9/10/04

$X, Y \in \mathbb{Q}$.

(1) $X^n + Y^n = 1$

(2) $x^n + y^n = z^n$

$X = \frac{a}{c}$, $Y = \frac{b}{d}$ $\gcd(a,c) = \gcd(b,d) = 1$.

can assume $c, d > 0$.

$$\frac{a^n}{c^n} + \frac{b^n}{d^n} = 1$$

$$a^n d^n + b^n c^n = c^n d^n$$

$$c \mid ad \Rightarrow c \mid d.$$

"By symmetry"

$$d \mid bc \Rightarrow d \mid c$$ (and both c,d positive)

so, $C = d$

$$\frac{a^n}{c^n} + \frac{b^n}{c^n} = 1$$

$$a^n + b^n = c^n \longrightarrow (2)$$

$$(x, y, z) = (0, 0, 0)$$

$a^n + b^n = c^n$

$(ta)^n + (tb)^n = (tc)^n \quad t \neq 0$

$t^n a^n + t^n b^n = t^n c^n.$

$x^n + y^n = 0$

$x^n = -y^n$

$x = -y$

$(1, -1, 0)$ solves (2)

$x \longrightarrow 1, \ y \longrightarrow -1, \ c \longrightarrow 0$

$X \to \frac{1}{0} \quad Y \to \frac{-1}{0}$

$X, Y \longrightarrow \infty.$

$\sim$: $[a, b, c] \sim [a', b', c']$ iff $a = ta'$, $b = tb'$, $c = tc'$

$(t \neq 0)$

$$\mathbb{P}^2 = \frac{\{ [a, b, c] \text{ s.t. } a, b, c \text{ are not all } 0 \}}{\sim}$$

$\sim$ $[a_0, a_1, \ldots, a_n] \sim [a_0', a_1', \ldots, a_n']$ iff

$a_i = t a_i' \quad \forall i$

$$\mathbb{P}^n = \frac{\{[a_0, \ldots, a_n] \text{ s.t. not all } a_i \text{ are } 0\}}{\sim}$$

<u>Lines</u> in $\mathbb{P}^2$ $[a, b, c]$ s.t. $\alpha X + \beta Y + \gamma Z = 0$

not all $\alpha, \beta, \gamma = 0$.

If $\alpha a + \beta b + \gamma c = 0$

$\alpha(ta) + \beta(tb) + \gamma(tc) = 0$.
