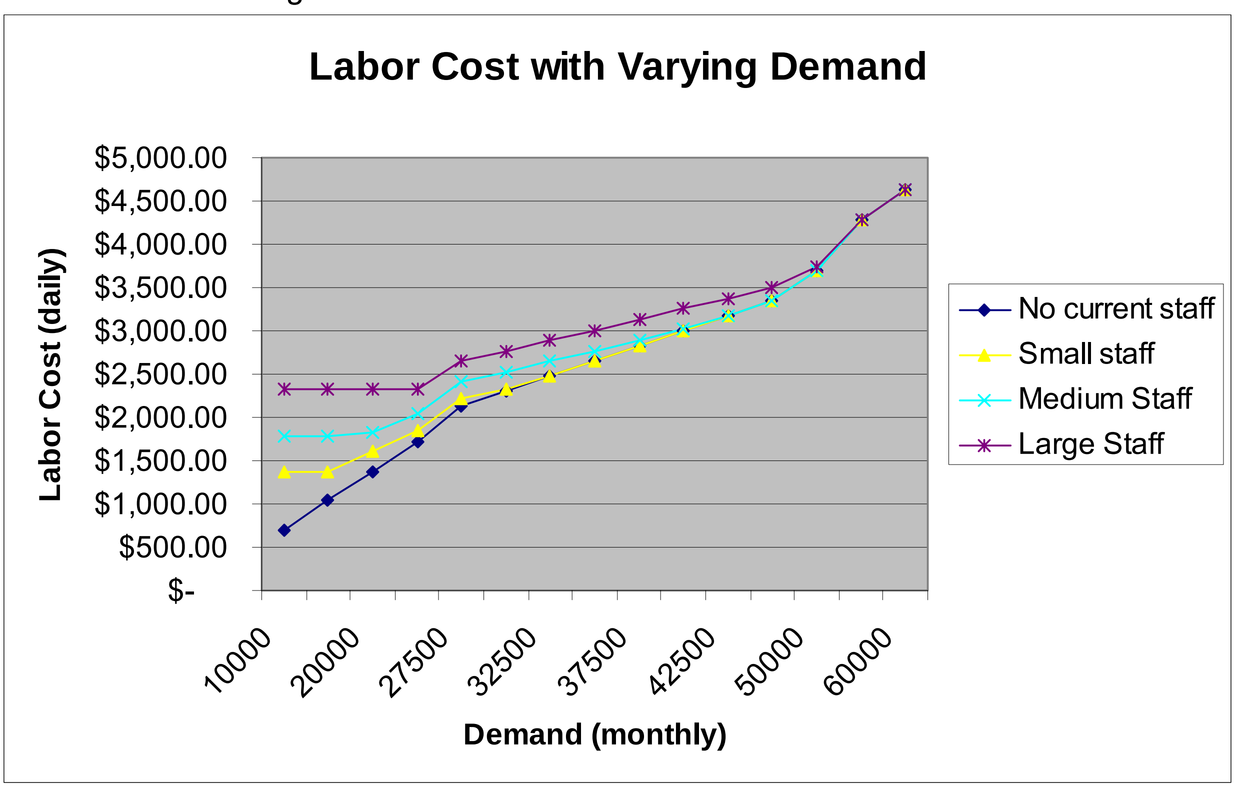

Exhibit 7: Factory Manager Input/Output Interface

Entered Data					
# Of Work Days In Month	22				
Monthly Demand	45000				
Existing Workforce	Paycode 1	Paycode 2	Paycode 3	Paycode 4	Paycode 5
Head Count	0	0	5	5	7

Daily Paycode Specific Staffing Hours

Operation	Avail Hours/Shift	1st Shift Staffing					2nd Shift Staffing					3rd Shift Staffing				
		Paycode 1	Paycode 2	Paycode 3	Paycode 4	Paycode 5	Paycode 1	Paycode 2	Paycode 3	Paycode 4	Paycode 5	Paycode 1	Paycode 2	Paycode 3	Paycode 4	Paycode 5
Sort Pins	40	4.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Weld contact to Header	8		0.00	0.00	0.00	8.00		0.41	0.00	0.00	2.95		0.00	0.00	0.00	0.00
Weld contact to Arm	8		8.00	0.00	0.00	0.00		3.36	0.00	0.00	0.00		0.00	0.00	0.00	0.00
Weld Arm to Header	8		0.00	0.00	0.00	8.00		3.36	0.00	0.00	0.00		0.00	0.00	0.00	0.00
Disc Assemble	8	2.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Calibrate	24					17.45					16.64					0.00
Laser Weld	7			0.00	4.05	2.95			2.50	1.86	0.00			0.00	0.00	0.00
Vacuum bake/tig weld	8		8.00	0.00	0.00	0.00		5.64	0.00	0.00	0.00		0.00	0.00	0.00	0.00
Leak check	8		6.82	0.00	0.00	0.00		0.00	0.00	0.00	0.00		0.00	0.00	0.00	0.00
Code	16		16.00	0.00	0.00	0.00		1.05	0.00	0.00	0.00		0.00	0.00	0.00	0.00
Temp Test	40			34.09	0.00	0.00			0.00	0.00	0.00			0.00	0.00	0.00
Creep Test	24	11.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Hypot Test	16	11.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bend Terminals	8	4.55	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Weld Wire Leads	16		6.82	0.00	0.00	0.00		0.00	0.00	0.00	0.00		0.00	0.00	0.00	0.00
Tin Dip	8			3.41	0.00	0.00			0.00	0.00	0.00			0.00	0.00	0.00
Weld Connector	8		8.00	0.00	0.00	0.00		2.23	0.00	0.00	0.00		0.00	0.00	0.00	0.00
Inspection/ Writeup	40				34.09	0.00				0.00	0.00				0.00	0.00
Totals Per Day		34.20	53.64	37.50	38.14	36.41	0.00	16.05	2.50	1.86	19.59	0.00	0.00	0.00	0.00	0.00

Daily Operation Specific Staffing Hours

Operation	Avail Hours/Shift	Shift 1	Shift 2	Shift 3
Sort Pins	40	4.09	0.00	0.00
Weld contact to Header	8	8.00	3.36	0.00
Weld contact to Arm	8	8.00	3.36	0.00
Weld Arm to Header	8	8.00	3.36	0.00
Disc Assemble	8	2.84	0.00	0.00
Calibrate	24	17.45	16.64	0.00
Laser Weld	7	7.00	4.36	0.00
Vacuum bake/tig weld	8	8.00	5.64	0.00
Leak check	8	6.82	0.00	0.00
Code	16	16.00	1.05	0.00
Temp Test	40	34.09	0.00	0.00
Creep Test	24	11.36	0.00	0.00
Hypot Test	16	11.36	0.00	0.00
Bend Terminals	8	4.55	0.00	0.00
Weld Wire Leads	16	6.82	0.00	0.00
Tin Dip	8	3.41	0.00	0.00
Weld Connector	8	8.00	2.23	0.00
Inspection/ Writeup	40	34.09	0.00	0.00

Staffing Cost	
Daily	$3,589
Monthly	$78,968

Daily Shift Staffing Hours

Shift 1	Shift 2	Shift 3
199.89	40.00	0.00

Daily Overall Paycode Staffing Hours

Paycode 1	Paycode 2	Paycode 3	Paycode 4	Paycode 5
34.20	69.68	40.00	40.00	56.00

Denotes No Work Required Given Current Demand

Denotes Pay code Does Not Having Training To Complete Operation