VII.D Two Dimensional Solids

The transition from a liquid to solid involves the breaking of translational symmetry. Here we shall examine the types of order that are present in a two dimensional solid. The destruction of such order upon melting is discussed next.

(a) A low temperature treatment of the problem starts with the perfect solid at $T = 0$. The equilibrium configuration of atoms forms a lattice, $\mathbf{r}_0(m,n) = m\mathbf{e_1} + n\mathbf{e_2}$, where $\mathbf{e_1}$ and $\mathbf{e_2}$ are basis vectors, and $\{m,n\}$ are integers. At finite temperatures, the atoms fluctuate away from their equilibrium position, moving to

$$\mathbf{r}(m,n) = \mathbf{r}_0(m,n) + \mathbf{u}(m,n). \tag{VII.81}$$

The low temperature distortions do not vary substantially over nearby atoms, enabling us to define a coarse-grained *distortion field* $\mathbf{u}(\mathbf{x})$, where $\mathbf{x} \equiv (x_1, x_2)$ is treated as continuous, with an implicit short distance cutoff of the lattice spacing a. The distortion $\mathbf{u}$, is the analog of the angle θ, is the XY model.

(b) Due to translational symmetry, the energy depends only on the *strain matrix*,

$$u_{ij}(\mathbf{x}) = \frac{1}{2}\left(\partial_i u_j + \partial_j u_i\right), \qquad \Longrightarrow \qquad u_{ij}(\mathbf{q}) = \frac{i}{2}\left(q_i u_j + q_j u_i\right). \tag{VII.82}$$

The elastic energy must respect the symmetries of the underlying lattice. For simplicity, we shall consider the triangular lattice whose elastic energy is fully isotropic, i.e. invariant under all rotations (see Landau and Lifshitz, *Theory of Elasticity*). In terms of the *Lamé coefficients* λ and μ,

$$\begin{aligned}\beta\mathcal{H} =& \frac{1}{2}\int d^2\mathbf{x}\left[2\mu u_{ij}u_{ij} + \lambda u_{ii}u_{jj}\right] \\ =& \frac{1}{2}\int \frac{d^2\mathbf{q}}{(2\pi)^2}\left[\mu q^2 u^2 + (\mu+\lambda)(\mathbf{q}\cdot\mathbf{u})^2\right].\end{aligned} \tag{VII.83}$$

The rotational invariance of energy is ensured by the implicit sum over the indices (i,j) in the above expression. In the Fourier representation, the energy depends on the quantities q^2, $|\mathbf{u}|^2$, and $(\mathbf{q}\cdot\mathbf{u})^2$ which are clearly independent of rotations. For other lattices, there are more elastic coefficients, since the energy should only be invariant under lattice rotations. For example, the symmetry of a square lattice permits a term proportional to $\left(q_x^2 u_x^2 + q_y^2 u_y^2\right)$.

(c) The Goldstone modes associated with the broken translational symmetry are *phonons*, the normal modes of vibrations. Equation (VII.83) supports two types of normal modes:

(i) *Transverse modes* with $\mathbf{q} \perp \mathbf{u}$, have energy

$$\beta\mathcal{H}_T = \frac{\mu}{2}\int \frac{d^2\mathbf{q}}{(2\pi)^2}\, q^2 u_T^2\,, \qquad \Longrightarrow \qquad \left\langle |u_T(\mathbf{q})|^2 \right\rangle = \frac{1}{\mu q^2}. \tag{VII.84}$$

(ii) *Longitudinal modes* with $\mathbf{q} \parallel \mathbf{u}$, with energy

$$\beta\mathcal{H}_L = \frac{2\mu+\lambda}{2}\int \frac{d^2\mathbf{q}}{(2\pi)^2}\, q^2 u_L^2\,, \qquad \Longrightarrow \qquad \left\langle |u_L(\mathbf{q})|^2 \right\rangle = \frac{1}{(2\mu+\lambda)q^2}. \tag{VII.85}$$

Combining the above results, we find that a general correlation function is given by

$$\langle u_i(\mathbf{q}) u_j(\mathbf{q}')\rangle = \frac{(2\pi)^2\delta^2(\mathbf{q}+\mathbf{q}')}{\mu q^2}\left[\delta_{ij} - \frac{\mu+\lambda}{2\mu+\lambda}\,\frac{q_i q_j}{q^2}\right]. \tag{VII.86}$$

