PROFESSOR: As you know, Professor Guth is away. I'm substituting for today, he didn't leave me with a particularly coherent game plan, so I'm going to begin with where he thinks we should start. Please jump in if I am just repeating something that he has already described to you guys, or if there's anything you like me go over a little bit more detail, I will do my best here. So, I'm working off of a fairly rough plan. But let me just quickly describe what-- based on what Alan has explained to me --what we're planning to talk about today, and if there's any adjustments you think I should be making that would be great.

So, the game plan for today. What I want to do very quickly is hit on a couple of the key points which I believe you talked about last week, which is a quick review of the essential features of symmetries of the gauge fields the make up the standard model. Now, I believe you guys did in fact talk about this last week, at least briefly. And you talked about how you can take these things and embed them in a larger gauge group, the group SU(5).

I'm not going to talk about that too much, but I want to just quickly hit on a few elements related to this before we get into that. From this we'll then talk about the Higgs mechanism-- really I'm going to talk about the Higgs field, I'm not going to talk about the Higgs mechanism quite so much as motivate why it is necessary-- and then talk about how the Higgs field behaves and why it's important for the next problem, which is what is called the cosmological monopole problem. To be more specific magnetic monopole problem.

I confess I feel a little bit awkward talking about this problem on behalf of Alan. This would be kind of like if you were planning on studying Hamlet and there was this guy W. Shakespeare who was listed as the instructor and you walk in and discover there's this guy Warren Shackspeare, who's actually going to be teaching or something like that. I kind of feel like Warren here. This stuff really is Allen's thing, so it's sort of, I'm probably going to leave this at the denouement of all this when you actually get into inflation to him. I may have a little bit of time at the end to just motivate it a little bit, but the grand summary will come from him.

OK, so, as discussed by Alan the standard model describes all the fundamental interactions between particles via gauge theories. OK, and these gauge theories all have a combined symmetry group that is traditionally written in a somewhat awkward form, SU(3) cross SU(2) cross U(1). U(1) could be an SU(1) for reasons which I'll elucidate a little bit more clearly in just a moment. There's really no point in putting the S on that one.

So each of these things essentially labels the particular symmetry group. So, the "S" an element of SU(n) is a matrix that is n x n, that is unitary-- that's the U. Unitary just means that the inverse and the transpose of the matrix at the same, actually the Hermitian conjugate because they can be complex, in fact, they generally are. And it has determinant of 1. That's what the special refers to, special, the S in SU(n) stands for special unitary n. So, the S means that the determinant is one-- that's what's special about it-- unitary is this idea that the inverse Hermitian conjugate are the same, and then n refers to all these things.

So, that tells us that the gauge degrees of freedom are related by a symmetry that looks like a 3 by 3 matrix with these properties, as listed there for the SU(3) piece of the symmetry. SU(2) means it's a 2 by 2 matrix. And U(1) means it's a one by one matrix, what's a 1 by 1 matrix? It's a number, its a complex number. And that's why we don't really need to put an "s" in front of it. If it's a complex number its determinant is 1 if it's just a complex number whose modules is one. That's why we don't bother with the S on the U(1). So, I think you've already hit on some of this but this is sort of useful to review because it's going to set up why we need to introduce a Higgs mechanism in a little bit.

Let me just quickly hit on what the details structure of this looks like for you want to think is the easiest one understand, So, as I just said, a one by one matrix is just a complex number. So that means that any element of this group is a complex number, which we can write in the form z equals ei theta, where theta is a real number.

Now, the thing which is I want to hit on in this, the reason I want to describe this a little bit is, this may not smell like the gauge symmetry that you're used to if you study classical E&M. Some of you here are in 807 with me right now, and we've gone over this quite a bit recently. How is this akin to the gauge group that we are normally used to when we talk about the gauge freedom of electricity and magnetism?

Well, it turns out there's actually a very simple relationship between one and the other, rather between this view of it and the way we learn about it when we study classical E&M. It's simply that we use a somewhat different language, because when we talk about it in this group theoretic picture we're doing it in the way that is sort of tuned to a quantum field theory. So, the way we have learned about electromagnetic gauge symmetry in terms of the fields sort of goes as follows.

We actually work with the potentials, and so what we do is we note that the potentials Amu, which you can write as a four vector, whose time-like component is the negative of the scalar potential, and whose spatial components are just the three components of the vector potential. So, this potential and this potential-- -- okay this is possibly module of factor of c somewhere in here but I'm going to imagine the speed of light has been set equal to 1. Both of those potentials generate the same E&B fields.

OK, again you still should be looking at this and thinking to yourself what the hell does this have to do with the U(1) as we presented it here. I've given you a bunch of operations that involve some kind of a scale or function of time and space. And I've added particular components of this four vector in this way, what does that to do with this multiplication by a complex number?

