

First Law

Sept 2005: changes reflect text: Woud
 Sept 2006: added examples

first law: during any cycle a system undergoes, the cyclic integral of the heat is proportional to the cyclic integral of the work

pg 83 van Wylen & Sonntag Fundamentals of Classical Thermodynamics 3rd Edition SI Version

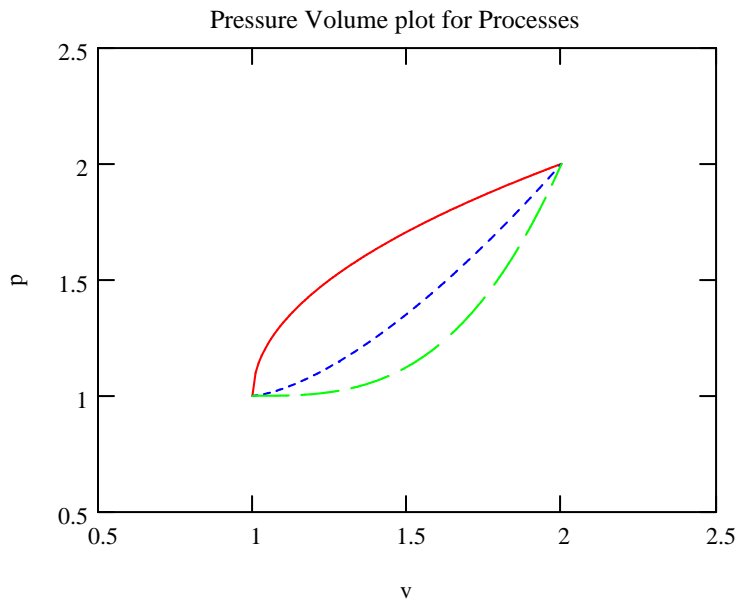
first law for cycle

$$\int 1 dQ = \int 1 dW \quad (5.2)$$

The net energy interaction between a system and its environment is zero for a cycle executed by the system.
 pg 2 Cravalho and Smith

$$\int 1 dQ - \int 1 dW = 0 \quad \text{where integral are cyclic and } dQ = \delta Q \quad dW = \delta W$$

plot data



— A process
 - - - B process
 - - - C process

$$\int 1 dQ = \int 1 dW \quad \text{apply first law to cycle A B} \quad \int_1^2 1 dQ_A + \int_2^1 1 dQ_B = \int_1^2 1 dW_A + \int_2^1 1 dW_B$$

$$\text{apply first law to cycle A C} \quad \int_1^2 1 dQ_A + \int_2^1 1 dQ_C = \int_1^2 1 dW_A + \int_2^1 1 dW_C$$

$$\text{subtract A C from A B} \quad \int_2^1 1 dQ_B - \int_2^1 1 dQ_C = \int_2^1 1 dW_B - \int_2^1 1 dW_C$$

rearrange ...

$$\int_2^1 1 dQ - \int_2^1 1 dW = \int_2^1 1 dQ - \int_2^1 1 dW \quad \text{i.e. } \delta Q - \delta W \text{ is a point function ... only dependent upon the end points } \Rightarrow \text{define as ...}$$

$$dE = \delta Q - \delta W \quad \text{energy, point function}$$

first law for system (Woud: Closed system) - change of state

N.B. Woud starts with rate equation and obtains this assuming steady state

rearrange and integrate ...

$$\delta Q = dE + \delta W$$

$$Q_{1,2} = E_2 - E_1 + W_{1,2}$$

$Q_{1,2}$

is the heat transferred TO system

E_1

E_2

are initial and final values of energy of system and ...

