18.950 Differential Geometry
Fall 2008

18.950 Homework 6

1. (6 points) Work out in detail the "hump reversal" method discussed in Wednesday's lecture; see also textbook p. 157, Remark 4.25(ii). Verify that the resulting surfaces really have the same first fundamental form, but generally different second fundamental forms.

2. (8 points) Let $f : U \to \mathbb{R}^3$ be a surface. Suppose that its first fundamental form satisfies $g_{12}(x) = 0$, $g_{11}(x) = g_{22}(x) = e^{\psi(x)}$. Work out the Gauss curvature. Check your results against surfaces of the form $f(x) = (f_1(x), f_2(x), 0)$, where (f_1, f_2) satisfy the Cauchy-Riemann equations

$$\partial_1 f_1 = \partial_2 f_2, \quad \partial_1 f_2 = -\partial_2 f_1.$$

3. (3 points) Write down another surface of revolution (besides the pseudosphere considered in class) which has $\kappa_{gauss} = -1$ everywhere.

4. (3 points) Take a hypersurface patch f, and define $\tilde{f}(x_1, \ldots, x_n) = f(x_1, \ldots, x_{n-1}, -x_n)$. How are the mean curvature, the scalar curvature, and the Gauss curvature of f related to those of $\tilde{f}$?