The little prince lives on his asteroid B-612.

And he really likes to watch the stars.

And what he really wishes for is to watch the stars while floating, to have the best possible view and to just immerse himself in the stars.

And so the little prince has to think if there is a position in space with respect to other celestial bodies where the gravitational force would just cancel, so that he can float.

Let's look at that.

So there is the planet of the businessman.

We're going to give it the mass m1.

And there is also the small planet from the lamplighter floating around with a mass m2.

And we know that gravitational forces act radially.

So we can already see here that if we put the asteroid B-612 somewhere here, then maybe we find a constellation where gravitational forces cancel.

So let's start by placing the little asteroid over here.

And what is this body experiencing in terms of gravitational forces?

Well, it's going to experience a gravitational force of object 1.

So we have object 1 or F1m, due to the interaction between object 1-- so the businessman planet and our little asteroid here.

And then we also, of course, have the same direction here due to the interaction of object 2 and our little asteroid, so that would be F2m.

And you can clearly see that if we tally up these forces, they're not going to add to 0.

So this is not a good location for the little prince's asteroid to be.

The same would be true if we put it out here.

It will experience those same forces again.

What do we have here-- 1m and 2m just going in the opposite direction.

But you have guessed it already.

We can find a spot here in the middle where one force goes in this direction.

So this is F2m.

And actually we should give those forces direction.

And the other body exerts a force going in the other direction.

And so we can see that we now just need to find that exact position here, in between these two objects with two different masses, where the little prince can free float while watching the stars.

We can say already that if these two masses were the same, it would obviously be in the middle.

But because they are not the same, we have to calculate what that distance is.

And so we're going to label this distance here x.

And we're going to say that the two bodies here are a distance d apart.

All right.

How are we going to go about this?

Well, we have to apply the universal law of gravitation in our F equals ma analysis.

Well, before we started with our F equals ma analysis, we actually have to pick a coordinate origin and a unit vector.

And so let's place the coordinate origin in here.

And let's have i hat go in this direction.

And we're going to label this position A, and position B, and position C. These are our three options for asteroid placement.

And well, let's look at position B.

And well, we have the universal law of gravitation between the mass of the object here.

And we'll have to describe both components.

So first this one F1m and then F2m.

And if we start with F1m, that goes in the negative i hat direction.

So minus Gmm1 on the distances x squared in the i hat direction.

And then the other one goes in the plus i hat direction.

And we have Gmm2.

And now we have d minus x gives us this portion-- d minus x squared also in the i hat direction.

And that needs to add up to 0, because that is what we want, right?

If it adds up to 0, then we have no gravitational forces acting on the asteroid.

So now we need to solve this for x here.

And in the first step, you see that actually G and m will fall out here.

And then we're left with m1 over x squared minus plus m2 over d minus x squared.

And we can write that as m2 x squared minus m2 d minus x squared.

And what you see here is that this will turn into a quadratic equation.

And if we do a few steps of arithmetic, and then write down the general solution to this quadratic equation, we will find this here.

x equals 2dm1 plus minus 2dm1 squared minus 4m1 minus m2m1d squared, and then the square root of that over m1 minus m2.

And actually we need two of those.

So we have this quadratic equation here-- well, the solution.

And we now need to consider one more thing-- namely, this equation here is only valid if-- well, can I write this here-- if x is between 0 and d.

Right?

In position C, my x is larger than d, which means this-- sorry, this force here flips sign, and then this would be different.

So this plus here refers to x being less than d.

And we have to decide which of the two signs here gives us the correct, which will fulfills this requirement here.

So this simplifies to x equals dm1 plus minus m1m2 and here we have the square root.

And then here, we have another bracket over m1 minus m2.

And we need to get this term here smaller than 1, which will be smaller than 1 if the x is between 0 and d.

And as it turns out, that is indeed true if we use the minus sign here.