Well, where it comes from is that when we study E&M, not as a classical field theory but as a quantum field theory, we have a field that describes the electron. So, where it comes from is that when you examine the Dirac field, which is the quantum field theory that governs the electron, when you change gauge the electron field acquires a local phase change.

So in particular, what we find is that if we have a field 5x, which those of you who have taken a little bit of quantum field theory should know this is actually a spinner field, but for now, just think of it as some kind of a field that under the field equations of quantum electrodynamics-- the Dirac equation or high order ones that have been developed by Feynman, Schwinger, and others-- under a change of gauge this goes over to si prime of x, which equals e to the-- terrible notation I realized-- 1e is obviously the root of natural logs, e sub 0 is the fundamental electric charge. OK, can everyone read that? I didn't block it too badly here I'm not used to this classroom.

So, here's the thing to note, is that this field lambda, which we learned about in classical E&M directly connects to the phase function of the Dirac field in quantum electrodynamics. So, our gauge symmetry is simply expressed in the quantum version of electrodynamics by a function of the form e to the i real number, where that real number is the fundamental electric charge times the classical gauge generator.

So, this is what is meant when people say that electrodynamics is a U(1) gauge theory. Now, I'm not going to go into this level of detail for the other two gauge symmetry that are built into the standard model. But, what I want you to understand is that the root idea is very, very similar.

It's just now, instead of my gauge functions looking like e to the i, some kind of a local gauge phase of x multiplying my functions, my quantities which generate the gauge transformation are going to become complex value matrices. So that makes them a lot more complicated, and it's responsible for the fact that the weak and the strong interactions are non-abelian paid which order you perform the gauge transformation in matters. Question.

AUDIENCE: What's the physical significance of them being non-abelian?

PROFESSOR: Yes. So, what is a physical significance of them being non-abelian? I'm trying think of a really simple way to put this, it's-- Alan would have an answer to this right off the top of his head, so I apologize for this-- this isn't the kind of thing that I work on every day so I don't have an answer right at the very top of my head, unfortunately. Let me get back to you on that one, OK, that's something I can't give you a quick answer to. It's an excellent question and it's an important question. Any other questions?

OK, so, here's a basic picture that we have. So, we find is that the strong interactions have a similar structure where my need to e to the i factor goes over to a 3 by 3 matrix, and the weak interactions in a similar structure with my e to the i factor going over to a 2 by 2 complex matrix.

OK, what does this have to do with cosmology? In fact, as an enormous amount to do with cosmology, as we'll see over the course of the rest of course. Part of the thing which is interesting about all this is that we have strong experimental reasons, and theoretical reasons to believe, that the different symmetries that these interactions participate in, the different symmetries that we see them having, that isn't the way things have always been.

So, in particular when the universe was a lot hotter and denser these different symmetries actually all began to look the same. In particular the one which is particularly important, and you guys have surely heard of this, is that the SU(2)-- if we just focus on electric and the weak piece of this-- SU(2) cross U(1). So, this is associated with the gauge boson that carry the weak force, OK, the z boson, the w plus, and the w minus.

And your U(1) ends up being associated with the photon. In many ways, when you actually look at the equations that govern these things, they seem very, very similar to one another except that the-- here's partly an answer to your question I just realized-- the gauge generators of these things have a mass associated with them.

That mass ends up being connected to the non-abelian nature of these things. That's not the whole answer, but it has a connection to that. That's one thing which I do remember, like I said I feel this is really Alan's perfect framework here and I'm just a posture in bad shoes.

So if we look at this thing, what we see is that these symmetry groups, what's particularly interesting is that U(1) can be regarded as a piece of SU(2). And we would expect that in a perfect world they would actually be SU(2) governing both the electric and the weak interactions. Whereby perfect I mean everything is a nice balmy 10 to the 16th GeV throughout all of space time, and all the different vector bosons happily exchange with one another, not caring with who is who.

It's actually not very perfect if you want to teach a physics class and have a nice conversation, but if you are interested in perfect symmetry among gauge interactions it's very, very nice. So, the fact that these are separate is now-- I was about to use the word believed but it's stronger and that, we now know this for sure thanks to all the exciting work that happened at the LHC over the past year or two-- the fact that these symmetries are separate is due to what is called spontaneous symmetry breaking.

So, let's talk very briefly about what goes into this spontaneous symmetry breaking. So SU(2) turns out to actually be isomorphic to the group of rotations on a sphere. So, when you think about something that has perfect SU(2) symmetry it's as though you have perfect symmetry when you move around through a whole host of different angles.

