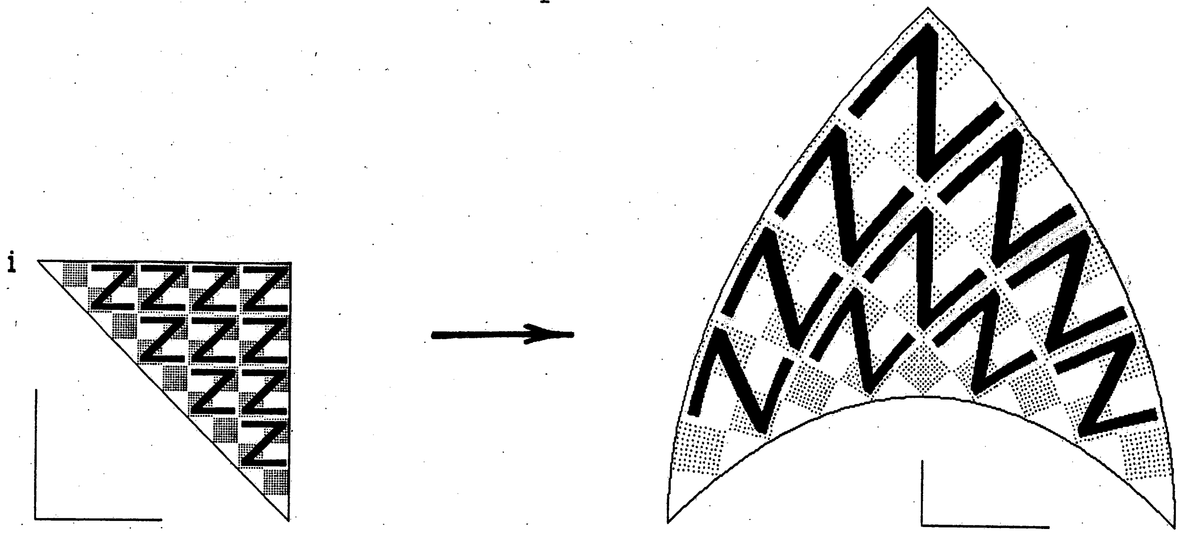
18.04 Handout Fri 12 Sep 03

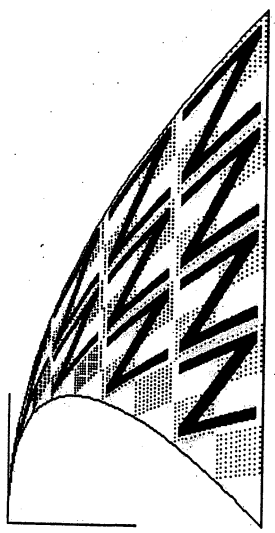


As these two diagrams reemphasize, the simple process of **squaring** via the complex function

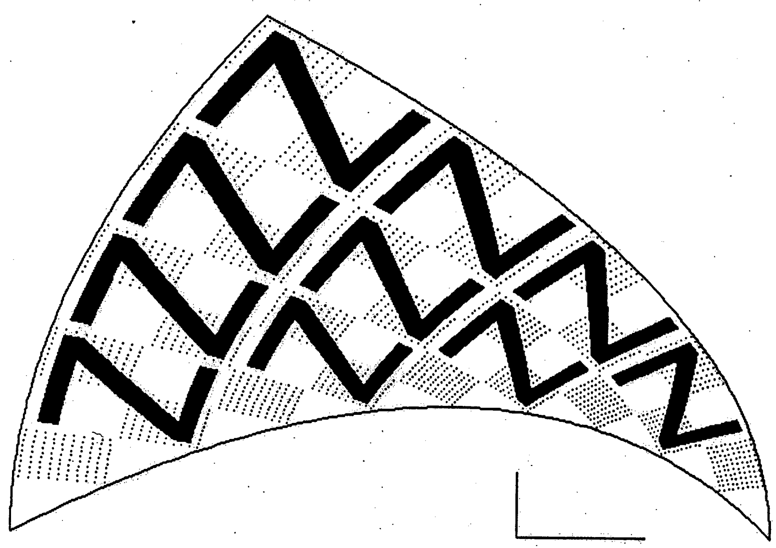
$$w = z^2 = (x^2 - y^2) + i\,2xy$$

maps the contents of the above z-plane triangle **conformally** into the region bounded by the three parabolas in the w-plane.

