

Let's analyze a one-dimensional collision, where we're in the laboratory reference frame.

And we have V_1 initial and V_2 initial is 0.

It's a frictionless surface.

But there's a collision here.

And now we're going to make this collision totally inelastic.

Now what that means is that the two particles stick together.

So if this is our initial state-- and let's, again, choose a direction \hat{i} -- then our final state will just stick those two particles together.

And they're going to move.

We only need one velocity for the final state, \hat{i} .

Now again, let's assume that there's no external forces.

There are clearly internal forces that are acting between the two particles as they collide.

But, remember, we know that internal forces cancel in pairs.

And if there's no external forces, the momentum of the system is constant.

So we can write down our momentum condition very simply.

And we'll write it again in terms of components.

We have some incoming momentum into the system.

And all of that incoming momentum is going out.

And now, is energy constant?

Well, a totally inelastic collision, energy cannot be constant.

And the reason is that these internal forces will cause deformations that are irreversible.

So the objects might be deformed, which is a loss of-- some of that kinetic energy will go into deformation.

There could be noise.

There could be heat generated in the collision, lots of sources.

When objects stick together, kinetic energy is not constant.

In fact, we can even figure out the change in kinetic energy, because we can see here that V_x final is m_1 over m_1 plus m_2 V_1 x initial.

And now let's ask ourselves, in a totally inelastic collision what is the change in the kinetic energy?

So our final kinetic energy is $\frac{1}{2} m_1$ plus m_2 V -- here, we can, again, right in terms of components-- V_x final, because the component's squared.

That's no problem.

And we just have this initial kinetic energy coming in.

Now we have a solution for our problems.

So let's just put that in.

m_1 plus m_2 times m_1 over m_1 plus m_2 times V_1 x initial squared minus $\frac{1}{2} m_1$ V_1 x initial squared.

And now let's pull out the $\frac{1}{2} m_1$ V_1 x initial squared-- $\frac{1}{2}$ one factor of m_1 V_1 x initial squared.

But what we're left with here is another factor of m_1 .

And downstairs, we have an m_1 plus m_2 outside but an m_1 plus m_2 .

Now remember, we didn't square that.

We have an m_1 plus m_2 inside.

So there's that m_1 plus m_2 minus 1.

Now this is the initial kinetic energy.

And when we subtract these terms, we have negative m_2 over m_1 plus m_2 .

And so what we see here is-- I'm just going to rewrite this again-- that the kinetic energy, the change in kinetic energy, just depends on the initial kinetic energy and this mass ratio.

In fact, if we asked ourselves, what is the ratio of that loss of kinetic energy to the initial kinetic energy, that just depends on the mass ratio of our two objects.

In particular, it should be a minus sign, because the kinetic energy is decreasing.

And so this is the fact that it's very transparent now that in a totally inelastic collision the kinetic energy is not constant.

Where did that energy go?

It went into other forms of energy.