(5.5)

$W_{1,2}$

is work done BY the system

energy E consists of internal energy + kinetic energy + potential energy

$$E = U + KE + PE$$

$$dE = dU + dKE + dPE$$

and first law can be restated ...

$$\delta Q = dE + \delta W = dU + dKE + dPE + \delta W$$

(5.4)

Closed System

$$\frac{d}{dt}U = \dot{Q} - \dot{W}$$

$$dU = \delta Q - \delta W \quad \dot{m}_e = \dot{m}_i = 0 \quad (W 2.3)$$

difference between d and δ

d and δ : VW&S: page 62 Woud page 11

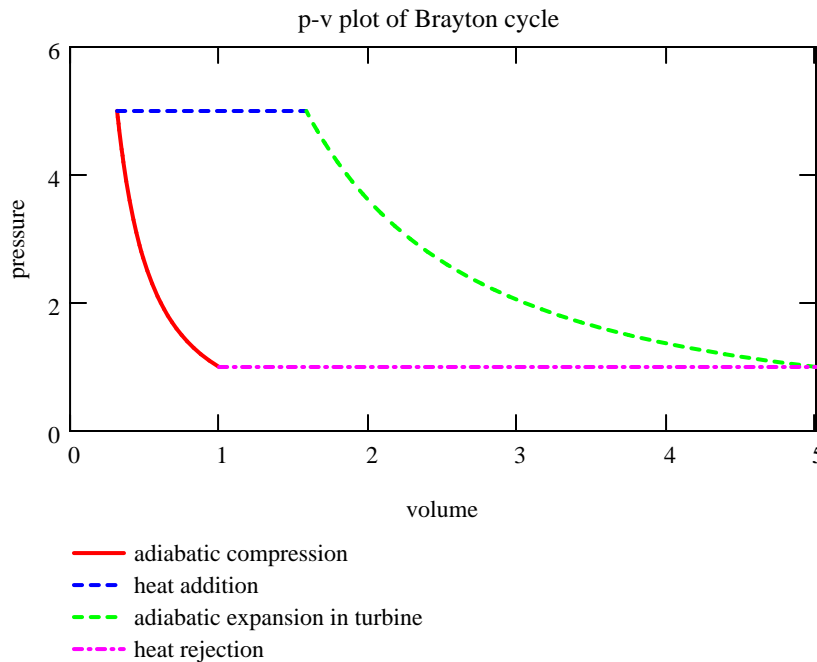
d = differential of point functions state variables

δ = differential of path functions - amount depends on path/process: diminutive

see discussion of cyclic process below

cycle may be considered a closed system; initial state and final state are identical, For example (detailed discussion later)

▶ set up limits and calculations



closed (cycle)

$$\frac{d}{dt}U = 0 = \dot{Q} - \dot{W}$$

$$\dot{Q} = \dot{W}$$

$$Q_{cycle} = W_{cycle} \quad (W 2.6)$$

this is where we started above using different approach to first law

example 5.3

Sonntag example 5.3: vessel with volume 5 m^3 contains 0.05 m^3 of saturated liquid water and 4.95 m^3 of saturated water vapor at 0.01 MPa . Heat is added until the vessel is filled with saturated vapor. Determine Q .

State 1: $V := 5 \text{ m}^3$ $V_{\text{vap}} := 4.95 \text{ m}^3$ $\text{MPa} := 10^6 \text{ Pa}$ $\text{kJ} := 10^3 \cdot \text{J}$

$V_{\text{liq}} := 0.05 \text{ m}^3$ $p := 0.1 \text{ MPa}$

constant volume and mass => constant v

$v_f := 0.001043 \frac{\text{m}^3}{\text{kg}}$

$u_f := 417.36 \frac{\text{kJ}}{\text{kg}}$

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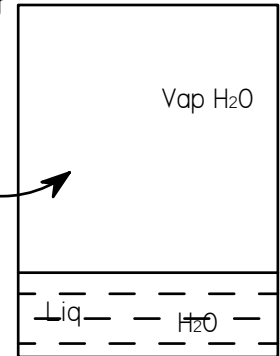
steam tables at $p = 0.1 \text{ MPa}$