The extent of fluctuations in position space is given by

$$\begin{aligned}\left\langle [\mathbf{u}(\mathbf{x})-\mathbf{u}(\mathbf{0})]^2 \right\rangle =& \int \frac{d^2\mathbf{q}}{(2\pi)^2}\,\frac{2-2\cos(\mathbf{q}\cdot\mathbf{x})}{\mu q^2}\,\left[\delta_{ii} - \frac{\mu+\lambda}{2\mu+\lambda}\,\frac{q_i q_i}{q^2}\right] \\ =& \frac{3\mu+\lambda}{\mu(2\mu+\lambda)}\,\frac{\ln(|\mathbf{x}|/a)}{\pi}.\end{aligned} \tag{VII.87}$$

The unbounded growth of fluctuations with $|\mathbf{x}|$ indicates the destruction of long-range translational order in two dimensions. A similar calculation in higher dimensions results in a *finite* maximum extent of fluctuations proportional to temperature. The *Lindeman criterion* identifies the melting point heuristically as the temperature for which these thermal fluctuations reach a fraction (roughly 0.1) of the perfect lattice spacing. According to this criterion, the two dimensional solid melts at any finite temperature. This is of coarse another manifestation of the general absence of true long-range order in two dimension. Is some form of quasi-long-range order, similar to the XY model possible in this case?

(d) The *order parameter* describing broken translational symmetry is $\rho_{\mathbf{G}}(\mathbf{x}) = e^{i\mathbf{G}\cdot\mathbf{r}(\mathbf{x})}$, where $\mathbf{G}$ is any *reciprocal lattice* vector. Since $\mathbf{G}\cdot\mathbf{r}_0$ is an integer multiple of 2π by definition, $\rho_{\mathbf{G}} = 1$ at zero temperature. Due to fluctuations $\langle\rho_{\mathbf{G}}(\mathbf{x})\rangle = \left\langle e^{i\mathbf{G}\cdot\mathbf{u}(\mathbf{x})}\right\rangle$, decreases at finite temperatures, and its correlations decay as

$$\begin{aligned}\langle\rho_{\mathbf{G}}(\mathbf{x})\rho^*_{\mathbf{G}}(\mathbf{0})\rangle =& \left\langle e^{i\mathbf{G}\cdot(\mathbf{u}(\mathbf{x})-\mathbf{u}(\mathbf{0}))}\right\rangle \\ =& \exp\left(-\frac{G_\alpha G_\beta}{2}\,\langle(u_\alpha(\mathbf{x})-u_\alpha(\mathbf{0}))(u_\beta(\mathbf{x})-u_\beta(\mathbf{0}))\rangle\right) \\ =& \exp\left\{-\frac{G_\alpha G_\beta}{2}\int\frac{d^2\mathbf{q}\,d^2\mathbf{q}'}{(2\pi)^2}\left(e^{i\mathbf{q}\cdot\mathbf{x}}-1\right)\left(e^{i\mathbf{q}'\cdot\mathbf{x}}-1\right)\langle u_\alpha(\mathbf{q})u_\beta(\mathbf{q}')\rangle\right\} \\ =& \exp\left\{-\frac{1}{2}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\,\frac{2-2\cos\mathbf{q}\cdot\mathbf{x}}{q^2}\,\left(\frac{G^2}{\mu}-\frac{\mu+\lambda}{\mu(2\mu+\lambda)}\,\frac{(\mathbf{G}\cdot\mathbf{q})^2}{q^2}\right)\right\} \\ \approx& \exp\left\{-\frac{G^2(3\mu+\lambda)}{2\mu(2\mu+\lambda)}\,\frac{\ln(|\mathbf{x}|/a)}{2\pi}\right\} = \left(\frac{a}{|\mathbf{x}|}\right)^{\eta_G}.\end{aligned} \tag{VII.88}$$

