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18.102 Introduction to Functional Analysis  
Spring 2009

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**TOPICS FOR THE FIRST TEST FOR 18.102, SPRING 2009**  
**TEST: THURSDAY 5 MARCH, 9:30-11:00.**

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Completeness of answers will be taken strong into account on the test. You are supposed to get things really right!

1. QUESTION 1

Will be on proving some property of integrable functions directly from the definition. Thus, just assuming that  $f$  is integrable (you will be asked to give the definition accurately) show that  $|f|$ ,  $\operatorname{Re} f$ ,  $\operatorname{Im} f$ ,  $(\operatorname{Re} f)_+$  (the positive part) or  $f + g$  for two integrable functions, or  $\max(f, g)$  for two real integrable functions or some such.

2. QUESTION 2

Monotonicity for step functions. I will ask you to prove one of the two basic monotonicity results for step functions – e.g. that if  $f_n$  is an increasing sequence with non-negative pointwise limit then  $\lim \int f_n \geq 0$  (including possibly  $+\infty$ .) Or the result on which this depends.

3. QUESTION 3

I suggested I might ask you to prove the continuity-in-the-mean of  $L^1$  functions. That is to show that

$$\lim_{t \rightarrow 0} \int |f_t - f| = 0$$

for any  $f \in L^1$ .

4. QUESTION 4

There probably will be no question 4, if there is, it would be about null functions.

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