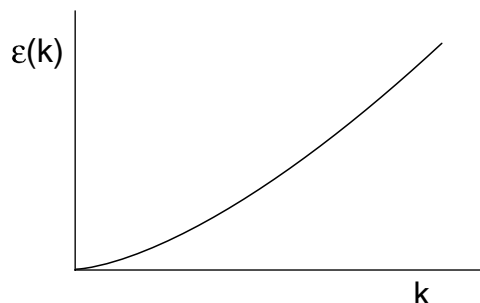


Practice Exam #4

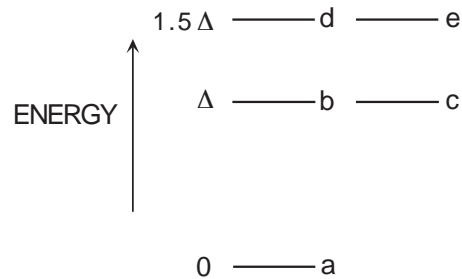
Problem 1: Ripplons (35 points)



We have seen that the bulk motion of a solid or liquid can be described by harmonic normal modes (that is, normal modes each having a harmonic oscillator Hamiltonian) known as phonons. In a similar manner the two dimensional waves on an interface between a liquid and its vapor can be described by harmonic normal modes known as “ripplons” each having a single direction of polarization perpendicular to the interface. The dispersion curve for these elementary excitations is isotropic and given by $\hbar\omega(k) \equiv \epsilon(k) = bk^{3/2}$ where $k = \sqrt{k_x^2 + k_y^2}$. For a rectangular sample with dimensions L_x and L_y , the wavevectors allowed by periodic boundary conditions are $\vec{k} = (2\pi/L_x)m\hat{x} + (2\pi/L_y)n\hat{y}$ where m and n can take on all positive and negative integer values.

- a) What is the density of allowed wavevectors $D(\vec{k})$ such that $D(\vec{k})dk_xdk_y$ gives the number of allowed wavevectors in the area dk_xdk_y around the point \vec{k} in k -space?
- b) Find an expression for the density of states as a function of energy $D(\epsilon)$ for the ripplons in terms of the parameter b and the area $A = L_xL_y$. Sketch your result.
- c) Find an expression for the ripplon contribution to the constant area heat capacity $C_A(T)$. Leave your result in terms of a dimensionless integral (do not try to evaluate the integral). How does $C_A(T)$ depend on T ? Sketch the result.
- d) Does the system exhibit energy gap behavior? Explain your reasoning.

Problem 2: Impurity Atom (35 points)

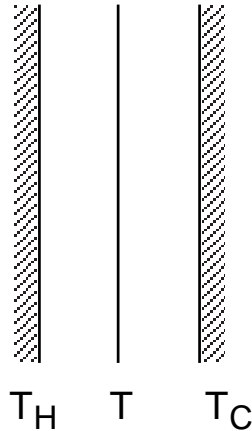


An impurity atom in a solid has 3 electrons (spin 1/2 Fermions) above a filled, inert electronic shell. These electrons have available to them 5 spatial single-particle states, $\psi_a, \psi_b, \psi_c, \psi_d$ and ψ_e with energies $\epsilon_a = 0, \epsilon_b = \epsilon_c = \Delta$, and $\epsilon_d = \epsilon_e = 3\Delta/2$. In what follows, assume that there is no interaction between the electrons. [Note that part g revisits b,c,d and f.]

- How many 3-particle states are available to the atom? Be sure to take into account both the spin and spatial variables when determining your number.
- Write down the terms in the partition function $Z(T)$ arising from the states corresponding to the two lowest 3-particle energies.
- Write down the terms in the partition function $Z(T)$ arising from the states corresponding to the two highest 3-particle energies.
- What is the entropy at $T = 0$?
- What value does the entropy approach asymptotically at very high T ?
- What is the asymptotic value for the heat capacity at very high T ?
- Repeat b), c), d) and f) [but not a) or e)] for the case where the three identical particles are spin 0 Bosons*.

*I will treat to a free dinner in the fall anyone who answers a) for spin 0 Bosons without resorting to brute force.

Problem 3: Realistic Super Insulation (20 points)



Two parallel plates of infinite extent are separated by a vacuum and maintained at temperatures T_H and T_C . The surface of each plate acts as a black body. A thin conducting sheet is suspended in the vacuum as shown in the figure. Heat can be transferred to the sheet only through the vacuum. The sheet has an absorptivity $\alpha < 1$, and a power reflectivity $r = 1 - \alpha$.

- a) Find the steady state temperature T of the sheet.
- b) Find the heat flow from the hotter plate to the colder plate as a fraction \mathcal{F} of that which would occur in the absence of the sheet.

Problem 4: Adiabatic Demagnetization (10 points)

Consider the extreme situation of an ideal paramagnet in thermal contact with a sample so small that the thermodynamics of the assembly is dominated by that of the paramagnet alone. The assembly is cooled adiabatically by reducing the applied magnetic field from 8 kilogauss to 20 gauss. What is the final temperature if the initial temperature was 1 Kelvin? [This does not require an extensive calculation.]

Work in simple systems

Hydrostatic system	$-PdV$
Surface film	$\$dA$
Linear system	$\mathcal{F}dL$
Dielectric material	$\mathcal{E}d\mathcal{P}$
Magnetic material	HdM

Thermodynamic Potentials when work done on the system is $dW = Xdx$

Energy	E	$dE = TdS + Xdx$
Helmholtz free energy	$F = E - TS$	$dF = -SdT + Xdx$
Gibbs free energy	$G = E - TS - Xx$	$dG = -SdT - xdx$
Enthalpy	$H = E - Xx$	$dH = TdS - xdx$

Statistical Mechanics of a Quantum Harmonic Oscillator

$$\begin{aligned}\epsilon(n) &= (n + \frac{1}{2})\hbar\omega & n = 0, 1, 2, \dots \\ p(n) &= e^{-(n+\frac{1}{2})\hbar\omega/kT} / Z(T) \\ Z(T) &= e^{-\frac{1}{2}\hbar\omega/kT} (1 - e^{-\hbar\omega/kT})^{-1} \\ \langle \epsilon(n) \rangle &= \frac{1}{2}\hbar\omega + \hbar\omega(e^{\hbar\omega/kT} - 1)^{-1}\end{aligned}$$

Radiation laws

Kirchoff's law: $e(\omega, T)/\alpha(\omega, T) = \frac{1}{4}cu(\omega, T)$ for all materials where $e(\omega, T)$ is the emissive power and $\alpha(\omega, T)$ the absorptivity of the material and $u(\omega, T)$ is the universal blackbody energy density function.

Stefan-Boltzmann law: $e(T) = \sigma T^4$ for a blackbody where $e(T)$ is the emissive power integrated over all frequencies. ($\sigma = 56.9 \times 10^{-9}$ watt-m⁻²K⁻⁴)

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8.044 Statistical Physics I
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