

7.8 QR Decomposition

$$Ax = b \tag{7.36}$$

$$QRx = b \tag{7.37}$$

$$Rx = Q^*b \tag{7.38}$$

$$\tilde{R}x = \tilde{Q}^*b \tag{7.39}$$

Therefore,

$$(\tilde{R} + \delta\tilde{R})\tilde{x} = (\tilde{Q} + \delta\tilde{Q})^*b \tag{7.40}$$

such that

(Continued on next page.)

$$\frac{\|\delta\tilde{R}\|}{\|\tilde{R}\|} = O(\epsilon_{\text{machine}}) \quad (7.41)$$

$$\|\delta\tilde{Q}\| \leq O(\epsilon_{\text{machine}}) \quad (7.42)$$

then

$$\begin{aligned} b &= (\tilde{Q} + \delta\tilde{Q})(\tilde{R} + \delta\tilde{R})\tilde{x} \\ &= (A + \delta A + \delta\tilde{Q}\tilde{R} + \tilde{Q}\delta\tilde{R} + \delta\tilde{Q}\delta\tilde{R})\tilde{x} \\ &= (Q + \Delta A)\tilde{x} \end{aligned} \quad (7.43)$$

$$\tilde{Q}\tilde{R} = A + \delta A \quad (7.44)$$

$$\frac{\|\tilde{R}\|}{\|A\|} \leq \|\tilde{Q}^*\| \frac{\|A + \delta A\|}{\|A\|} = O(1) \quad (7.45)$$

$$\frac{\|\delta\tilde{Q}\tilde{R}\|}{\|A\|} \leq \|\delta\tilde{Q}\| \frac{\|\tilde{R}\|}{\|A\|} = O(\epsilon) \quad (7.46)$$

$$\frac{\|\tilde{Q}\delta\tilde{R}\|}{\|A\|} \leq \|\tilde{Q}\| \frac{\|\delta\tilde{R}\| \cdot \|\tilde{R}\|}{\|\tilde{R}\| \cdot \|\tilde{A}\|} = O(\epsilon) \quad (7.47)$$

$$\frac{\|(\delta\tilde{Q})(\delta\tilde{R})\|}{\|A\|} \leq \|\delta\tilde{Q}\| \frac{\|\delta\tilde{R}\|}{\|A\|} = O(\epsilon^2) \quad (7.48)$$

$$\frac{\|\Delta A\|}{\|A\|} = O(\epsilon) \quad (7.49)$$

then

$$\frac{\|\tilde{x} - x\|}{\|x\|} = O(\kappa(A)\epsilon) \quad (7.50)$$