$v_g := 1.694 \frac{\text{m}^3}{\text{kg}}$

$u_g := 2506.1 \frac{\text{kJ}}{\text{kg}}$

$u_{fg} := 2088.7 \frac{\text{kJ}}{\text{kg}}$

Q_2



State 2: $V_2 = V$ $v = v_g$ $u = u_g$

first law: $Q_{1,2} = E_2 - E_1 + W_{1,2}$ $W_{1,2} = 0$ $Q_{1,2} = U_2 - U_1$ $\Delta V = 0$ $\Delta z = 0$ $E = U$

have volume, determine mass of each $V_n = m_n \cdot v_n$ $\text{mass}_f = \frac{V_f}{v_f}$

$\text{mass}_{1_liq} := \frac{V_{liq}}{v_f}$ $\text{mass}_{1_liq} = 47.9386 \text{ kg}$ $\text{mass}_{1_vap} := \frac{V_{vap}}{v_g}$ $\text{mass}_{1_vap} = 2.9221 \text{ kg}$

$x = \text{quality}(\text{of_steam})$ $x_1 := \frac{\text{mass}_{1_vap}}{\text{mass}_{1_vap} + \text{mass}_{1_liq}}$ $x_1 = 0.0575$ $x\%_1 := x_1 \cdot 100$ $x\%_1 = 5.7453$

$U_1 := \text{mass}_{1_liq} \cdot u_f + \text{mass}_{1_vap} \cdot u_g$ $U_1 = 2.7331 \times 10^4 \text{ kJ}$

intensive property = not dependent on mass (x, v, u, ρ)
extensive does (U, V, mass)

or ... using average specific properties

$u_1 := u_f + x_1 \cdot u_{fg}$ $u_1 = 537.3611 \frac{\text{kJ}}{\text{kg}}$ $U_{11} := u_1 \cdot (\text{mass}_{1_vap} + \text{mass}_{1_liq})$ $U_{11} = 2.7331 \times 10^4 \text{ kJ}$

which can be shown by ...

$U_1 = \text{mass}_{1_liq} \cdot u_f + \text{mass}_{1_vap} \cdot u_g$ $u_g = u_f + u_{fg}$ $u_{fg} = 2.0887 \times 10^3 \frac{\text{kJ}}{\text{kg}}$ $u_g - u_f = 2.0887 \times 10^3 \frac{\text{kJ}}{\text{kg}}$

$u_{1a} = \frac{\text{mass}_{1_liq} \cdot u_f + \text{mass}_{1_vap} \cdot u_g}{\text{mass}_{1_liq} + \text{mass}_{1_vap}}$ substitute for u_g

$\frac{\text{mass}_{1_liq} \cdot u_f + \text{mass}_{1_vap} \cdot (u_f + u_{fg})}{\text{mass}_{1_liq} + \text{mass}_{1_vap}} = \frac{(\text{mass}_{1_liq} + \text{mass}_{1_vap}) \cdot u_f + \text{mass}_{1_vap} \cdot u_{fg}}{\text{mass}_{1_liq} + \text{mass}_{1_vap}} = u_f + x_1 \cdot u_{fg}$

state 2: need 2 properties, e.g. quality = 100%, can calculate v (specific volume). T and p will be rising as heat is added.

$$\text{mass_total} := \text{mass}_{1_liq} + \text{mass}_{1_vap}$$

$$v_2 := \frac{V}{\text{mass_total}} \quad v_2 = 0.0983 \frac{\text{m}^3}{\text{kg}} \quad \text{and ... is saturated vapor so we need to look up (interpolate) steam tables for } v_g = v_2$$

$$\text{my_interp}(x_2, x_1, y_2, y_1, vx) := y_1 + \frac{vx - x_1}{x_2 - x_1} \cdot (y_2 - y_1) \quad \begin{array}{l} \text{an interpolation statement} \\ v \text{ is value between } x_1 \text{ and } x_2 \\ \text{result is } y \text{ at } v \end{array}$$

could use mcd function linterp(vx,vy,x) where:

vx is a vector of real data values in ascending order.

