0.2.1 Komlós Conjecture

We start with a fascinating problem in Discrepancy Theory.

Open Problem 0.1 (Komlós Conjecture) *Given n, let $K(n)$ denote the infimum over all real numbers such that: for all set of n vectors $u_1, \ldots, u_n \in \mathbb{R}^n$ satisfying $\|u_i\|_2 \leq 1$, there exist signs $\epsilon_i = \pm 1$ such that*

$$\|\epsilon_1 u_1 + \epsilon_2 u_2 + \cdots + \epsilon_n u_n\|_\infty \leq K(n).$$

There exists a universal constant K such that $K(n) \leq K$ for all n.

An early reference for this conjecture is a book by Joel Spencer [Spe94]. This conjecture is tightly connected to Spencer's famous *Six Standard Deviations Suffice* Theorem [Spe85]. Later in the course we will study semidefinite programming relaxations, recently it was shown that a certain semidefinite relaxation of this conjecture holds [Nik13], the same paper also has a good accounting of partial progress on the conjecture.

- It is not so difficult to show that $K(n) \leq \sqrt{n}$, **try it!**

References

[Nik13] A. Nikolov. The komlos conjecture holds for vector colorings. *Available online at arXiv:1301.4039 [math.CO]*, 2013.

[Spe85] J. Spencer. Six standard deviations suffice. *Trans. Amer. Math. Soc.*, (289), 1985.

[Spe94] J. Spencer. *Ten Lectures on the Probabilistic Method: Second Edition.* SIAM, 1994.

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