PROFESSOR: Hi. Welcome to recitation. My name is Martina, and I'll be your recitation instructor for some of these linear algebra videos.

Today's problem is a straightforward solve the following linear system with four equations and four unknowns, using the method of elimination. The system is x minus y minus z plus u equals 0, 2x plus 2z equals 8, minus y minus 2z equals -8, and 3x minus 3y minus 2z plus 4u equals 7.

And although you might know different ways to solve the system at this point, the method of elimination is going to show up a million times during these videos, so it's really important to get it right. So I suggest you try solving this system now, using the method of elimination as it was described in class.

I'm going to leave you alone with the problem. You should pause the video, solve it yourself, and then come back and compare your solution with mine.

And we're back. So the method of elimination, if you remember it from class, consisted of replacing this system with an equivalent system-- equivalent meaning they have the same solution-- by a series of row operations. Row operations are not supposed to change the solution to the system. And they're, for example, exchange the order of two equations. Multiply an equation with a nonzero number, and add a nonzero multiple of one equation to the other. So let's do that.

As we're going to do this series of arithmetic operations, we don't really want to copy the names of the variables and the equality signs every time. So we're going to keep the important information which are these numbers. these coefficients here, we're going to keep that information in a matrix.

So let's write a matrix. Each row is going to correspond to an equation, and each column is going to correspond to an unknown.

So the first row is 1, -1, -1, 1. The second row, corresponding to the second equation, is 2, 0, 2, 0. And you want to be very careful to put 0's on the right spots here. The third equation is 0-- I haven't left myself enough room-- 0, -1, -2, 0. And the fourth row, corresponding to the fourth equation, is 3, -3, -2, 4. And as we care about the right-hand side as well, we're going to copy this information as well, and get the augmented matrix of the system. 0, 8, -8, 7.

OK, and now let's try reducing this matrix to an upper triangular one. We start with the first column, and we're going to use this number, called a pivot, to get rid of all the numbers under it, so to get 0's here and here. A way to do it is-- well, to get rid of this 2, I have to multiply the first row by -2, and add it to the second one.

Writing this here is not strictly necessary, but I like to do a bit of bookkeeping, because I'm prone to make a lot of errors while doing this simple arithmetic operations. And then if I get to the end, figure out I made a mistake somewhere, this bookkeeping makes it easier to backtrack and find the place where I made this mistake.

So we replace this matrix with another matrix. The first row stays the same. 1, -1, -1, 1, 0. The second row gets replaced by the second row minus 2 times the first row. The aim of that is to get a 0 here, so that's good.

Next, on this position here, we get -2 times -1, which is 2, plus 0 which is 2. -1 times -2, which is 2, plus 2, which is 4. -2 plus 0 which is -2. And 8 minus 2 times 0 which is 8. The third row already has a 0 here, so I can just copy it over. 0, -1, -2, 0, -8. And to get a 0 here, I'm going to multiply the first row by -3 and add it to the fourth row and get 0, -3 times -1 is 3, minus 3 is 0. -3 minus 1-- oh, sorry-- 3 minus 2 is 1, and -3 plus 4 is 1.

And there we go. The first column looks like a first column of an upper triangular matrix. Now let's do the same to the second column. This is going to be our pivot, the number that we use to get rid of numbers under it. And we see that to get rid of this number here, we will need to multiply it with 1/2. So multiply the whole second row with 1/2, and add it to the third row.

The matrix that we get will have the first row the same. It stays the same. 1, -1, -1, 1, 0. The second row stays the same. 0, 2, 4, -2, 8. The third row gets replaced by the third row plus 1/2 times the second row and becomes 0, 0, 2 minus 2 which is 0, -1 plus 0 which is -1, and 4 minus 8 which is -4. And the fourth row already has a 0 here so I just copy it over. 0, 0, 1, 1, 7.

And now let's look at this matrix. It has the first two columns as they're supposed to be, 0's under the diagonal. And now we want to get a 0 here. Normally what I would do is to circle this number here, multiply it by something so that I get a -1, and add it to this row to get a 0 here.

But that's not going to work. You might remember from lecture that 0's can never be pivots. Or you can just try finding a number such that 0 times this number equals -1, and seeing that such a number doesn't exist, because you're always going to get 0. So we can proceed as we did until now.

But is there another way to get a 0 here? There is a very simple row operation, which consists just of switching the third and the fourth row. It certainly doesn't change the solution of the system. So let's do that. And let's get the next matrix which is 1, -1, -1, 1, 0. 0, 2, 4, -2, 8. Then we put the fourth row here. 0, 0, 1, 1, 7. And we put the third row here, 0, 0, 0, -1, -4. And there it is. This is an upper triangular matrix.

So in the same way as at the beginning, we had a system and then wrote the matrix representing it, this matrix also represents a system. And this system has the same solutions as the initial system but is much easier to solve. Now let's write this back as a system, and let me do that not starting from the first equation, but starting from the last equation. So the last equation here reads -u equals -4, which is, as equations go, a pretty easy one to solve. The solution is u equals 4.

Now let's go back to the third equation. The third equation reads y-- no, I'm sorry-- z-- the third column corresponds to z-- plus u equals 7. But we know what's u now. So, it reads z plus 4 equals 7, which just becomes z equals 3.

The second equation is 2 times y plus 4 times z, but z is 3, minus 2 times u, but u is 4, equals 8. And from this, one can easily compute that y is equal to 2.

And finally, the first equation reads x minus y minus z plus u equals 0, or x equals 1. And this is our solution. x, y, z, and u equal 1, 2, 3, and 4.

This finishes the problem, but I would very strongly encourage you now to take this solution and plug it back into the original system and check if it's really a solution. And that's all I wanted to say today.