vy is a vector of real data values having the same number of elements as vx.

x is the value of the independent variable at which to interpolate a result. For best results, this should be in the range encompassed by the values of vx.

using Table A.1.1	T	v_g	u_g	p_g
values at 1	$T_1 := 210$	$v_{1g} := 0.10441 \frac{\text{m}^3}{\text{kg}}$	$u_{1g} := 2599.5 \frac{\text{kJ}}{\text{kg}}$	$p_{1g} := 1.9062 \text{MPa}$
values at 2	$T_2 := 215$	$v_{2g} := 0.09479 \frac{\text{m}^3}{\text{kg}}$	$u_{2g} := 2601.1 \frac{\text{kJ}}{\text{kg}}$	$p_{2g} := 2.104 \text{MPa}$

interpolated values at

$$vx := v_2$$

$$T_2 := \text{my_interp}(v_{2g}, v_{1g}, T_2, T_1, vx) \quad T_2 = 213.1717$$

$$p_2 := \text{my_interp}(v_{2g}, v_{1g}, p_{2g}, p_{1g}, vx) \quad p_2 = 2.0317 \text{MPa}$$

$$u_2 := \text{my_interp}(v_{2g}, v_{1g}, u_{2g}, u_{1g}, vx) \quad u_2 = 2.6005 \times 10^3 \frac{\text{kJ}}{\text{kg}}$$

$$\text{total internal energy at state 2:} \quad U_2 := \text{mass_total} \cdot u_2 \quad U_2 = 1.3226 \times 10^5 \text{kJ}$$

$$\text{heat added ...} \quad Q_{1_2} := U_2 - U_1 \quad Q_{1_2} = 104933 \text{kJ}$$

same result can be obtained using Table A.1.2 Pressure Tables

$$\text{my_interp}(x_2, x_1, y_2, y_1, vx) := y_1 + \frac{vx - x_1}{x_2 - x_1} \cdot (y_2 - y_1)$$

	p	v_g	u_g	T
values at 1	$p_{1g} := 2.0 \text{MPa}$	$v_{1g} := 0.09963 \frac{\text{m}^3}{\text{kg}}$	$u_{1g} := 2600.3 \frac{\text{kJ}}{\text{kg}}$	$T_1 := 212.42$
values at 2	$p_{2g} := 2.25 \text{MPa}$	$v_{2g} := 0.08875 \frac{\text{m}^3}{\text{kg}}$	$u_{2g} := 2602.0 \frac{\text{kJ}}{\text{kg}}$	$T_2 := 218.45$

interpolated values at

$$v_x := v_2$$

$$T_2 := \text{my_interp}(v_{2g}, v_{1g}, T_2, T_1, v_x) \quad T_2 = 213.1529$$

$$p_2 := \text{my_interp}(v_{2g}, v_{1g}, p_{2g}, p_{1g}, v_x) \quad p_2 = 2.0304 \text{ MPa}$$

$$u_2 := \text{my_interp}(v_{2g}, v_{1g}, u_{2g}, u_{1g}, v_x) \quad u_2 = 2.6005 \times 10^3 \frac{\text{kJ}}{\text{kg}}$$

total internal energy at state 2: $U_2 := \text{mass_total} \cdot u_2 \quad U_2 = 1.3226 \times 10^5 \text{ kJ}$

heat added ... $Q_{1_2} := U_2 - U_1 \quad Q_{1_2} = 104933 \text{ kJ}$

example 5.3

first law as a rate equation

from above ... $Q_{1_2} = E_2 - E_1 + W_{1_2} = U_2 - U_1 + KE_2 - KE_1 + PE_2 - PE_1 + W_{1_2}$

$\delta Q = \delta U + \delta KE + \delta PE + \delta W$ in small time interval δt