The integration over $\mathbf{q}$ is performed after replacing $(\mathbf{G}\cdot\mathbf{q})^2$ with the angular average of $G^2q^2/2$. This is not strictly correct because of the presence of the term $\cos\mathbf{q}\cdot\mathbf{x}$ which breaks the rotational symmetry. Nevertheless, this approximation captures the correct asymptotic growth of the integral. Correlations thus decay algebraically with a power

$$\eta_G = \frac{G^2(3\mu+\lambda)}{4\pi\mu(2\mu+\lambda)}. \tag{VII.89}$$

The translational correlations are measured in diffraction experiments. The scattering amplitude is the Fourier transform of $\rho_{\mathbf{q}}$, and the scattered intensity at a wave-vector $\mathbf{q}$ is proportional to the structure factor

$$\begin{aligned} S(\mathbf{q}) = \left\langle |A(\mathbf{q})|^2 \right\rangle &= \left\langle \left| \sum_{m,n} e^{i\mathbf{q}\cdot\mathbf{r}(m,n)} \right|^2 \right\rangle = N \sum_{m,n} \left\langle e^{i\mathbf{q}\cdot(\mathbf{r}(m,n)-\mathbf{r}(0,0))} \right\rangle \\ &= N \sum_{m,n} e^{i\mathbf{q}\cdot(\mathbf{r}_0(m,n)-\mathbf{r}_0(0,0))} \left\langle e^{i\mathbf{q}\cdot(\mathbf{u}(\mathbf{x})-\mathbf{u}(\mathbf{0}))} \right\rangle, \end{aligned} \tag{VII.90}$$

where N is the total number of particles. At zero temperature, the structure factor is a set of delta-functions (Bragg peaks) at the reciprocal lattice vectors. Even at finite temperature the sum is zero due to varying phases, unless $\mathbf{q}$ is in the vicinity of a lattice vector $\mathbf{G}$. We can then replace the sum with an integral, and

$$S(\mathbf{q}) \approx N \sum_{\mathbf{G}} \int d^2\mathbf{x} e^{i(\mathbf{q}-\mathbf{G})\cdot\mathbf{x}} \left(\frac{a}{|\mathbf{x}|}\right)^{\eta_{\mathbf{G}}} \approx N \sum_{\mathbf{G}} \frac{1}{|\mathbf{q}-\mathbf{G}|^{2-\eta_{\mathbf{G}}}}. \tag{VII.91}$$

The Bragg peaks are now replaced by power law singularities. The strength of the divergence decreases with temperature and increasing $|\mathbf{G}|$. The peaks corresponding to sufficiently large $|\mathbf{G}|$ are no longer visible, and gradually more of them disappear on increasing temperature. In three dimensions, the structure factor is still a set of delta-functions, but with magnitudes diminished by the so called *Debye-Waller* factor of $\exp\left(-\frac{G^2}{12\pi a}\,\frac{5\mu+2\lambda}{\mu(2\mu+\lambda)}\right)$.

(e) The crystal phase is also characterized by a broken rotational symmetry. We can define an *orientational order parameter* $\Psi(\mathbf{x}) = e^{6i\theta(\mathbf{x})}$, where $\theta(\mathbf{x})$ is the angle between local lattice bonds and a reference axis. (The factor of 6 results from the equivalence of the 6 possible directions on the triangular lattice. The appropriate choice for a square lattice is $e^{4i\theta(\mathbf{x})}$.) The order parameter has unit magnitude at $T=0$, and is expected to decrease due to fluctuations at finite temperature. The distortion $\mathbf{u}(\mathbf{x})$ leads to a change in bond angle given by

$$\theta(\mathbf{x}) = -\frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) = -\frac{1}{2}\hat{z}\cdot\nabla\times\mathbf{u}. \tag{VII.92}$$

The decay of orientational fluctuations is now calculated from

$$\begin{aligned}
\langle \Psi(\mathbf{x})\Psi^*(0)\rangle &= \left\langle e^{i6(\theta(\mathbf{x})-\theta(\mathbf{0}))}\right\rangle \\
&= \exp\left[-\frac{6^2}{2}\,\frac{1}{4}\left\langle [\nabla\times\mathbf{u}(\mathbf{x})-\nabla\times\mathbf{u}(\mathbf{0})]^2\right\rangle\right] \\
&= \exp\left\{-\frac{9}{2}\int \frac{d^2\mathbf{q}\,d^2\mathbf{q}'}{(2\pi)^4}\,(e^{i\mathbf{q}\cdot\mathbf{x}}-1)(e^{i\mathbf{q}'\cdot\mathbf{x}}-1)\,\varepsilon_{ijk}\varepsilon_{ij'k'}q_jq'_{j'}\;\langle u_k(\mathbf{q})u_{k'}(\mathbf{q}')\rangle\right\} \\
&= \exp\left\{-\frac{9}{2}\int \frac{d^2\mathbf{q}}{(2\pi)^2}\,(2-2\cos\mathbf{q}\cdot\mathbf{x})\left(q^2\left\langle|u(\mathbf{q})|^2\right\rangle-\left\langle(\mathbf{q}\cdot\mathbf{u})^2\right\rangle\right)\right\} \\
&= \exp\left\{-\frac{9}{2\mu}\int \frac{d^2\mathbf{q}}{(2\pi)^2}\,(2-2\cos\mathbf{q}\cdot\mathbf{x})\right\} \approx \exp\left(-\frac{9}{a^2\mu}\right).
\end{aligned} \tag{VII.93}$$