OK, so you move through all of your different angles and everyone looks exactly identical to all the others. If you break that symmetry it may mean you're picking out one angle as being special, and then you only retain a symmetry with respect to the other angle. And essentially, that is what happens when SU(2) breaks off in a U(1) piece of it. Something has occurred that picked out one of these directions.

And by the way, you have to think very abstractly here. This is not necessarily a direction in physical space we're talking about here but it's a direction in the space of gauge fields. So, if we imagine that all of these, my gauge fields in some sense the different components of them defined in some abstract space direction, initially these things are completely symmetric with respect to rotations in some kind of an abstract notion of a sphere.

And then something happens to freeze one of the directions and only symmetries with respect to one of the angles remains the same. Let's just write that out, when SU(2)'s symmetry is broken so one of the directions in the space of gauge fields is picked out as special. That direction then ends up being associated with your U(1) symmetry.

So, what is the mechanism that actually breaks the symmetry and causes this to happen. Well, this is what the Higgs field is all about. The idea is there is some field that fills all of space time. It has the property that at very high energies it is extremely symmetric, with to respect all these gauge fields, all directions and sort of gauge field space look exactly the same. And then as things cool, as the energy density goes down by the temperature of the expanding universe, cooling everything off, the Higgs field moves to a particular place that picks out some direction in the space of gauge fields as being special. So let's make this a little bit more concrete.

OK. You guys have probably heard quite a lot about the Higgs field over the past couple years, months-- what actually is it? Well, the field itself is described by a complex doublet. So, if you actually see someone write down a Higgs field what they will actually write down is h, being a two components spinner, whose components are h1 of x, h2 of x-- where x really stands for space time coordinates, so that's time and all of your spatial coordinates-- and both h1 and h2 are complex fields.

The thing which is particularly key to understanding the importance of this thing is that h transforms, under gauge transformations, with elements of SU(2). So, if you want to change gauge the way you're going to do it is you're going to have some new Higgs field.

So remember, if U(2) is an element of SU(2) we call it the two by two matrix. This is what they look like in a new gauge OK-- pardon me a second I don't see a clock in this room, I just want to make sure I know the time, thank you. OK, so, what are we going to do with this? Well, there's a couple features which it must have, so the Higgs field fills all of space time and it has an energy density associated with it, which we will call just the potential energy. It's really an energy density, but, whatever. The energy density that is associated with this thing must be gauge invariant.

OK, even when you're working with strong fields and weak fields, the lesson of gauge invariance from E&M still holds. OK, one of the key points was that the gauge fields affect potentials, they allow us to manipulate our equations to put things into a form where the calculation may be easier. But at the end of the day, there are certain things it actually exert forces that cause things to happen, those must be invariant to the gauge transformation.

Energy density is of those things. If you were to get into your spaceship and go back to the early universe and actually take a little scoop of early universe out and measure the energy density, A, that would be cool, but B it would be something that couldn't actually depend on what gauge you were using to make your measurements. That is something that is a complete artifice of how you want to set up the convenience of your calculation.

So, in order for the energy density to be gauge invariant we have to find a gauge invariant quantity that is constructed from this, which is the only thing the energy density can depend on. This means, let's call our energy density V, it's the potential energy density. So, it can only depend on the following combination of the fundamental fields Pretty much just what you'd expect.

This is sort of the equivalent to saying that if you're working in spherical symmetry the electrostatic potential can only depend on the distance from a point charge. This is a very similar kind of construct here, where I'm taking the only quantity that follows in a fully symmetric way, of calling the fact that this is a special unitary matrix that I can construct from these things. So then, where all the magic comes in is in how the Higgs field potential energy density varies as a function of this h, this magnitude of h.

So, as I plot v as a function of h, in order to get your spontaneous symmetry breaking to happen what you want is for the minimum of V, the minimum potential energy, to occur somewhere out at a non-zero value of the Higgs field H.

Now, why is that so special? The thing that is so special about that is that when I constructed this magnitude of h, I actually lost a lot of information about the Higgs field. OK, let's just say for the sake of argument that this minimum occurs at a place where the Higgs field in some system of units has a value of 1.

So, all I need to do is as my universe cools what I'm going to want is energetically, my potential is going to want to go down to its minimum. So, that just means that as the universe is cooling, maybe at very, very early times when everything is extremely hot and dense, I'm up here where the potential energy is very high. As the universe expands, as everything cools, it moves over to here, it just moves to someplace where the Higgs field takes on a value of 1. And that's exactly correct, that is what ends up happening.

But remember, the minimum occurs at some value in which the magnitude of this field does not equal zero, but given that value-- where again let's just say for this for sake of specificity that we set it equal to the magnitude of this thing equal to 1 in some units-- there's actually an infinite number of configurations that correspond to that because this is a complex number, this is a complex number.

