

18.100A Fall 2012: Assignment 6

Directions: List collaborators, don't consult solutions to problem sets from previous semesters.

Reading: Mon.: 7.1-.2, 7.4-.5 Wed.: 7.3, 7.6-.7

Infinite series; convergence tests; absolute and conditional convergence.

Problem 1. (4: 1, 1.5, 1.5)

- Work 7.2/1, citing theorems used.
- Work 7.2/2; you will need two theorems from this section.
- Work 7.2/5; again, you will need to use the definitions and more than one theorem from earlier chapters..

Problem 2. (2) Prove the ratio test for the case $L > 1$, but assume the a_n are non-negative, so absolute value signs are not required in the statement, or in its proof.

Follow the hints given in the text for this case; in particular, for full credit do not make use of a "buffer" M between L and 1 analogous to the one used in the book's proof for the case $L < 1$; only half-credit if you use an M and imitate the book's proof for $L < 1$.

Problem 3. (1: .5, .5) Using any of the tests in section 7.4 (without the absolute value signs since $a_n > 0$ in what follows), work: a) 7.4/1b b) 7.4/1e .

Problem 4. (1) In the same way as in Problem 3, work 7.4/1h, but replace 2 with an arbitrary constant $a > 0$; determine with proof for what values of a the series converges, and for what values it diverges.

(One value is to be omitted. Note the limit in the book's directions for working 7.4/1 .)

Problem 5. (2) A famous problem in number theory, proposed by Euler, but first solved by Dirichlet about 80 years later, was to determine whether the series of prime reciprocals below converges or diverges:

$$\sum \frac{1}{p_n} = \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{11} \dots ; \quad p_n = \text{the } n\text{-th prime}$$

If primes are sparsely distributed, the series should converge; if thickly, it should diverge.

Dirichlet's proof uses complex analysis and is rather roundabout; 70 years later, the harder Prime Number Theorem was finally proved after 150 years of effort: there are various equivalent forms in which it can be stated, but the simplest for the present purpose is

$$p_n \sim n \ln n ,$$

where the symbol \sim (read: "is asymptotic to" or "twiddles") has the meaning given in the Asymptotic Comparison test 7.5B.)

Use the prime number theorem to determine (with proof, and citing theorems) whether the series of reciprocal primes converges or diverges.

Problem 6. (2: .5, 1.5) This problem asks you to prove the Absolute Convergence Theorem 7.3 another way, one that works also for series of complex numbers (the proof in the book only works for reals).

- Work 7.1/2 (an exercise in translating sequence-talk into series-speak).
- Work Problem 7-5, using part (a)..

Problem 7. (1) Do 7.6/1a,b. (For conditional convergence, you have to show two things.)

Problem 8. (2: .5,1.5) Problem 7-6. (There are similar ideas in Chapter 5.)

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18.100A Introduction to Analysis
Fall 2012

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