1. If FT{g(x)} = G(k), show that FT{g(x-a)} = exp(-ika) G(k).
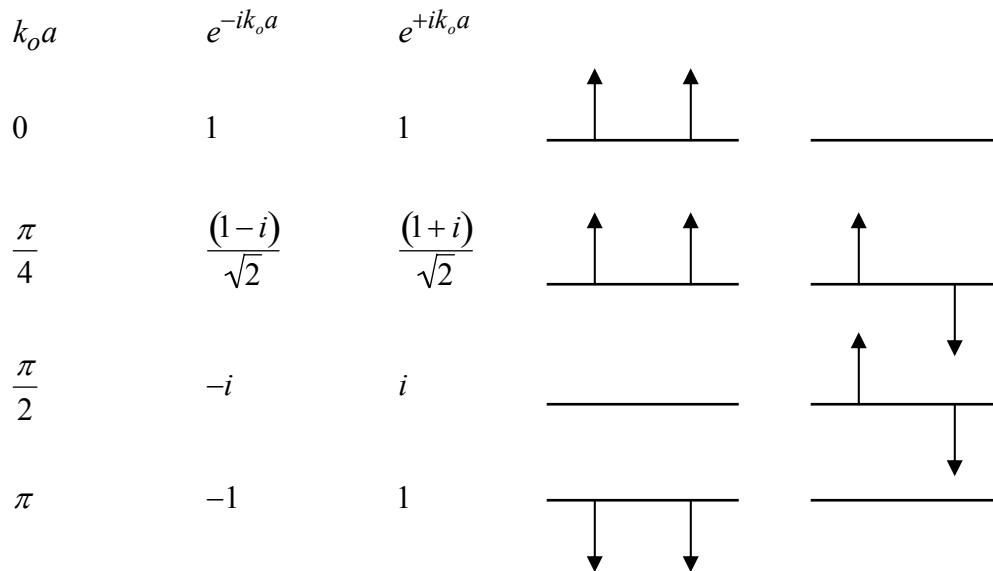
$$g(x) \Leftrightarrow G(k)$$

$$g(x-a) \Leftrightarrow \int_{-\infty}^{\infty} g(x-a)e^{-ikx}dx = e^{-ika}\int_{-\infty}^{\infty} g(x-a)e^{-i(k(x-a))}dx$$

$$= e^{-ika}G(k)$$

2. Calculate the Fourier Transform of A Cos(k_0 x + a), for a/ k_0 ={0, π/4, π/2, π}. Plot the result.

$$\cos(k_o x) \Leftrightarrow [\delta(k-k_o) + \delta(k+k_o)]$$

$$\cos(k_o x + a) \Leftrightarrow e^{-ika}[\delta(k-k_o) + \delta(k+k_o)]$$

$k_o a$	$e^{-ik_o a}$	$e^{+ik_o a}$	
0	1	1	
$\dfrac{\pi}{4}$	$\dfrac{(1-i)}{\sqrt{2}}$	$\dfrac{(1+i)}{\sqrt{2}}$	
$\dfrac{\pi}{2}$	$-i$	i	
π	-1	1	

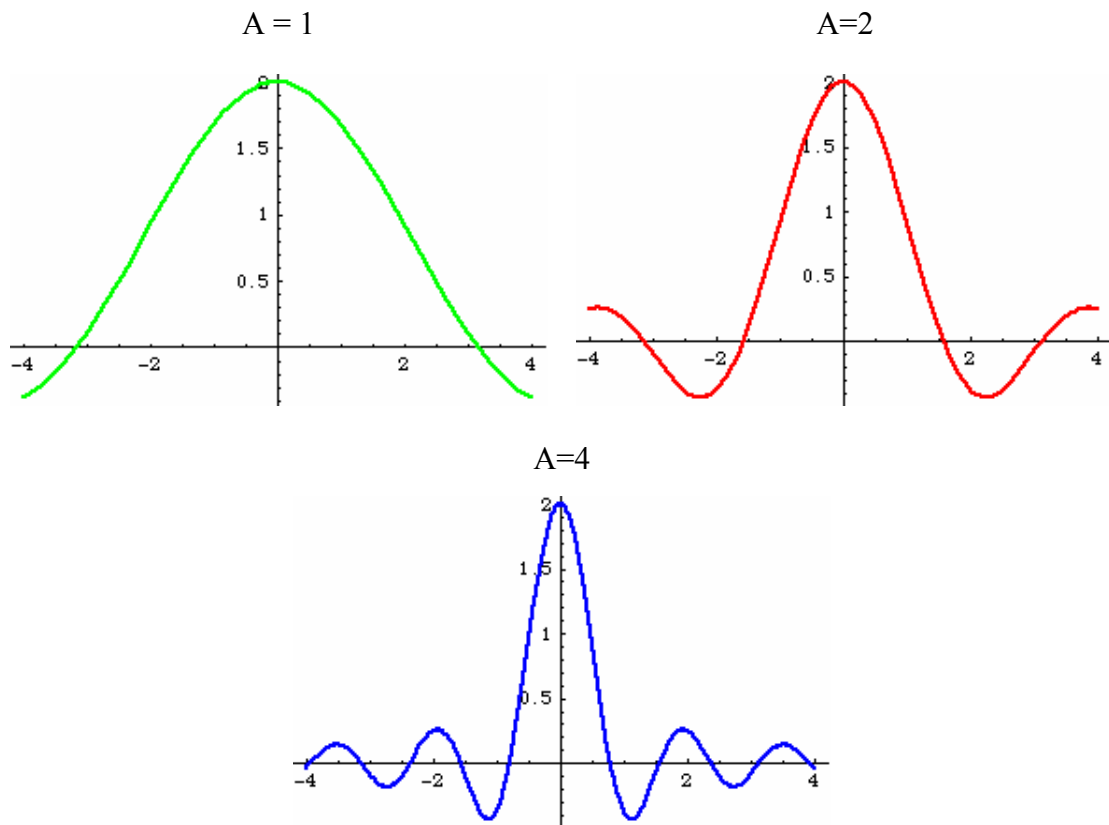3. If FT$\{g(x)\}$ = G(k), show that FT$\{g(ax)\}$ = (1/a) G(k/a).