I could put it all into little h1, and I could set into the value where that thing is completely real, or I could put it all into little h2 being completely imaginary or all on to h1 being all imaginary, halfway into h1, halfway into h2. There are literally an infinite number of combinations that I can choose which are consistent with this value of the magnitude of H. So, yeah--

AUDIENCE: So, I don't know if I'm putting too much physical significance on the gauge, but with the other cases of spontaneous symmetry, briefly, that we discussed you can always measure. OK, I've broken my symmetry, and now it's lined up this way, or there's something measurable. Now, the field has to be physical because the fact that you have gauge symmetry gives you some concerned quantity, right? But, how can I measure what direction in gauge space that I picked out?

PROFESSOR: So, that is, let me talk about this just a little bit more. I think answering your question completely is not really possible, but there is a residue of that is in fact very interesting, and let me just lay out a couple more facts about what actually happens with this gauge symmetry, and it's not going to answer your question but it's going to give you something to think about.

OK, so that's an excellent and very deep question, and there are really interesting consequences. And this is a case where my failure to answer the previous one is because there's details I can't remember, in this case, I think it's because there's details we actually don't understand fully. Research into the mechanism of electroweak symmetry breaking, which is what this is all about, is one of the hot topics in particle physics right now.

AUDIENCE: I was just wondering if gravity has any gauge symmetry associated with it.

PROFESSOR: It does, but it fits in a very, very different way, and with the exception of the fairly speculative framework of string theory-- which I think is very, very promising, but it's just sufficiently removed from experimental verification that I'm going to have to label it speculative-- it doesn't quite tie in in the same way. And that's the best I can say right now. The gauge symmetries of general relativity are, at the classical level, they correspond to coordinate transformations, at a quantum level, there's not such a simple way to put it. All right, where was I, OK, sorry I didn't get to your question.

So, the point we made here is that we have spontaneously, when we actually choose which one of these infinite number of values we're going to have, we just randomly break the symmetry. OK, and you guys apparently have already talked a little bit about spontaneous symmetry breaking. The analogy that people often make is to the freezing of water, OK, prior to the water entering its solid phase its completely rotationally symmetric, then at a certain point crystalline planes start to form, the water forms, all the molecules get set into a particular orientation, you lose that rotational symmetry.

In this case, we started out with a theory, with a set of interactions that were completely symmetric in sort of gauge field space. And now by settling down and picking a particular special value of h1 and h2 we have at least nailed down one direction. It's like we've defined a crystalline plane, and so now things, suddenly, aren't as symmetric. And we start to pick out preferred directions in our gauge fields. What we can do with this is really a topic for a whole other course, and that course is called quantum field theory, but I will sketch a couple of the consequences and this gets directly to the answer your questions.

So, one of the consequences of this is that once we have picked out a particular direction, electrons and neutrinos are different. When the Higgs field is equal to zero there is no difference between an electron and a neutrino. They obey exactly the same equation, there's literally no difference between them. Once we have actually settled on an h1 and an h2 some combination of the fundamental underlying fields comes together, acquires a mass, acquires an electric charge, and we say A-HA thou beist an electron. It wasn't like that in the original unbroken symmetry.

AUDIENCE: Also, [INAUDIBLE]?

PROFESSOR: Presumably, but I'm going to stick with just these for now, but I've I'm pretty sure that's the case, yeah. That gets into even more complications of course because the additional generations are actually consequence presumably of some broken higher level symmetry, which is even poorly, more poorly understood. But you raise a good point. So, that's one partial answer your question. How one can actually walk that backwards to understand this thing about the initial state? That's hard to say.

I actually think this particular one is one of the profound and interesting aspects of this, in part because we now know the neutrino has a mass. We have no idea what that is, and in fact we only really have bounds on the mass, such that we know it is non-zero, and we have upper limits that are set by very indirect measurements. But the actual values of the mass are very, very poorly constrained. Within the standard model you just take the electroweak interaction, introduce a Higgs coupling and allow the symmetry to be spontaneously broken, the neutrino mass is zero. Full stop zero.

So something's not right, we're actually missing something here. People have kind of jury rigged the standard model to put in the masses by hand, and it works OK, but it's not completely satisfying. And a lot of experiments going on right now to explore the neutrino sector are hopefully going to open us up to a deeper understanding of this and may say a lot about all this physics, which is at present, pretty poorly understood.

The consequence, which has received the most popular press, and what you guys have certainly seen about in newspapers, given the results that came out from the LHC over the past year is that quarks and leptons have mass, or put more specifically, rest mass. To understand what this actually means I think you really need to ask yourself what is mass meant to be.

