Let's now explore an example of a force in which the work done it is not path independent.

And the classic example is the friction force.

So let's consider the following setup.

Suppose we have a horizontal surface with friction and we have an object.

And we're moving this object.

So let's choose an origin.

We'll call this plus x, our i hat direction, it's all going to be one-dimensional motion.

And we're going to move this object from an initial to a final state.

And were going to move it directly in a straight line from the initial to the final state.

And this will be our path 1.

And in our second case, what we'd like to contrast with that, is that we'd like to move the object out to a point xa and then back to the final point.

So this is our path 2.

Has two legs.

And we'd like to compare the work done on these two paths.

So for path 1 we'll begin by calculating-- our force here is the kinetic friction force.

And the kinetic friction force remember is, in this case, it's going to oppose the motion.

So we have force kinetic for path 1, and that is minus mu k mg in the i-hat direction.

And so when we do the integral for the work from x initial to x final, this is path 1, then we have minus mu k mg i-hat dotted into-- Now what is the ds for this path?

It's simply dx i-hat, so dx i-hat.

Notice we're not putting any sign into dx.

The sign will show up in terms of our end points of our integral.

So we do the dot product here, we have i-hat dot i-hat, that's 1.

And so this interval, we can pull out all the constants, mu k mg.

We're just integrating dx from x initial to x final.

And so we get mu k mg times x final minus x initial.

Now for path 2 we have two separate integrals.

So for path 2 we'll just show the first part where we're going from x initial to xa.

Then the friction force is opposing the motion.

And we always just write dx in terms of the coordinate system, dx i-hat because you'll see that the signs show up in the end points of the integral.

And then when we're coming back-- I'll put that in a different color and I'll put it below it.

So when we come back, notice the friction force is going to change direction.

ds will still be written that way but pay close attention to the end points of the integral.

So now what we have is two integrals.

So W is the integral from x initial to xa.

And now I'm going to take the dot products here directly.

It's the same friction force, we still have this same integral, which is minus mu k mg dx.

Now here's where it's a little bit tricky.

Notice on this path fk is plus mu k mg i-hat.

And so when we dotted into dx we have a plus sign, we'll just continue that integration here, of mu k mg dx from xa to x final.

Both of these integrals are straightforward integrals to do.

This is minus mu k mg xa minus x initial.

And over here, we have a plus mu k mg x final minus x initial.

Notice x final minus xa, rather, is negative.

And so both of these integrals are negative, as we expect.

And so what we see here is that we have two pieces, so minus 2 mu k mg xa.

And then we have that other piece, mu k mg x final minus x initial.

Hang on, this is actually a plus sign.

So our answer is very different because the displaced the amount that we've traveled is different.

So what we see here is an example of a force which is the work done is not path independent but depends on the path taken from the initial to the final states.