(Note that $\int d^2\mathbf{q}/(2\pi)^2$ is the density $n = N/L^2 = 1/a^2$.) The final result is *independent* of $\mathbf{x}$ at large distances, and asymptotes to a constant that decays exponentially with temperature (since $\mu \propto 1/T$). The two dimensional solid is thus characterized by quasi-long-range translational order, and true long-range orientational order.

VII.E Two Dimensional Melting

There are qualitative similarities between the melting of a two dimensional solid and the disordering of the XY spin system. Expanding on the work of Kosterlitz and Thouless, Halperin and Nelson, and independently, Young, developed a theory of two dimensional melting. The resulting, so called KTHNY theory, is briefly sketched here.

(a) The topological defects of a solid are *dislocations*. A single dislocation corresponds to an extra half lattice plane, and is characterized by performing a *Burger's circuit:* The circuit is a series of steps from site to site that on a perfect lattice returns to the starting point (e.g. 5 steps to the right, 4 down, followed by 5 left, and 4 up on a square lattice.) If the circuit encloses a region with dislocations, it will fail to close. The difference between the initial and final sites of the circuit defines $\mathbf{b}$, the *Burger's vector* of the dislocation. This closure failure is allowed, since the distortion field $\mathbf{u}(\mathbf{x})$, is defined only up to a lattice vector. In the continuum limit this degeneracy is described by

$$\oint \nabla u_\alpha \, d\mathbf{s} = b^\alpha. \tag{VII.94}$$

Thus each component of the distortion field behaves like the angle $\theta(\mathbf{x})$ is the XY model. Using this analogy, the distortion field due to a collection of dislocations $\{\mathbf{b}_i\}$, at locations $\{\mathbf{x}_i\}$, has a singular part

$$\nabla \tilde{u}^*_\alpha = -\nabla \times \left(\hat{z} \sum_i \frac{b^\alpha_i}{2\pi} \ln |\mathbf{x} - \mathbf{x}_i| \right). \tag{VII.95}$$

While $\tilde{u}^*_\alpha$ correctly describes the singularities due to the presence of vortices, it is not a local equilibrium configuration, i.e. not a solution to $2\mu\tilde{u}_{\alpha\beta} + \lambda\delta_{\alpha\beta}\tilde{u}_{\gamma\gamma} = 0$. Allowing the particles to equilibrate in the presence of the dislocations leads to a solution $\tilde{u}_\alpha$, which differs from $\tilde{u}^*_\alpha$ only by a regular part. The total strain field can now decomposed as $u_{\alpha\beta} = \phi_{\alpha\beta} + \tilde{u}_{\alpha\beta}$, where $\tilde{u}_{\alpha\beta}$ results from dislocations, and $\phi_{\alpha\beta}$ is the contribution of phonons. After substituting this form in the elastic energy $\beta\mathcal{H} = \int d^2\mathbf{x} \left[2\mu u_{\alpha\beta}u_{\alpha\beta} + \lambda u_{\alpha\alpha}u_{\beta\beta}\right]/2$, manipulations similar to the XY model lead to $\beta\mathcal{H} = \beta\mathcal{H}_0 + \beta\mathcal{H}_1$, where

$$\beta\mathcal{H}_0 = \frac{1}{2}\int d^2\mathbf{x} \left[2\mu\phi_{\alpha\beta}\phi_{\alpha\beta} + \lambda\phi_{\alpha\alpha}\phi_{\beta\beta}\right], \tag{VII.96}$$

and

$$\beta\mathcal{H}_1 = -\bar{K}\sum_{i<j} b^\alpha_i b^\beta_j C_{\alpha\beta}(\mathbf{x}_i - \mathbf{x}_j) - \sum_i \ln[y_0(\mathbf{b}_i)]. \tag{VII.97}$$