Well, the idea is you calculate the spectrum of oscillations associated with the fields of your theory, and then if your theory predicts a discrete spectrum of oscillations, it doesn't even have to be discrete but predict some spectrum of oscillations, then for every oscillation frequency omega there's an associated mass that is just H bar omega over c squared. If your omega has some lower bound that is greater than zero, then your theory has particles with nonzero rest mass.

Without going into the details-- and this again is something which those of you who are going to go on to study this in more detail in a higher level course, which is fairly standard stuff is done in probably the first or maybe late in the first or early in the second semester of a typical quantum field theory course-- what you'll find is that when the Higgs field is zero then quarks and leptons have, the field that describes quarks and leptons-- and yes including mu and tau, so including all the leptons, this one I'm very confident on-- the spectrum goes all the way to zero if the Higgs field is zero.

But when the Higgs field becomes non-zero, roughly speaking, it shifts the spectrum over for these particles. There's an interaction between the things like the electron field in the Higgs field or the up quark field and the Higgs field, which shifts the spectrum over just enough so that the frequency is never allowed to go below some minimum.

AUDIENCE: Going back a bit, I'm confused about how picking a specific value to the Higgs field is breaking SU(2) symmetry and not U(1), because it seems like we're fixed on a circle, right?

PROFESSOR: That's right what U(1) is a symmetry on a circle, SU(2) is kind of like symmetry on a sphere, essentially.

AUDIENCE: Right, so how are we not picking a specific value [INAUDIBLE] circle [INAUDIBLE]?

PROFESSOR: Well, what we're doing is, think of it this way, imagine SU(2) is a symmetry on a sphere, and then when we break the SU(2) symmetry it's like we're picking some circle on that sphere. So, we've broken one circle, we've picked one circle, but now we're allowed to go anywhere on that remaining circle, which is a U(1) symmetry. Does that help? Yeah, OK good.

And it comes down to the fact if you sort of count up your degrees of freedom, it has to do with the fact you you've got four, you have two complex numbers, so there's four real parameters associated with this thing, and they are isomorphic to sort of rotations in a three space and you're adding one constraint.

OK, so let me just finish making this point here again. So, when h does not equal zero, spectrum get shifted for the quarks and leptons, so everything picks up a little bit of a mass. And the final one, final consequence which we're going to talk about today, is that the universe is filled with magnetic monopoles.

We all remember studying Maxwell's equations learning that del dot b is equal to 4 pi times the density of magnetic charge-- this all makes perfect sense, right? Well, this is actually something that when it first sort of came out and people begin to appreciate this thing with sort of a "Um, well everything else works so well, maybe we're just not looking hard enough. " So, it was a bit of a surprise.

So, where do these magnetic monopoles come from? And essentially, the magnetic monopoles are going to turn out to be a consequence of the fact that when spontaneous symmetry breaking happens it doesn't happen everywhere simultaneously. So, think again about-- yeah?

AUDIENCE: Doesn't that bring up possibility that the symmetry could break in different ways in different places?

PROFESSOR: That is in fact exactly what this is going to be. Magnetic monopoles are in fact exactly a consequence of this, yes. Give me a few moments to step ahead to fill in a couple of the gaps, but you're basically already there.

So, think about crystalline crystal formation again. Imagine you have, we could do ice if you like or choose something that's got a little bit more of an interesting crystalline structure. Imagine you have a big bucket full of molten quarts, OK. So, if you have a big thing of quartz that you want to sort of freeze into a single gigantic crystal, what you typically do if you'd like to do this is you actually seed it with a little bit of a starter crystal.

So, you put a little bit of crystal into this thing, and what that does is it sort of defines a preferred orientation of the crystal axes, so that as things start to cool in the vicinity of that they have a preferred orientation to grab on to. And that seed then gradually gets bigger and bigger and bigger, and all the little crystals as they form near it tend to latch onto the preexisting crystalline structure, and that allows you to grow actually extremely large crystals.

I don't know if anyone here is doing a year off with the LIGO project but these guys have to make these sort of 100 kilogram mirrors of very pure either Sapphire or silicon dioxide, and when you make 100 kilograms of crystal you need to build it really, really carefully. It's extremely important for the optical purposes that all the axes associated with the crystal will be pointing in the right direction. Otherwise you spend $100,000 on this thing and it ends up being the world's prettiest paperweight.

So, similar things happen when the Higgs field cools. Let's imagine that we've got our universe, time going forward like this, and at some point over here the universe cools enough that's the Higgs field condenses into some particular direction. And symmetry is spontaneously broken right at this one point over here. So, I'm going to draw my diagram over there and put some words over here. I shouldn't say Higgs field cools enough, the universe cools enough so that the Higgs field breaks the symmetry. So, just to be concrete, let's imagine that at 0.1 over here it takes on a field of the value one for h1 and I for h2. So just for concreteness imagine it looks something like this at this point.