$\frac{\delta Q}{\delta t} = \frac{\delta U}{\delta t} + \frac{\delta KE}{\delta t} + \frac{\delta PE}{\delta t} + \frac{\delta W}{\delta t}$ divide by δt

first law as a rate equation

$$\frac{d}{dt} Q = \frac{d}{dt} U + \frac{d}{dt} KE + \frac{d}{dt} PE + \frac{d}{dt} W = \frac{d}{dt} E + \frac{d}{dt} W \quad (5.31 \text{ and } 5.32)$$

first law as a rate equation - for a control volume (Woud: system boundary)

$$Q_{1_2} = E_2 - E_1 + W_{1_2} \quad \Rightarrow \quad \frac{\delta Q}{\delta t} = \frac{E_2 - E_1}{\delta t} + \delta W \quad (5.38)$$

E_t = energy in control volume at time t

$E_{t+\delta t}$ = energy in control volume at time $t + dt$

$E_1 := E_t + e_i \cdot \delta m_i$ = the energy of the system at time t

$E_2 := E_{t+\delta t} + e_e \cdot \delta m_e$ = the energy of the system at time $t + dt$

system consists of control volume and differential entities δm_i each with e_i, v_i, T_i, p_i where $i = \text{input}$ and differential entities δm_e each with e_e, v_e, T_e, p_e where $e = \text{output}$

$$E_2 - E_1 \rightarrow E_{t+\delta t} + e_e \cdot \delta m_e - E_t - e_i \cdot \delta m_i \quad E_2 - E_1 = E_{t+\delta t} - E_t + e_e \cdot \delta m_e - e_i \cdot \delta m_i \quad (5.39)$$

$e_e \cdot \delta m_e - e_i \cdot \delta m_i$ represents flow of energy across boundary during δt as a result of δm_i and δm_e crossing the control surface

now consider work associated with masses δm_i and δm_e

work done ON mass δm_i is ... $p_i \cdot v_i \cdot \delta m_i$ ON as work must be done to make it enter system

work done BY mass δm_e is ... $p_e \cdot v_e \cdot \delta m_e$ BY as leaving represents work done

work done BY system in δt is then ... $\delta W + p_e \cdot v_e \cdot \delta m_e - p_i \cdot v_i \cdot \delta m_i$ (5.41)

divide by δt and substitute into first law ... (5.38) and combining and rearranging $\frac{\delta Q}{\delta t} = \frac{E_2 - E_1}{\delta t} + \delta W$ (5.38)

$$\frac{\delta Q}{\delta t} + \frac{\delta m_i}{\delta t} \cdot (e_i + p_i \cdot v_i) = \frac{E_{t_2} - E_{t_1}}{\delta t} + \frac{\delta m_e}{\delta t} \cdot (e_e + p_e \cdot v_e) + \frac{\delta W_{c_v}}{\delta t} \quad (5.42)$$

$$e + p \cdot v = u + pv + \frac{V^2}{2} + g \cdot z = h + \frac{V^2}{2} + g \cdot z \quad (5.43)$$

$$H = U + p \cdot V$$

enthalpy defined - is a property (5.12)

$$h = u + p \cdot v$$

therefore ...

$$\frac{\delta Q}{\delta t} + \frac{\delta m_i}{\delta t} \cdot \left(h_i + \frac{V_i^2}{2} + g \cdot z_i \right) = \frac{E_{t_2} - E_{t_1}}{\delta t} + \frac{\delta m_e}{\delta t} \cdot \left(h_e + \frac{V_e^2}{2} + g \cdot z_e \right) + \frac{\delta W_{c_v}}{\delta t} \quad (5.44)$$

δm_i and δm_e are summed over all inputs and outputs ...

example 5.4

Sonntag example 5.4 a cylinder fitted with a piston has volume 0.1m³ and contains 0.5 kg steam at 0.4 MPa. Heat is transferred until the temperature is 300 deg_C while pressure is constant

What are the heat and work for this process?

