We would now like to compare the moment of inertia for a rigid body.

Let's take an arbitrary rigid body about the center of mass.

So let's say the rigid body is rotating about this axis.

And what we'd like to compare that is to the moment of inertia, say, about a parallel axis that's also going through the rigid body.

Now, let's recall how we define moment of inertia.

We first choose a mass element dm.

What I'd like to show is if this object is rotating about this axis, then what is that object doing.

Well, that object is undergoing a little bit of circular motion.

And this distance here is what we call the perpendicular distance about that axis.

And let's indicate this is for our element dm.

So this perpendicular distance is what shows up in our definition for the center of mass moment of inertia about that axis-- it's the interval of dm r cm perp.

Now again, quantity squared.

What is this distance?

This is the perpendicular distance from our dm and to the axis of rotation.

Imagine it's doing a circle and that's the radius of that circle.

So if we were to calculate the moment of inertia about another axis, then about this axis the perpendicular distance here that I'll write as rs perp.

And you can see these perpendicular distances are not the same.

And the moment of inertia about that other axis is equal to the integral of dm rs perp quantity squared.

Now, how do we relate those perpendicular distances?

Well, there's a couple of ways to do it.

And notice that the distance between these axes is given by d.

And I'm going to call this the distance r cm.

Now, let's just call this-- the x direction-- I'll call that x.

So how do I relate these distances?

Well, d is a fixed distance.

And you can see from my diagram that rs perp is equal to d plus r cm x.

And if I square this, I get d-squared plus 2d rcm x plus rcm x-squared.

And that r cm x-squared is precisely what we're calling perpendicular distance.

So when I put those into my moment of inertia Is, I get dm times d-squared plus 2d times rcm x plus, parentheses, rcm perp squared.

Now, I'll separate this into three terms.

The first term is dm times d-squared.

This is an integral over the body.

The second term is 2d-- and I'm going to hold off on the interval, because the 2d is the same for every piece-- dm rcm x.

And the third piece is integral over the body of dm r-- since rcm x is the r perp, I'll write it as r perp squared.

And you can see that this term is precisely the moment of inertia about the center of mass.

Now, what I'd like to focus on is this terribly, in particular, dm rcm x that appears in our integral expression.

Recall, that we define center of mass.

We had the condition that the sum of mj rcmj was 0.

Now for an integral relationship, this is dm rcmj cm equal to 0.

So when you sum up the position of every object with respect to the vector from the center of mass to your dm element, 0.

What does this say in terms of components?

In terms of components, each component separately vanishes so we have the condition that cm x is 0.

So that term is 0, which is precisely this term-- that's 0.

And so we can conclude that Is-- now in this term, where d is the same piece for every object-- so we're just pulling out the total mass.

So it's m total d-squared.

And let's remind ourselves that d is the distance between the two parallel axes plus the moment of inertia about the center of mass.

And this result is called the parallel axis theorem.