And so what happens is as the university continues to expand other areas are going to cool off. The bits that are closest to it are going to see that there is already a preferred orientation defined by the Higgs field. And so it's energetically favorable for those regions of the universe to fall into the same alignment and so there'll be a region in space times that grows here as the universe cools, in which the Higgs field all falls into this configuration, which I will call h1.

But suppose somewhere over here at 0.2, and the key thing is that initially 0.2 is going to be so far away from 0.1 that these points are out of causal contact with one another. I can not send a message from event one to event two. The Higgs field also reaches a point that the universe cools enough that at 0.2, just you know, it's a system that's not in thermal equilibrium. So, some places are going to be a little bit hotter than others, some are going to be a little bit cooler.

And so, at these two points it just so happened that the Higgs field got to the point where it could spontaneously break the symmetry. So at 0.2 the Higgs field also got to the point where it could spontaneously break its symmetry. And the only thing that's got to happen is, remember the only constraint we have is that the magnitude of the Higgs field be equals to some value-- I should normalize that to root 2 in units I want to use but whatever.

Let's say on this one my h1 is equal to y, and h2 is equal to minus 1. So, it's basically the same thing but all the fields are multiplied by i. It's the same magnitude, so it's going to have the same potential energy. So that's cool. Clearly this is allowed, and now all the regions in the universe that are close to the this are going to sort of smell this particular arrangement of the Higgs field and say OK, that's preferred arrangement I want to go into. So, we have two separate values of the Higgs field that are happily swooping out space time here. This gets to the excellent question I was just asked a moment ago-- what happens when they collide?

As the universe expands and gets cooler, all of it is going to end up getting swooped into either the field that was seeded at event one, or the field that was seeded in to event two, but at a certain point we're going to get the bits where they're smashing into one another. So what happens when these different domains come into contact with one another? The absolutely full and probably correct answer is we don't know. The reason is that we don't really, to be perfectly blunt, fully understand every little detail about the symmetry breaking, or about the structure of whatever grand unified theory brings all these things together at the temperatures at which this is happening. Because this is happening when the universe has a temperature of like 10 to the 16th GeV. And so it's way beyond the domain of where we can push things.

But we can, as physicists are fond of doing, we can paramaterize our ignorance, and we can ask ourselves, well what happens if these various parameters that characterize my grand unified theory take on the following plausible kinds of parameters. And what we find is that generically, when you have two different domains where the Higgs field takes on different values like this, when these domains come into contact you get what are called topological defects.

The topological defects come in three different flavors. To understand something about those flavors you have to know a little bit about what happens in general when you have phase transitions, and different regions of your medium go through a phase transition with different values of the parameters. So, it's a general case that whenever you have some kind of a phase transition and you have domains of different phase that come into contact with one another, your field will attempt to smoothly match itself across the boundary.

But that can be very difficult. So if you imagine these particular two cases that I have here, that's essentially saying that when these two domains coming to contact with one another there's going to be sort of a transition zone where the field is attempting to rotate from one value of the Higgs to the other. And it's going to pick some value that is in some sense intermediate to those two things.

So that, let's say we continue these up here, so that the collision is occurring right in this place here, in this little locus of events in space time. I have Higgs field 2 over here, Higgs field 1 over here, and I've got some crazy intermediate field that goes between the two of them, which is trying to sort of force itself to smoothly transition from one to the other.

In so doing, I might end up pushing my field away from the minimum, in which case there will then be some energy trapped in that layer. And there's a reason we do this level the class in a bit of a hand wavy way, I mean it's very, very complicated to get the details right. But the key thing we see is that in doing this match, the field has to do some pretty silly shenanigans order to make everything kind of match up and we can be left with odd observable consequences from the energy associated with the Higgs field getting pinned down at that boundary here. Now, the details of the forms of this boundary vary a lot depending upon to the specific assumptions you make about your underlying grand unified theory.

OK, so I should back up for a bit. I'm sort of assuming here when I discuss all this that there is some underlying SU(5) theory which describes the strong weak and electromagnetic interactions are very, very high temperatures as one gigantic thing. And we're getting to the point now where all the different interactions are beginning to just sort of crystallize out of it. There's a lot of different ways you can pack your underlying, fundamental, what we now think of as our standard model, into SU(5) grand unified theories. And so the ways in which we can get different topological defects depend upon how we choose to do that. So defect flavor one is you get something called a domain wall.

When we do this the fields attempts to make itself smoothly match from one region of Higgs field, say from Higgs 1 to Higgs 2. It succeeds, but you end up with kind of a two dimensional structure-- a wall-- in which there's some kind of anomalous field that is just pinned down there. And so we end up with a big sheet.