$V_1 := 0.1 \text{ m}^3$ $m_{\text{tot}} := 0.5 \text{ kg}$ I think by definition, steam = water + vapor at quality = x

$T_2 := 300 \text{ deg_C}$ $p := 0.4 \text{ MPa}$ $\Rightarrow v_1 := \frac{V_1}{m_{\text{tot}}}$ $v_1 = 0.2 \frac{\text{m}^3}{\text{kg}}$

quality = $\frac{\text{mass_of_vapor}}{\text{total_mass}} = x$ an intensive property $1 - x = \frac{\text{mass_of_liquid}}{\text{total_mass}}$

$v_f := 0.001084 \frac{\text{m}^3}{\text{kg}}$ $v_g := 0.4625 \frac{\text{m}^3}{\text{kg}}$ $v_{fg} := v_g - v_f$ $v_{fg} = 0.4614 \frac{\text{m}^3}{\text{kg}}$

$v_1 = v_f + x \cdot v_{fg}$ $x := \frac{v_1 - v_f}{v_{fg}}$ $x = 0.4311$

constant pressure

$Q_{1_2} = E_2 - E_1 + W_{1_2}$ $W_{1_2} = p \cdot (V_2 - V_1) = p \cdot m_{\text{tot}} \cdot (v_2 - v_1)$

$E_2 - E_1 = U_2 - U_1$ as ΔV^2 and $\Delta z = 0$

$$E_2 - E_1 = U_2 - U_1 = m_{\text{tot}} \cdot (u_2 - u_1)$$

$$Q_{1-2} = E_2 - E_1 + W_{1-2} = m_{\text{tot}} \cdot (u_2 - u_1) + p \cdot m_{\text{tot}} \cdot (v_2 - v_1) = m_{\text{tot}} [u_2 + p \cdot v_2 - (u_1 + p \cdot v_1)]$$

$$h_f := 604.74 \frac{\text{kJ}}{\text{kg}} \quad h_{fg} := 2133.8 \frac{\text{kJ}}{\text{kg}} \quad h_1 := h_f + x \cdot h_{fg} \quad h_1 = 1.5246 \times 10^3 \frac{\text{kJ}}{\text{kg}} \quad h_2 := 3066.8 \frac{\text{kJ}}{\text{kg}}$$

$$Q_{1-2} := m_{\text{tot}} \cdot (h_2 - h_1)$$

$$Q_{1-2} = 771.0904 \text{ kJ}$$

Table A.1.2 Saturated
Steam Pressure Table

$$v_2 := 0.6548 \frac{\text{m}^3}{\text{kg}} \quad \text{Table A.1.2 Saturated Steam Pressure Table}$$

$$v_1 = 0.2 \frac{\text{m}^3}{\text{kg}}$$

$$W_{1-2} := p \cdot m_{\text{tot}} \cdot (v_2 - v_1)$$

$$W_{1-2} = 90.96 \text{ kJ}$$

$$Q_{1-2} = E_2 - E_1 + W_{1-2} = U_2 - U_1 + W_{1-2}$$

$$U_2 - U_1 = Q_{1-2} - W_{1-2} = Q_{1-2} - p \cdot m_{\text{tot}} \cdot (v_2 - v_1)$$

