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Pak

9/30/05

(Still) Chromatic + Tutte Polynomial

Thm (Stanley)

$\chi_G(-1) = (-1)^n \cdot \#$ acyclic orientations of G (where $n = |V(G)|$)

Pf: By induction, ETST # a.o. of G = # a.o. of $G-e$ + # a.o. of G/e ✓

(b/c acyclic orientation of $G-e$ ⇒ can orient e at least one way to get a.o. G, if both ways ⇒ a.o. for G/e) ('cause if no orientation for e ⇒ ∃ cycle already)

Tutte polynomial

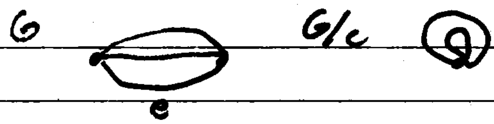
G - graph (w/ loops + multiple edges)

dichromatic, where di is classical for 2

$T_G(x,y) = T_{G-e}(x,y) + T_{G/e}(x,y)$

when e is neither bridge nor loop.

New definition of contraction by an edge ("work with me people")



If e is a bridge or loop, then

$T_G(x,y) = x\,T_{G-e}(x,y)$ e is a bridge

$T_G(x,y) = y\,T_{G-e}(x,y)$ if e is a loop

And for empty graph $T_{O_n}(x,y) = 1$

Thm $T_G(x,y) = T_{G^*}(y,x)$ where G^* = dual of G if G is planar (it all makes perfect sense if you know matroid theory)

Proposition: $T_G(x,y)$ is well-defined

Let $G=(V,E)$, $H \subset G$ $H = (V,F)$ $F \subseteq E$

Def'n $T_G(x,y) = \sum\limits_{\substack{F \subseteq E \\ \text{i.e. } H \subseteq G}} (x-1)^{c(H)-c(G)} (y-1)^{n(H)}$

where $c(H) = \#$ conn. comp. of H

$n(H) = |F| - |V| + c(H)$ "nullity"

note that this satisfies recursive def'n