The dislocations behave as a grand-canonical gas of particles with vector charges $\{\mathbf{b}_i\}$. The fugacity y_0 comes from the core energy of each dislocation. The charges interact via a vectorial generalization of the Coulomb interaction,

$$C_{\alpha\beta}(\mathbf{x}) = \frac{1}{2\pi}\left[\delta_{\alpha\beta}\ln\left(\frac{|\mathbf{x}|}{a}\right) - \frac{x_\alpha x_\beta}{x^2}\right]. \tag{VII.98}$$

The strength of the interaction in eq.(VII.97) is $\bar{K}a^2 = 2\mu(\mu+\lambda)/(2\mu+\lambda)$. Implicit in the above derivation is an overall neutrality condition, $\sum_i \mathbf{b}_i = \mathbf{0}$.

The bare interaction between dislocations is screened by other dislocations in the medium. The effective strength can be calculated perturbatively and diverges for $\pi\bar{K}a^2 < 2$. This signals the unbinding of paired neutral dislocations. Under coarse graining, the parameters of the theory evolve as

$$\begin{cases} \dfrac{d\bar{K}^{-1}}{d\ell} = Ay^2 + By^3, \\ \dfrac{dy_0}{d\ell} = \left(2 - \pi\bar{K}\right) y + Dy^2, \end{cases} \tag{VII.99}$$

where A, B, and D are constants. In contrast to the XY model, additional terms appear at order of y_0^3. This is because in a scalar Coulomb gas the only neutral configurations have an even number of particles. For dislocations on a triangular lattice, it is possible to construct a neutral configuration from three dislocations at 120° angles.

The low temperature phase maps onto a line of fixed points characterized by renormalized Lamé coefficients, μ_R and λ_R, and $y_0^* = 0$. At high temperatures y_0 and $\bar{K}$ both diverge, indicating the vanishing of the shear modulus μ_R, and a finite correlation length ξ. On approaching the transition from the low temperature side, the effective shear modulus undergoes a discontinuous jump, with a singular behavior, $\mu_R \simeq \mu_c + c(T_c - T)^{\bar{\nu}}$. The correlation length diverges from the high temperature side as $\xi \simeq a \exp\left(c'/(T - T_c)^{\bar{\nu}}\right)$. Due to the cubic terms appearing in eqs.(VII.99), the value of $\bar{\nu} = 0.36963\cdots$ is different from the 1/2 that appears in the XY model, indicating a difference between transitions in vector and scalar Coulomb universality classes.

(b) Does the vanishing of the shear modulus imply that the high temperature phase with unbound dislocations is a liquid? As discussed earlier, the crystalline phase has both translational and orientational order. A distortion $\mathbf{u}(\mathbf{x})$ results in a rotation in bond angles according to eq.(VII.92). The net rotation due to a collection of dislocations is

$$\tilde{\theta}(\mathbf{x}) = -\frac{1}{2}\hat{z}\cdot\nabla\times\tilde{u} = \frac{1}{2\pi}\sum_i \frac{\mathbf{b}_i\cdot(\mathbf{x}-\mathbf{x}_i)}{|\mathbf{x}-\mathbf{x}_i|^2}. \tag{VII.100}$$

In terms of a continuum dislocation density $\mathbf{b}(\mathbf{x}) = \sum_i \mathbf{b}_i\delta^2(\mathbf{x}-\mathbf{x}_i)$,

$$\tilde{\theta}(\mathbf{x}) = \frac{1}{2\pi}\int d^2\mathbf{x}'\,\frac{\mathbf{b}(\mathbf{x}')\cdot(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}. \tag{VII.101}$$

Alternatively, in Fourier space,

$$\tilde{\theta}(\mathbf{q}) = \int d^2\mathbf{x}d^2\mathbf{x}'e^{i\mathbf{q}\cdot\mathbf{x}}\frac{b(\mathbf{x}')\cdot(\mathbf{x}-\mathbf{x}')}{2\pi|\mathbf{x}-\mathbf{x}'|^2} = i\,\frac{\mathbf{b}(\mathbf{q})\cdot\mathbf{q}}{q^2}. \tag{VII.102}$$