$$\Delta U := Q_{1-2} - p \cdot m_{\text{tot}} \cdot (v_2 - v_1)$$

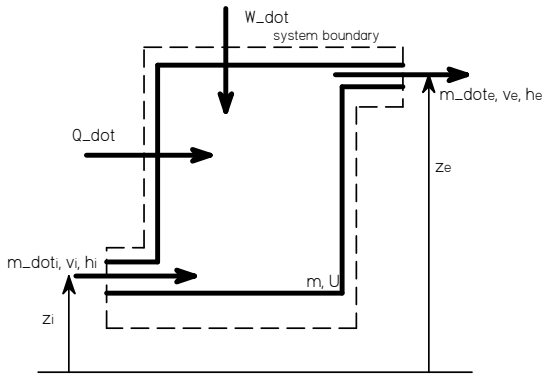
$$\Delta U = 680.1304 \text{ kJ}$$

example 5.4

first law as a rate equation - for a control volume

$$\frac{d}{dt}Q_{c_v} + \sum_n \left[m_{dot_i} \cdot \left(h_i + \frac{V_i^2}{2} + g \cdot z_i \right) \right] = \frac{d}{dt}E_{c_v} + \sum_n \left[m_{dot_e} \cdot \left(h_e + \frac{V_e^2}{2} + g \cdot z_e \right) \right] + \frac{d}{dt}W_{c_v} \quad (5.45)$$

this is where Woud starts



- Q_dot = heat_flow
- W_dot = work_flow
- m_dot = mass_flow
- v = velocity
- h = specific_enthalpy
- z = elevation
- i = inlet
- e = exit
- m = mass_system
- U = internal_energy_system

First law: change of energy within the system equals the heat flow into the system, minus the work flow delivered by the system, plus the difference in the enthalpy, H, kinetic energy \$E_{kin}\$ and potential energy \$E_{pot}\$ of the entering and exiting mass flows.

assuming energy $E = U + E_{kin} + E_{pot}$ and ... $E_{kin} = E_{pot} = 0$ $E = U$

$$\frac{d}{dt}U = Q_{dot} - W_{dot} + m_{dot_i} \cdot \left(h_i + \frac{V_i^2}{2} + g \cdot z_i \right) - m_{dot_e} \cdot \left(h_e + \frac{V_e^2}{2} + g \cdot z_e \right) \quad \text{N.B. dot } \Rightarrow \text{rate not } d(\)/dt$$

W (2.1)

enthalpy = $H = U + p \cdot V$ enthalpy = $h = U + p \cdot v$ a defined property

steady state, steady flow process ... Woud: open systems steady state (stationary)

assumptions ...

1. control volume does not move relative to the coordinate frame
2. the mass in the control volume does not vary with time
3. the mass flux and the state of mass at each discrete area of flow on the control surface do not vary with time and .. the rates at which heat and work cross the control surface remain constant.

$$\frac{d}{dt}m_{c_v} = 0 \quad \frac{d}{dt}E_{c_v} = 0$$

steady state, steady flow process ...

$m_{\dot{}} = \text{flow_rate}$

$$\sum_n m_{\dot{i}_n} = \sum_n m_{\dot{e}_n} \quad (5.46)$$

$$\frac{d}{dt} Q_{c_v} + \sum_n \left[m_{\dot{i}_n} \cdot \left(h_i + \frac{V_i^2}{2} + g \cdot z_i \right) \right] = \sum_n \left[m_{\dot{e}_n} \cdot \left(h_e + \frac{V_e^2}{2} + g \cdot z_e \right) \right] + \frac{d}{dt} W_{c_v} \quad (5.47)$$

$$W_{\dot{}} = Q_{\dot{}} + m_{\dot{}} \cdot \left[h_i - h_e + \frac{V_i^2 - V_e^2}{2} + g \cdot (z_i - z_e) \right]$$

(W 2.8) see text for examples of application to steam turbine, boiler or heat exchanger, nozzle and throttle

$$q = \frac{\frac{d}{dt} Q_{c_v}}{m_{\dot{}}}$$

$$w = \frac{\frac{d}{dt} W_{c_v}}{m_{\dot{}}}$$

are heat transfer and work per unit mass flow (5.51)

steady state steady flow ... - single flow stream

$$q + h_i + \frac{V_i^2}{2} + g \cdot z_i = h_e + \frac{V_e^2}{2} + g \cdot z_e + w \quad (5.50)$$

uniform state, uniform flow process USUF

1. control volume remains constant relative to the coordinate frame
2. state of mass within the control volume may change with time, but at any instant of time is uniform throughout the entire control volume - I define this as $f(t)$ but not of space
3. the state of mass crossing each of the areas of flow on the control surface is constant with time although the mass flow rates may be changing

at time t, continuity equation ...

