GENERAL FRAMEWORK. Discrete to continuum limits.

- Densities and fluxes.
- Issues relating to the meaning of the "continuum" limit.
- EXAMPLES: Car densities and flux.
    - Interstellar media (where do super-nova shocks travel on?).
    - River density and flux.
    - Pressure in a gas. Other fluid properties.
- Grade "curve" (idealization versus reality: histograms).

EXAMPLE: Lighthill-Whitham model for Traffic Flow.

Derive the model equation $\rho_t + q_x = 0$, where q is a function of ρ.

Brief discussion of the continuum limit involved:

- What is ρ and under what assumptions one can define it.
- What is q and under what assumptions one can define it.
- Point out analogy with continuum hypothesis in fluids and solid matter. The difference between 10^{20} and $10^{1.5}$.
- Conservation of cars: Integral and differential forms.

CLOSURE PROBLEM: more unknowns than equations.
1) Quasi-equilibrium approximation leading to $q = Q(\rho)$.

Assumption: there is a stable equilibrium to which system adjusts on a time scale faster than the time scales of interest in problem. Explain how drivers, at steady state, and given a velocity, adjust to an optimal separation between cars. When changes slow enough, they have time to adjust to the optimal separation, so one can use $q = Q(\rho)$ to fill-in for the "missing" equation in the conservation of cars [one equation for two unknowns: ρ and q]. Typical time scale here is seconds [dominated not by human reaction time, but car inertia!]. Compare this with the adjustment times in thermodynamics.

2) Discuss generic form of Q. Existence of a maximum throughput $q_m = Q(\rho_m)$.

Lighthill-Whitham model for Traffic Flow.
The flux function $q = Q(\rho)$:
Discuss practical significance of the existence of $q_m = Q(\rho_m)$.
Leads to entrance lights at highway access at peak time.
Explain purpose, and limitations (no dynamics and not enough space to have cars wait till optimal ρ_m is reached).