The angular fluctuations are thus related to correlations in dislocation density via

$$\left\langle|\tilde{\theta}(\mathbf{q})|^2\right\rangle = \frac{q_\alpha q_\beta}{q^4}\left\langle b^\alpha(\mathbf{q})b^\beta(\mathbf{q})\right\rangle, \tag{VII.103}$$

where

$$\left\langle b^\alpha(\mathbf{q})b^\beta(\mathbf{q})\right\rangle = \int d^2\mathbf{x}\,e^{i\mathbf{q}\cdot\mathbf{x}}\left\langle b^\alpha(\mathbf{x})b^\beta(\mathbf{0})\right\rangle. \tag{VII.104}$$

After the dislocations are unbound for $T > T_c$, they interact via a screened Coulomb interaction, and

$$\lim_{|\mathbf{x}|\to\infty} \left\langle b^\alpha(\mathbf{x}) b^\beta(\mathbf{0}) \right\rangle \propto \delta_{\alpha\beta} e^{-|\mathbf{x}|/\xi}, \quad \Longrightarrow \quad \lim_{\mathbf{q}\to\mathbf{0}} \left\langle b^\alpha(\mathbf{q}) b^\beta(\mathbf{q}) \right\rangle \propto \delta_{\alpha\beta} \xi^2. \tag{VII.105}$$

Substituting into eq.(VII.103) leads to

$$\lim_{\mathbf{q}\to\mathbf{0}} \left\langle |\tilde{\theta}(\mathbf{q})|^2 \right\rangle \propto \frac{\xi^2}{q^2}. \tag{VII.106}$$

(In the low temperature phase, the neutrality of the charges at a large scale $|\mathbf{q}|^{-1}$ requires the vanishing of $\mathbf{b}(\mathbf{q})$ as $\mathbf{q} \to \mathbf{0}$, and $\lim_{\mathbf{q}\to\mathbf{0}} \left\langle b^\alpha(\mathbf{q}) b^\beta(\mathbf{q}) \right\rangle \propto q_\alpha q_\beta$, leading to a finite $\langle |\tilde{\theta}(\mathbf{q})|^2 \rangle$.)

Eq.(VII.106) implies that after the unbinding of dislocations the orientational fluctuations are still correlated. In fact such correlations would result from a Hamiltonian

$$\beta\mathcal{H} = \frac{K_A}{2} \int d^2\mathbf{x} \, (\nabla\theta)^2, \qquad \text{with} \qquad K_A \propto \xi^2. \tag{VII.107}$$

The angular stiffness K_A is known as the Frank constant. The bond angle order correlations now decay as

$$\langle \Psi(\mathbf{x}) \Psi^*(\mathbf{0}) \rangle = e^{-\frac{36}{2} \left\langle \left[\tilde{\theta}(\mathbf{x}) - \tilde{\theta}(\mathbf{0})\right]^2 \right\rangle} = \left(\frac{a}{|\mathbf{x}|} \right)^{-\eta_\Psi}, \tag{VII.108}$$

with $\eta_\Psi = 18/(\pi K_A)$. The quasi-long-range decay of orientational fluctuations leads to the appearance of a six fold intensity modulation in the diffraction pattern. The dislocations are thus not effective in completely destroying order, and their unbinding leads to the appearance of an orientationally ordered phase known as a *hexatic*. The stiffness of the hexatic phase diverges at the transition to a solid according to eq.(VII.107).

(c) Orientational order disappears at a higher temperature due to the unbinding of a new set of topological defects known as *disclinations*. These are very similar to the vortices in the XY model, except that since the bond angle is defined only up to $2\pi/6$, they satisfy

$$\oint \nabla\theta \cdot d\vec{s} = \frac{2\pi}{6}. \tag{VII.109}$$

The energy cost of a disclination grows with system size as $\mathcal{E}_1 = \pi K_A \ln(L/a)/36$. Considering the entropy of $2\ln(L/a)$, we find a disclination unbinding transition for $K_A < 72/\pi$. This transition is in the universality class of the scalar Coulomb gas. The resulting high

temperature phase has neither orientational or translational order and is a conventional liquid.

This scenario predicts that the melting of a two dimensional solid proceeds through an intermediate hexatic phase. In fact, computer simulations suggest that simple two dimensional solids, e.g. particles interacting via Lennard-Jones potentials, undergo a direct first order melting transition as in three dimensions. More complicated molecular systems in three dimensions, e.g. long polymers, are known to have intermediate liquid crystal phases. Liquid crystals have order intermediate between solid and fluid and are three dimensional analogs of the hexatic phase. Thin films made up of a few mono-layers of liquid crystals are the best candidates for examining the KTHNY melting scenario.

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