So in a theory like this, it would predict that somewhere out in the universe if there were regions in which the Higgs field had taken on a different value than the one that we encounter around us right now, it could be somewhere out gigaparsecs away, essentially a giant sheet of some kind. And there would be weird, anomalous behavior associated with it. People have really looked long and hard to try to find things like this and in fact it would be expected to leave interesting residuals in the cause of microwave background. My understanding of the literature is that there are actually now very strong bounds on the possibility of having a grand unified theory that leads to domain walls. And so this kind of a topological defect is observationally disfavored. So this, I should mention, only occurs in some grand unified theories.

Basically, As we move on to the other flavors of defects we end up just going down a step in dimensionality associated with the little kinks that are left over when the different domains come into contact with one another. Flavor two, we would get what's called a cosmic string.

Some of you may have heard of this. This is essentially, at its core, just a one dimensional, it could be gigparsecs long, but one dimensional, truly one dimensional-- essentially just a point in the other two dimensions-- string of mismatch Higgs field with some kind of an energy density associated with it when the different domains get in contact.

AUDIENCE: Do we have any estimate of how close in actual space these different regions would have started?

PROFESSOR: We do and I'm actually going to get to that. So, let me give you two answers to that. One of them is you are going to estimate that apparently on PSET 10, according to the notes that Alan left for me. But I'm going to spell out for you the arguments that go into it in the last 10 minutes of a class. But yeah, so let me just quickly finish up this one because this again-- so a cosmic string is sort of like a one dimensional analog of a domain wall.

And because it would be this sort of long one dimensional structure, that has actually up a lot of energy sort of pinned down to it by the fact it has a Higgs anomaly associated with it, it would be strongly gravitating and so it would leave really interesting signatures. It was thought for a while that cosmic strings might have been the sort of original gravitational anomalies that seeded some of the structures we see in the universe today. Again, it's now pretty highly disfavored.

If cosmic strings exist, they don't appear to contribute very much to the budget of mass in our universe. I should also mention that this is only predicted by some grand unifying theories. If you guys are curious about this I suggest when Alice back you ask him what the difference between these sums, why some predict a domain wall, some predict the cosmic strings.

Flavor three is where you end up with the Higgs field essentially being able to smoothly transition without leaving any defect anywhere except at a zero dimensional point. So you end up with just a little knot in the Higgs field. And for reasons that I will outline very soon, it turns out that this little not must carry magnetic charge, and so it must be a magnetic monopole.

The domain walls and the cosmic strings are, as I've emphasized, only predicted by certain specific grand unified theories. Magnetic monopoles are actually predicted by all of them. Question.

AUDIENCE: What does it mean to have a one dimensional domain wall, because there's no different region separated by one [INAUDIBLE].

PROFESSOR: That's right. So what ends up happening, and this is where I think you're going to have to ask Alan to sort of follow up on this a little bit. So, as the domains come into contact with one another. The fields do their best to smoothly transition from one to the other. And grand unified theories that predict a cosmic string, they succeed pretty much everywhere. They're able to actually smoothly make it all go away so you don't end up with feel being pinned down anywhere, except in a little one dimensional singularity that is somewhere along where the two dimensional services originally met.

And that is-- there's details there that I'm not even pretending to explain. And as I say, those are only predicted by certain kinds of grand unified theories. All of them will then predict that even if you don't have that, that cosmic string will then shrink itself down and it'll just be left with a little knot of Higgs field, where there's a little bit of residual mismatch between the two regions.

AUDIENCE: Do all three types of defects carry a magnetic charge, or only the knots?

PROFESSOR: I think only the knots. They do carry other kinds of fields, though, in particular the other ones gravitate, in fact all them gravitate, and so that's one of the ways in which people have tried to set observational limits on these things. In particular there have recently been a fair amount of work of people trying to set limits on cosmic strings from gravitational lensing, and there was really a lot of excitement because people thought they discovered want a couple years ago.

And they saw basically two quasars that looked absolutely identical, that were separated or scale that was just right to be a cosmic string. And then people actually looked at with better telescopes, and saw they had absolutely nothing to do with one another. They were not cosmic, they were not lenses, it's just every now and then God is screwing with you.

OK, so without going into some of the details what you have, these little point like defects-- and I'm short on time so I'm going to kind of go through this a little bit in a sketchy way enough so that I can pay for you how to do some calculations you're going to need to do. So the point like defects end up being regions, where at that point the Higgs field actually takes the value zero.

So remember I was describing how when you have two regions where the Higgs fields are both taking on values such as there at the minimum of the Higgs potential energy, and they come in to match one another, and what we have a boundary condition that very far away the Higgs field has values such as the energy is minimized. And there is a theorem, which in his notes Alan-- the way he describes it is he gives you a figure and outlines the various things that are necessary for the theorem to be true, and invites you to think deeply for a moment and until insight comes to you, I guess.

