

Topics: Current, Resistance, and DC Circuits

Related Reading:

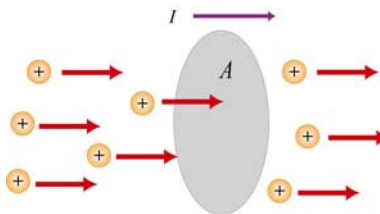
Course Notes (Liao et al.): Chapter 6; Sections 7.1 through 7.4
 Serway and Jewett: Chapter 27; Sections 28.1 through 28.3
 Giancoli: Chapter 25; Sections 26-1 through 26-3

Topic Introduction

In today's class we will define *current*, *current density*, and *resistance* and discuss how to analyze simple DC (constant current) circuits using Kirchoff's Circuit Rules.

Current and Current Density

Electric currents are flows of electric charge. Suppose a collection of charges is moving perpendicular to a surface of area A , as shown in the figure



The electric current I is defined to be the rate at which charges flow across the area A . If an amount of charge ΔQ passes through a surface in a time interval Δt , then the current I is given by $I = \frac{\Delta Q}{\Delta t}$ (coulombs per second, or amps). The current density \vec{J} (amps per square meter) is a concept closely related to current. The magnitude of the current density \vec{J} at any point in space is the amount of charge per unit time *per unit area* flowing pass that point. That is, $|\vec{J}| = \frac{\Delta Q}{\Delta t \Delta A}$. The current I is a scalar, but \vec{J} is a vector.

Microscopic Picture of Current Density

If charge carriers in a conductor have number density n , charge q , and a drift velocity \vec{v}_d , then the current density \vec{J} is the product of n , q , and \vec{v}_d . In *Ohmic* conductors, the drift velocity \vec{v}_d of the charge carriers is proportional to the electric field \vec{E} in the conductor. This proportionality arises from a balance between the acceleration due the electric field and the deceleration due to collisions between the charge carriers and the “lattice”. In steady state these two terms balance each other, leading to a steady drift velocity (a “terminal” velocity) proportional to \vec{E} . This proportionality leads directly to the “microscopic” Ohm’s Law, which states that the current density \vec{J} is equal to the electric field \vec{E} times the conductivity σ . The conductivity σ of a material is equal to the inverse of its resistivity ρ .

Electromotive Force

A source of electric energy is referred to as an electromotive force, or emf (symbol \mathcal{E}). Batteries are an example of an emf source. They can be thought of as a “charge pump” that moves charges from lower potential to the higher one, opposite the direction they would normally flow. In doing this, the emf creates electric energy, which then flows to other parts of the circuit. The emf \mathcal{E} is defined as the work done to move a unit charge in the direction of higher potential. The SI unit for \mathcal{E} is the volt (V), i.e. Joules/coulomb.

Kirchhoff's Circuit Rules

In analyzing circuits, there are two fundamental (Kirchhoff's) rules: (1) The junction rule states that at any point where there is a junction between various current carrying branches, the sum of the currents into the node must equal the sum of the currents out of the node (otherwise charge would build up at the junction); (2) The loop rule states that the sum of the voltage drops ΔV across all circuit elements that form a closed loop is zero (this is the same as saying the electrostatic field is conservative).

If you travel through a battery from the negative to the positive terminal, the voltage drop ΔV is $+\mathcal{E}$, because you are moving against the internal electric field of the battery; otherwise ΔV is $-\mathcal{E}$. If you travel through a resistor in the direction of the assumed flow of current, the voltage drop is $-IR$, because you are moving parallel to the electric field in the resistor; otherwise ΔV is $+IR$.

Steps for Solving Multi-loop DC Circuits

- 1) Draw a circuit diagram, and label all the quantities;
- 2) Assign a direction to the current in each branch of the circuit--if the actual direction is opposite to what you have assumed, your result at the end will be a negative number;
- 3) Apply the junction rule to the junctions;
- 4) Apply the loop rule to the loops until the number of independent equations obtained is the same as the number of unknowns.

Important Equations

Relation between \vec{J} and I :

$$I = \iint \vec{J} \cdot d\vec{A}$$

Relation between \vec{J} and charge carriers:

$$\vec{J} = nq\vec{v}_d$$

Microscopic Ohm's Law:

$$\vec{J} = \sigma\vec{E} = \vec{E}/\rho$$

Macroscopic Ohm's Law:

$$V = IR$$

Resistance of a conductor with resistivity ρ ,
cross-sectional area A , and length l :

$$R = \rho l / A$$

Resistors in series:

$$R_{\text{eq}} = R_1 + R_2$$

Resistors in parallel:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Power:

$$P = \Delta V I$$