Hour Test

The 50-minute test in class covers Sections 1.1 through 3.3 of the text by Adams and Guillemin, except Section 2.8. It also covers the notes that have been posted. Here is a rough description of the types of problems, followed by last year's test. In all cases of proofs you should be ready to state carefully not only the theorem you are proving but also the ingredients in the proof.

1. Establish the main step in the Lebesgue measure construction, that is, confirm that μ^* equals the volume measure on the rectangle ring.

2. Prove one of these convergence theorems. (I will pick one.)

   a) The monotone convergence theorem

   b) Fatou's lemma

   c) The dominated convergence theorem

3. Prove or establish a step or two in the proof of one of the following theorems.

   a) Borel-Cantelli Lemma 1 or 2

   b) Bounded measurable functions are uniform limits of simple functions.

   c) Bounded Riemann integrable functions are Lebesgue integrable with the same value.

   d) $L^1(X, \mu)$ is complete.

4 + 5. Questions in which you have to decide what's true and why. You will be given statements related to limit theorems, Fubini's theorem, or issues of integrability or measurability. Most students find these to be the trickiest questions because of the uncertainty. They are not designed to be devious, but you need to come to the test armed with examples related to where the hypotheses of the theorems do and don't work.

6. The last question is up for grabs. It may include a computation or the evaluation of a limit as an application of any of our theorems. For example,

   a) A computation with Rademacher functions.

   b) Use of $\mathbf{E}(f_1 f_2 \cdots f_n) = \mathbf{E}(f_1)\mathbf{E}(f_2) \cdots \mathbf{E}(f_n)$ for independent random variables.

   c) Computation of the probability distribution μ_f given an explicit function f.

   d) Evaluation of a limit using a convergence theorem, Fubini's theorem, or density of smooth functions in L^p functions, $1 \leq p < \infty$.

   e) Evaluation of a probability using a Borel-Cantelli lemma

Last year's test, on the next page, follows the rubric above.

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