Daily Assignment for Lecture #37

Let U_1, U_2 be open subsets of $\mathbb{R}^n$, and let $f : U_1 \to U_2$ be an orientation preserving diffeomorphism. Suppose that f maps $U_1 \cap \mathbb{H}^n$ onto $U_2 \cap \mathbb{H}^n$. Taking $\omega \in \Omega_c^n(U_2)$, we can write

$$\omega = \rho(x) dx_1 \wedge \cdots \wedge dx_n. \tag{0.1}$$

We know that

$$\int_{\mathbb{H}^n \cap U_2} \omega = \int_{\mathbb{H}^n \cap U_2} \rho(x) dx = \int_{\mathbb{H}^n} \rho(x) dx. \tag{0.2}$$

Taking the pullback

$$f^*\omega = \rho(f(x)) \det(Df) dx_1 \wedge \cdots \wedge dx_n, \tag{0.3}$$

we get

$$\int_{\mathbb{H}^n \cap U_1} f^*\omega = \int_{\mathbb{H}^n \cap U_1} \rho(f(x))(\det Df) dx. \tag{0.4}$$

Show that

$$\int_{\mathbb{H}^n \cap U_2} \omega = \int_{\mathbb{H}^n \cap U_1} f^*\omega. \tag{0.5}$$

Hint: Look at Exercises #1 and #2 in section 5 of the Supplementary Notes.