HOMEWORK 3

DUE: MONDAY, FEB 28

1. Let X be an arbitrary pointed space, and let $X_w = X \cup_{\{0\}} I$ have basepoint $1 \in I$. Show that
(a) X is a deformation retract of X_w.
(b) X_w is well-pointed.

2. For a CW pair (X, A), is there an isomorphism $\pi_*(X, A) \cong \pi_*(X/A)$? Justify your answer.

3. Compute the homotopy groups of the quasi-circle (the "circle" containing $\sin(1/x)$ defined on p79, problem 7 of section 1.3 of Hatcher). Deduce that the inclusion of a point on the quasicircle is a weak equivalence. Show the inclusion is not a homotopy equivalence.

4. Show that if $A \hookrightarrow X$ is a cofibration, then $A \times Y \hookrightarrow X \times Y$ is a cofibration.

5. Suppose that $A \hookrightarrow X$ is a cofibration. Show that the inclusion

$$X \times S^{n-1} \cup_{A \times S^{n-1}} A \times D^n \hookrightarrow X \times D^n$$

is a cofibration.

6. Suppose that $A \hookrightarrow X$ is a cofibration.
(a) Show that the canonical map $\mathrm{Cone}(i) \to X/A$ is a homotopy equivalence. Here $\mathrm{Cone}(i)$ is the unreduced mapping cone.
(b) Deduce that there is an isomorphism $H^*(X, A) \cong \widetilde{H}^*(X/A)$.

7. Show that if X is well pointed, then the quotient map

$$\mathrm{Susp}(X) \to \Sigma X$$

is a homotopy equivalence. (I found problems 4,5 and 6a helpful, but they might be completely unnecessary.)