We will now like to generalize the work energy theorem to more than one dimension.

Remember that we defined our work as an integral from a to b of a force dot ds.

And graphically let's look at that.

If we start at A and we go to a point B, , and if at some point on this path we have a force F, and the displacement is always tangent to the force for an object M, we would like to now figure out how this integral is related to the change in kinetic energy.

So first off, it's always important when analyzing a new result to understand where Newton's second law comes in.

And for our point particle Newton's second law is that the force is equal to ma, which we can write as mass times the derivative of the velocity.

And so our first step is to rewrite the work integral and now to incorporate Newton's second law into this integral.

So this is no longer simply a concept of physics but it's combining the second law and our definition of work.

Now what I'd like to do here is to note that the displacement of a particle is always equal to the velocity times dt.

So if we put that result in here, AB of M dv dt dot v dt, you can see that we can cancel dt's, which is essentially taking the time out of this integral and writing the work entirely in terms of the small change in velocity, dv, scalar product with the velocity.

And this is the quantity we would now like to calculate.

So one way, when we ever want to calculate these dot products, we choose a coordinate system.

So let's choose a Cartesian coordinate system, plus x plus y, with our unit vectors i-hat and j-hat.

Now our task is to write down these various vectors.

So for instance, dv can be written as dv x i-hat plus dv y j-hat.

Now if we were to go into three dimensions we would have dv z k-hat but I'm just going to do this in two dimensions.

Similarly, the velocity is vx i-hat plus vy j-hat.

And now when we take the dot product, as we've learned in Cartesian coordinates, it's just the components dv x vx, and I'll just do a little parentheses there, plus dv y vy.

And so now, our work integral, AB, is equal to the mass times dv x vx plus the mass times dv y vy.

Now what we have here is to a sum in the [? integrant. ?] But by the rules of integration we can rewrite this integral as two separate integrals from A to B of M dv x vx and from A to B of M dv y vy.

And now we can integrate each of these separately.

And if you recall, in one dimension we used our integral formula, this is just vx squared.

And when we integrate them what we have to look at is the end points of our integral.

What we're integrating is a velocity, and we're going from some initial value to some final value.

So our first integral simply becomes M vx final squared.

I'll pull out the factor 2 minus vx initial squared, where A vx initial is the initial x component velocity of A in vx final is the final component of B.

And I have a similar term here, which is vy final squared minus the vy initial squared.

And when I combine our terms, vx final squared plus vy final squared minus one-half M vx initial squared plus vy initial squared.

We've already shown that our kinetic energy, 1/2 M v dot v is 1/2 M vx squared plus vy squared.

And so we see that this is just the final kinetic energy minus the initial kinetic energy.

And so we can call that delta k.

And what we have now is the work energy theorem.

And I'll come back to my original statement that the work is equal to the change in kinetic energy.

And this is a combination of our definition of work and Newton's second law.