$$g(x) \Leftrightarrow G(k)$$

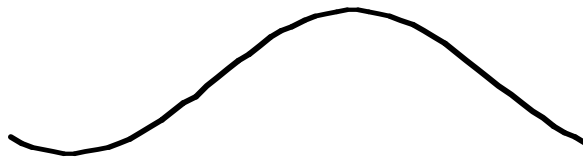$$g(ax) \Leftrightarrow \int_{-\infty}^{\infty} g(ax)e^{-ikx}\,dx = \frac{1}{a}\int_{-\infty}^{\infty} g(ax)e^{-ikax}\,dx$$

$$\Leftrightarrow \frac{1}{a}G\left(\frac{k}{a}\right)$$

4. Calculate the Fourier Transform of (A) TopHat(x/A), for A={1, 2, 4}. Plot the result.

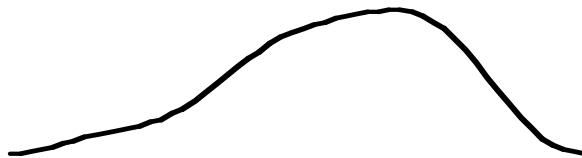$$A\ TopHat\left(\frac{x}{A}\right) \Leftrightarrow 2\,\mathrm{sinc}(kA)$$

A = 1

A=2

A=4

5. Approximate the shape of a TopHat function from the Fourier Transform given in 4. Use 4, 8, 16, and 32 Fourier components in your approximation. Plot the result. You can calculate this in Matlab or Mathematica.
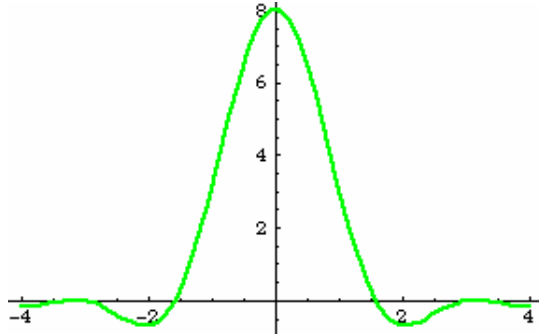

8 Fourier Components

16 Fourier Components

32 Fourier Components

6. Calculate in real space the convolution of TopHat(x) with TopHat(x/2).

7. Repeat the calculation in 6 using Fourier convolution.

$$\underbrace{TopHat(x)}_{\updownarrow} \quad \otimes \quad \underbrace{TopHat\left(\frac{x}{2}\right)}_{\updownarrow}$$

$$2sinc(k) \quad \otimes \quad 4sinc(2k)$$
$$= 8sinc(k)sinc(2k)$$

8. Calculate the convolution of TopHat(x/a) with Comb(x/b) for 2a<b, 2a>b, 2a=b.

9. Define the Instrument Response Function and the Point Spread Function and explain their differences.

The instrument response function describes the true mapping of every point in the source (x_s, y_s) to every point in the detector (x_D, y_D), $IRF(x_D, y_D | x_s, y_s)$

The point spread function describes the average broadening of any point in the source when mapped onto the detector, $P(x_D - x_s, y_D - y_s)$.

The definition of a point spread function permits the imaging process to be described as a convolution, $Image = Object \otimes PSF + noise$

10. What is the relationship between the modulation transfer function, the optical transfer function and the point spread function? How is the system resolution defined?

$OTF \qquad = F\{PSF\}$

$MTF \qquad = |OTF|$

$resolution \qquad = full\ width\ at\ half\ maximum\ of\ the\ point\ spread\ function.$