I would now like to calculate the moment of inertia for a very thin disk.

So we have a thin disk.

And the radius of that disk is r.

And it has a mass m.

And I would like to calculate the moment of inertia for this disk.

Now, let's just remind what point we're calculating it about, about the center of mass.

So our definition of moment of inertia was take a small element, mass element to the disk.

In fact, we're going to see it doesn't have to be small.

Take a mass element to the disk that's useful, and multiply it by the perpendicular distance squared from the point we're calculating it.

So the way I'll do it is I will choose a ring.

I'm gonna choose a ring of radius r.

And now I'll make the ring a certain thickness.

And this thickness is dr.

Now, in this calculation, we're going to take a limit as dr goes to zero.

So even though the ring has some finite thickness, its radius-- we'll eventually treat treated as all of the mass element a distance r from the center.

So r will be our integration variable.

And that will be equal to rcm, what we're calling rcm in the abstract result.

Now, the dm is the tricky part.

So what is the mass that's contained in this area disk of radius r and thickness dr?

Well, one way to think about that is it's-- here we didn't say this, but our disk is going to be uniform.

And so we can describe the mass per unit area as the total mass divided by the area of the whole disk.

And then we can say that the mass in that ring is equal to sigma mass per area times the area of the outer ring minus the area of the inner ring.

Now, when we expand this out, dm, m over pi r squared, we get pi r squared plus 2rdr plus dr quantity squared minus pi r squared.

And you can see those terms cancel.

And so what I get is m times pi r squared.

And in here I have 2 pi r dr. Now, this is only order dr, plus a second term that goes like pi dr squared.

And so, when I take this limit as dr goes to 0, this term is much, much smaller than that term.

And so I can say my mass element is m pi r squared times 2 pi r dr.

Now, let's think about this term, why it makes sense.

When we're shrinking our ring, so taking a limit as dr goes to 0, and the ring just becomes an extremely thin ring at radius r, then this piece is a circumference, and this piece is just the width.

And so it's no surprise that area is 2 pi r times d pi r dr in the limit.

And now that enables us to write the moment of inertia about the center of mass, icm.

Let's pull out these constants, m pi r squared.

Now we're integrating over the body.

Let's hold off on the limits for the moment, and put our values for dm.

That's 2 pi r dr. And we have our distance squared, which was, again, the radius of r squared.

And so the pis will cancel.

I have 2m over r-squared times the integral of r cubed dr.

Now, we're supposedly integrating over the body, but what does that body integral actually mean?

Well, what we're doing is we're taking a series of rings and adding them up as we go from the origin out to the radius of the whole disk.

So the limits of our body integral with respect to our integration variable, we start with rings that essentially have no width.

And we're integrating these, we're adding up the contribution of every ring until we get to rings of radius r.

And our integration variable, r cubed, dr.

Now, this is an integral that's easy to do.

That's r to the forth over 4 between 0 and r equals r.

And when we put that in, the 2 cancels the 4.

And oh, the pi we lost.

So let's make sure this pi should be in m.

So we have the 2 over the 4 is one half, and r squared.

And that is the moment of inertia of it does about an axis passing through the center perpendicular to the plane of the disk.