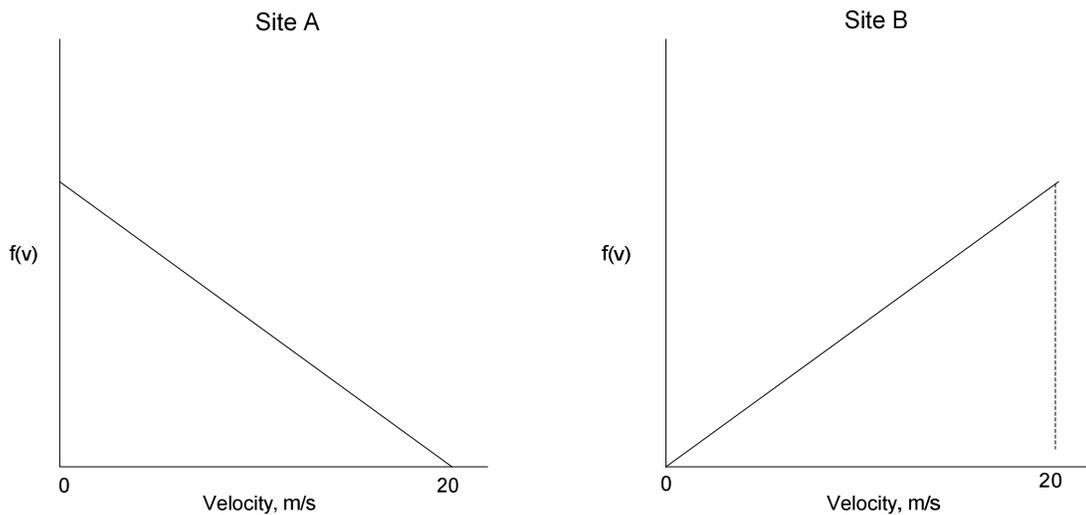# Sustainable Energy Take-Home Exam 1

Solutions

Question 1 (50 Points):

A 3 MWe wind turbine can be placed at either Site A or B, which have respective probability density functions for the wind velocity at the site (assume that the power output of the turbine varies as the cube of the velocity) as shown in Fig 1.

The land for Site A costs $1.0 million and that at Site B costs $2.0 million. The turbine capital cost is $2 million. O&M costs for a turbine is $100,000/yr. Electricity from a wind turbine can be sold to the grid for $60/MWhre. The turbine lifetime is 20 years. The discount rate is 0.04 annually. Inflation and taxes may be ignored. Which site offers the expectation of a better investment?

Solution:

We can assess each of these sites by comparing their net present values—the difference between the net present values of the expenditures and revenues.

Since we are to ignore inflation and taxes, the revenues from electricity generation and the costs associated with O&M are fairly straightforward to compute.

The revenue associated with either site can be estimated as the price per MWhre times the average power produced by the project.

If the turbines have their maximum electric power output at v=20 m/s, then the power produced by the turbines goes as P=(3 MWe/8000) v^3.

In order to find the average power produced by each site, we need to find an expression for this and then perform a change of variables. Since the area under each of the probability curves must integrate to one, the slopes of the velocity probability curves must be -0.005 and 0.005, respectively.

For Site A: f(v)=0.1-0.005v

For Site B: f(v)=0.005v

The average values of the power outputs for each site can be found using

$$\int_a^b Pf(P)dP.$$

Basic probability theory says that the incremental area under related distributions must be the same, so:

f(v) dv = f(P) dP

Using this with our equation for P, we get:

$$\overline{P} = \int_0^{20} \frac{3}{8000} v^3 f(v)$$

Site A: $\overline{P} = \int_0^{20} \frac{3}{8000} \left(0.1v^3 - 0.005v^4\right)dv = \frac{3}{8000}\left(\frac{0.1}{4}20^4 - \frac{0.005}{5}20^5\right) = 0.3\,MWe$

Site B: $\overline{P} = \int_0^{20} \frac{3}{8000} 0.005v^4 dv = \frac{3}{8000}0.001\cdot 20^5 = 1.2\,MWe$

Without values for inflation provided, we're only able to compare the total un-inflated revenue with the initial investment and un-inflated capital costs.

Revenues for Site A: (0.3MWe)(24 h/day)(365 d/y)(20 y)($60/MWhe)= $3.1536 million

Revenues for Site B: (1.2 MWe)(24 h/d)(365d/y)(20 y)($60 MWhe)= $12.6144 million

The expenditures associated with O&M are the same for either site, and can be treated as a lump sum paid out once per year. In the absence of inflation, the total expenditure for either site is (20 years)($100,000/year)=$2 million

The present value of the capital cost for each site can be evaluated using:

$$PV_{cap} = \frac{P_{over}}{T} \sum_{0}^{T-1} \left(\frac{1+i_I}{1+i_R} \right)^n$$

If we assume that the capital costs are paid off over the 20-year lifetime of the facility, and that the money for future payments is not being invested ($i_R=0$), the present value of the capital costs is $4.466 million for Site A and $5.955 million for Site B.