Notice how soon this conformality is lost once we tamper just a bit with the functions that govern the mapping:

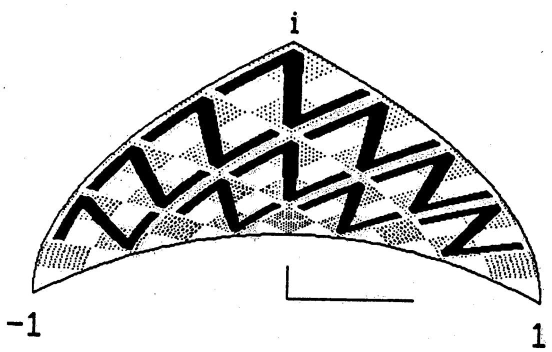


$w = x^2 + i\,2xy$

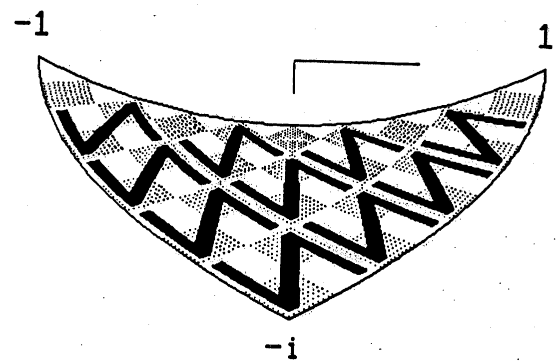


$w = (x^2 - 2y^2) + i\,2xy$

Likewise observe these sadly down-squashed alternative mappings:



$w = (x^2-y^2) + i\ xy$

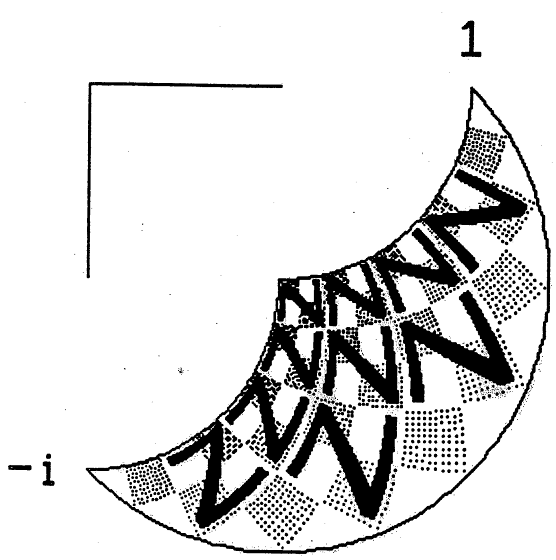


$w = (x^2-y^2) - i\ xy$

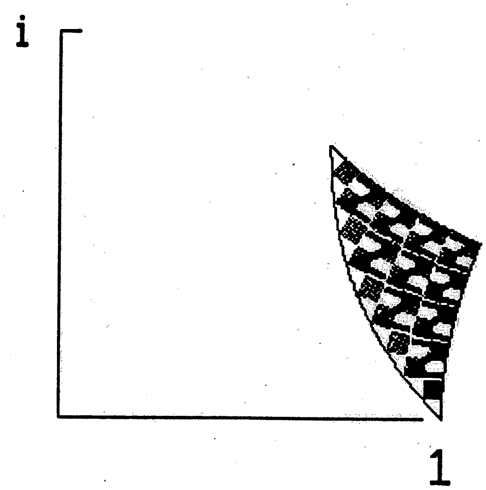
Such conformality — or the strict preservation of shapes and relative directions in each infinitesimal vicinity, as we saw for $w = z^2$ but in none of the later examples — is the very essence of **analytic functions**, since their rates of change (a.k.a. derivatives) in the limit of vanishing separations near any given z-location are by definition totally independent of the direction in which one chooses to approach that location.

In the large, even the conformal maps may look badly distorted, but in any given small neighborhood of the "output" plane they really are just a near-uniformly magnified or shrunken (and usually also rotated) facsimile of the corresponding small patch of the "input" plane. Also beware that the conformal images NEVER include reflections, unlike the "anti-Z"s visible in the non-conformal map above on the right.

Lastly, here are two additional images of the original triangle; they resulted from the **inversion** and **square-root** mappings:



$w = 1/z$



$w = \mathrm{sqrt}(z)$