$$\frac{d}{dt} m_{c_v} + \sum_n m_{\dot{e}_n} - \sum_n m_{\dot{i}_n} = 0$$

integrating over time gives change in mass of the control volume

$$\int_0^t \frac{d}{dt} m_{c_v} dt = m_{2_{c_v}} - m_{1_{c_v}}$$

mass entering and leaving

$$\int_0^t \sum_n m_{\dot{i}_n} dt = \sum_n m_{i_n}$$

$$\int_0^t \sum_n m_{\dot{e}_n} dt = \sum_n m_{e_n}$$

continuity for USUF process ...

$$m_{2_c_v} - m_{1_c_v} + \sum_n m_{e_n} - \sum_n m_{i_n} = 0 \quad (5.53)$$

apply first law at time t (5.45)

$$\frac{d}{dt} Q_{c_v} + \sum_n \left[m_{\dot{i}} \cdot \left(h_i + \frac{V_i^2}{2} + g \cdot z_i \right) \right] = \frac{d}{dt} E_{c_v} + \sum_n \left[m_{\dot{e}} \cdot \left(h_e + \frac{V_e^2}{2} + g \cdot z_e \right) \right] + \frac{d}{dt} W_{c_v} \quad (5.45)$$

since at time t c.v. is uniform ...

$$\begin{aligned} \frac{d}{dt} Q_{c_v} + \sum_n \left[m_{\dot{i}} \cdot \left(h_i + \frac{V_i^2}{2} + g \cdot z_i \right) \right] &= \frac{d}{dt} \left[m \cdot \left(u_{c_v} + \frac{V_{c_v}^2}{2} + g \cdot z_{c_v} \right) \right] \dots \\ &+ \sum_n \left[m_{\dot{e}} \cdot \left(h_e + \frac{V_e^2}{2} + g \cdot z_e \right) \right] + \frac{d}{dt} W_{c_v} \end{aligned}$$

now integrate over time t

$$\begin{aligned} \int_0^t \frac{d}{dt} Q_{c_v} dt &= Q_{c_v} & \int_0^t \sum_n \left[m_{\dot{i}} \cdot \left(h_i + \frac{V_i^2}{2} + g \cdot z_i \right) \right] dt &= \sum_n \left[m_i \cdot \left(h_i + \frac{V_i^2}{2} + g \cdot z_i \right) \right] \\ \int_0^t \frac{d}{dt} \left[m \cdot \left(u_{c_v} + \frac{V_{c_v}^2}{2} + g \cdot z_{c_v} \right) \right] dt &= m_2 \cdot \left(u_2 + \frac{V_2^2}{2} + g \cdot z_2 \right) - m_1 \cdot \left(u_1 + \frac{V_1^2}{2} + g \cdot z_1 \right) \\ \int_0^t \sum_n \left[m_{\dot{e}} \cdot \left(h_e + \frac{V_e^2}{2} + g \cdot z_e \right) \right] dt &= \sum_n \left[m_e \cdot \left(h_e + \frac{V_e^2}{2} + g \cdot z_e \right) \right] & \int_0^t \frac{d}{dt} W_{c_v} dt &= W_{c_v} \end{aligned}$$

uniform state, uniform flow process USUF

$$\begin{aligned} Q_{c_v} + \sum_n \left[m_{i_n} \cdot \left(h_i + \frac{V_i^2}{2} + g \cdot z_i \right) \right] &= \sum_n \left[m_{e_n} \cdot \left(h_e + \frac{V_e^2}{2} + g \cdot z_e \right) \right] \dots \\ &+ m_2 \cdot \left(u_2 + \frac{V_2^2}{2} + g \cdot z_2 \right) - m_1 \cdot \left(u_1 + \frac{V_1^2}{2} + g \cdot z_1 \right) + W_{c_v} \end{aligned} \quad (5.54)$$