The net present values are then:

Site A: $3.1536 million - $4.466 million - $2 million = -$3.3124 million

Site B: $12.6144 million - $5.955 million -$2 million = $4.6594 million

Site B is clearly the better choice.

Extra Credit (up to 5 Points): Once the more attractive site is identified what additional factors should a potential investor take into account prior to deciding whether to fund the wind turbine?

While the average is useful for making a first estimate of the worth of the investment, the investor should also consider the impact of the uncertainty introduced by the wind speed distribution. In addition, they should consider how the availability of subsidies, and the associated uncertainty, affects their ability to recoup their costs.

Question 2 (50 Points):

An ocean thermal gradient electric generating system is proposed for siting in Hawaii. This system would operate a heat engine driven by heat flow from warm ocean water (cooled from 27 °C to 25 °C in the heating heat exchanger of the system), and cooled by heat flow to cold ocean water (which is heated from 3 °C to 5 °C in the cooling heat exchanger of the system). The ocean water used in the different parts of the heat engine is obtained from different depths of the surrounding sea. Using this heat engine electricity can be produced at 90% of the corresponding Carnot efficiency of the heat engine.

Solution:

a. (10 points) What is the value of the Carnot efficiency of the heat engine? Explain the basis for selecting the parameter values used in calculating this value.

   The Carnot efficiency of the heat engine is $1-T_C/T_H$, where T_c and T_h are the maximum temperature of the cold reservoir, and the minimum temperature of the hot reservoir, respectively.

   The minimum absolute temperature of the hot reservoir is 298 °C, and the maximum absolute temperature of the cold reservoir is 276 °C, making the Carnot efficiency

   $1-(276/298)=0.074$.

b. (15 points) Should the cold ocean water flow through the engine at a rate of 100 million kg/min what is the electric power output of the engine

   The heat carried away from the system is $Q_{out}=Q_{in}-W$.

   The work done by the system is $W=Q_{in}*efficiency$.

   This means that $Q_{out}=W(1/efficiency - 1)$.

   Since the heat transferred to the working fluid is defined by its flow rate, temperature change, and heat capacity as:

   $Q_{out}=mc_p\Delta T$,

   We find $W=mc_p\Delta T/(1/efficiency-1)$.

   $W=(100 \times 10^6$ kg/min$)($min$/60$ s$)(4.184$ kJ/kg K$)(2$ °C$)/(1/0.074/0.9 -1)$

   $= 0.995$ GWe.

c. (15 points) What is the corresponding flow rate of warm ocean water?

In order to find the flow rate of the warm ocean water, we need to balance the heat rejected to the cold water, the heat rejected by the cold water, and the work done by the system.

$Q_{in}=Q_{out}+W=m_{out}c_p\Delta T+0.995\ GWe$

$Q_{in}=m_{in}c_p\Delta T$

$m_{in}=(m_{out}c_p\Delta T+0.995\ GWe)/c_p\Delta T$

$=[(100\ x\ 10^6\ kg/min)(min/60\ s)(4.184\ kJ/kg)(2\ ^{\circ}C)+0.995\ GWe)]/[(4.184\ kJ/kg\ K)(2\ ^{\circ}C)]$

$=107.134\ million\ kg/min$

d. (10 points) Friction losses arising in pumping seawater through the engine account for 0.33 of the engine's irreversibilities. Over time we can expect such friction losses to double, due to befouling in the flow circuits.   What would be the resulting value of the heat engine's efficiency?

Irreversibility is the difference between the maximum theoretical work which can be done by the thermodynamic cycle and the actual work done.

The Carnot efficiency calculated for this cycle leads us to a maximum theoretical work of

*$W_{max}=Q_{in}*0.074=m_{in}c_p\Delta T*0.074=(107.134\ x\ 10^6\ kg)(min/60s)(4.184\ kJ/kg)(2\ ^{\circ}C)$
$*0.074=1.105\ GWe.$*

The irreversibility is then 1.105 GWe-0.995 GWe=0.110 GWe.

If over time, the 33% of this value representing frictional loss doubles, we add an extra 33% to the irreversibility, for a total of 0.147 GWe lost work.

This makes the efficiency of the system (1.105 GWe-0.147 GWe)/Q_{in} =0.064.

22.081J / 2.650J / 10.291J / 1.818J / 2.65J / 10.391J / 11.371J / 22.811J / ESD.166J
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