MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.717J/MAS.857J Optical Engineering **Spring '02**
Problem Set #4   Posted Mar. 16, 2002 — Due <u>Wednesday Mar. 27, 2002</u>

1. Goodman 3-5.

2. A space-domain linear, shift-invariant system has impulse response

$$h(x) = \text{rect}\left(\frac{x}{a}\right).$$

   The system is driven by white noise. Find the autocorrelation function of the output process.

3. Consider the random process

$$X(t) = a e^{j(\Omega t - \Phi)},$$

   where a is a fixed amplitude, the frequency Ω is a random variable with probability density function $p_\Omega(\omega)$, and the phase delay Φ is independent of Ω and uniform in the interval $(-\pi, \pi)$. Show that $X(t)$ is wide-sense stationary with zero mean and power spectrum

$$\mathcal{G}_X(\nu) = 2\pi a^2 p_\Omega\left(2\pi\nu\right).$$

4. A harmonic source of electromagnetic waves, emitting signal $e^{j\omega_0 t}$, is at distance r_0 from a stationary observer at $t = 0$. The oscillation frequency ω_0 is a deterministic constant. The source moves with constant but random velocity V relative to the observer. The random variable V has probability density function $p_V(v)$. The observer receives a Doppler-shifted signal

$$S(t) = a \exp\left\{j\omega_0\left(t - \frac{r_0 + Vt}{c}\right)\right\}.$$

   4.a) What is the power spectrum of the received signal? (<u>Hint:</u> Use the result of the previous problem.)

   4.b) What do you conclude about the power spectrum of the light produced by a collection of gas molecules moving at random and emitting at identical frequencies?

   4.c) What is an appopriate probability density function for V for this case?
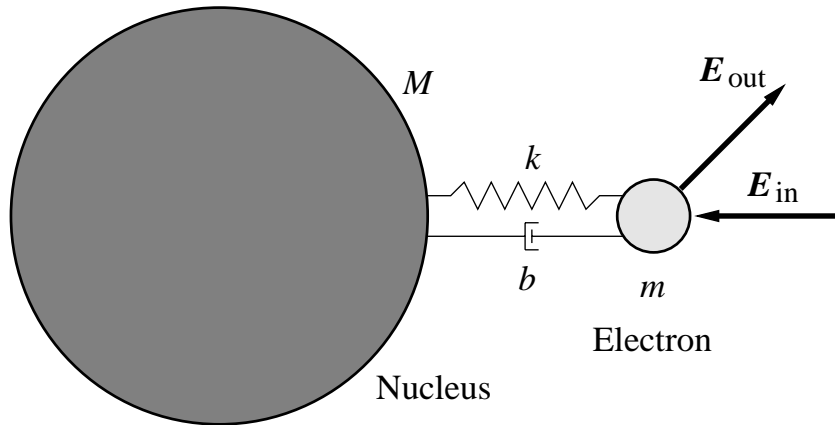
5. **Why is the sky blue?** In 1899, Lord Rayleigh observed that when we look at the sky, we see light scattered from particles in the atmosphere, primarily nitrogen. He then proposed the model shown above in order to quantify the scattering process. The figure shows an electron with mass m bound to the nucleus with a spring with spring constant k and friction coefficient b. The nucleus has mass $M \gg m$. A force is applied to the electron due to the electric field of the incident sunlight. The scattered field is then proportional to the acceleration of the electron.

5.a) Formulate a one-dimensional model for the scattering process described above. (<u>Hint:</u> model the nucleus as immobile.)

5.b) Assuming that the power spectral density of sunlight is pretty much constant over the entire electromagnetic spectrum, derive an expression for the power spectrum of the scattered light.

5.c) Explain the blue color of the sky given that the spring constant for nitrogen is $k \approx 140 \text{N/m}$ and the mass of the electron is $m = 9.11 \times 10^{-31} \text{kg}$.