And when you put this ingredient that the Higgs field has this asymptotic, very far away value that drives you to the minimum of the field, and yet it must change value somewhere in the middle, the theorem requires that there be one point at which H equals 0. And apparently, this is a consequence in all grand unified theories.

So, recall, H equals 0. This is a point at which the potential energy density can be huge. So, when you have a little point like defect like this, it looks like a massive nugget, little massive particle. You can in fact calculate the total amount of energy associated with this particle. If you do so just including the influence of the Higgs field, the calculation basically goes like this.

It's very similar to the way we calculate the energy associated with electric and magnetic fields in electrodynamics. Ask yourself, how much energy is contained in a sphere of radius, capital R, centered on this little knot of Higgs field. Well, it's going to look like 4pi times an integral of the gradient of the Higgs field squared r squared dr It turns out, when you calculate the [INAUDIBLE] of the Higgs field around one of these little defects, it's actually very complicated close to the defect, but as you get far away it has a very simple form. The gradient goes as 1 over r, it tells you the field itself actually goes something like log.

That means your energy looks something like, R squared, 1 over R squared dr which goes as R, which diverges as you make the sphere bigger and bigger and bigger. So, what's the mistake we made? Well, the Higgs field doesn't always just sit there and operate on itself. The Higgs field actually couples pretty strongly to all of our vector bosons. Particularly, it couples pretty strongly to electric and magnetic fields. So, we have to repeat this calculation including the interaction of the Higgs field with the E&D field.

And in Alan's notes he gives you some references on this because this is not the kind of calculation you can really sketch out very easily in an undergraduate class. To make this integral convergent, the only way it can be done is if that little nugget of Higgs field is endowed with magnetic charge. You need to have a monopolar magnetic field that ends up putting in interaction terms, that make the divergence of this integral go away.

So, I at last get to the punchline of all this, we are left inevitably, if we accept the whole foundation story of particle physics that the different interactions were unified in some high energy scale and then froze out. We are driven inevitably to the story that defects in the Higgs field create magnetic monopoles.

Now, I realize I'm out of time, so let me just quickly sketch a few interesting facts about this and there's a few exercises that you guys are apparently going to look at in your homework assignment. When we do this calculation, one which is I believe just referenced in the notes that Alan has for the class, we learn a couple of things about this magnetic charge. One of them is that if you work in the fundamental unit, say CGS units, the value of the magnetic charge, we'll call that g, is exactly 1 over 2 alpha where alpha is a fine structure constant, times the electric charge.

So if you have two magnetic monopoles they attract each other with a force that is-- so 1 over 2 alpha is approximately 68.5 I think-- and so it would be 68.5 squared times the force of two electric charges at that same distance. We also end up learning the mass. It turns out to be 1 over alpha times the scale of GUT symmetry breaking.

Anyone recall what the scale of GUT symmetry breaking is? 10 to the 16 GeV. So, this is a particle, 1 over alpha is approximately 10 to the two, so this is a particle that has a mass of about 10 to the 18th GeV, in other words it's a single particle with a mass of 10 to the 18th protons. This is approximately one microgram. If you put one of these things on a scale it could measure it, that's bloody big.

So getting to the last bit of the class, which I am just going to very basically quote the answer. The question becomes how often do these things get created and here

I'm going to refer to Alan's notes. What you'll find is that, remember when we sketched our original picture of this thing we looked at regions of the universe where the Higgs field was initially seeded with different values. In order for the Higgs field to take on different values, initially, these regions had to be out of causal contact with one another.

So we are going to require that the initial seed areas be separated by a distance, which is the correlation length, which has to be less than or of order the horizon distance. You can get a lower bound on this thing by imagining that it's-- sorry let me say one other thing. If you do that, then you can estimate that the number density associated with these things, the number density of these monopoles will be 1 over the correlation length cubed.

To get a lower bound on the number density of these things, set the correlation length exactly to the horizon distance, and then do the following exercise. So first, let's set up the correlation length equal to the horizon distance. Set the density in monopoles equal to the mass of a monopole over rH cubed, normalized to the critical density. If you do this, you will find that just due to magnetic monopoles alone, the density of the universe.

PROFESSOR 2: Excuse me, professor

PROFESSOR: Yes, I'm wrapping up right this second.

PROFESSOR 2: It's seven minutes. You were supposed to end at 10:55.

PROFESSOR: I'm substitute teaching, I'm sorry.

OK, so this tells us that we are at 10 to the 20 of the critical density. And a consequence is that the universe is approximately two years old. I will let Alan pick it up from there.