18.01 EXAM 4
NOVEMBER 4, 2003

Name: _____

Problem 1: _____ /25

Problem 2: _____ /25

Problem 3: _____ /25

Problem 4: _____ /25

Please write the hour of your recitation.

Total: _____ /100

Hour: _____

Instructions: *Please write your name at the top of every page of the exam.* The exam is closed book, calculators are not allowed, but you are allowed to use your prepared index card. You will have approximately 50 minutes for this exam. The point value of each problem is written next to the problem – use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown.

You may use either pencil or ink. If you have a question, need extra paper, need to use the restroom, etc., raise your hand.

Date: Fall 2003.

Name: _____ **Problem 1**: _____ /25

Problem 1(25 points)

(a):(20 points) Find the volume of the solid of revolution obtained by revolving about the y-axis the region in the first quadrant of the xy-plane bounded by the x-axis, the line $x = r$, and the curve

$$y = h\left(\frac{x}{r}\right)^3.$$

Here r and h are positive constants.

(b):(5 points) This solid is contained in a cylinder with base radius r and height h. What fraction of the volume of the cylinder is taken up by the solid in (a)?

Name: _____ **Problem 2**: _____ /25

Problem 2(25 points) Compute the surface area of the surface of revolution obtained by revolving about the y-axis the curve,

$$y = \frac{1}{2}x^2,\ 0 \leq x \leq 1.$$

Name: _____ **Problem 3**: _____ /25

Problem 3(25 points)

(a): (10 points) Sketch the curve with polar equation,

$$r = \sin(2\theta),\ 0 \leq \theta \leq \frac{\pi}{2}.$$

In particular, label the following on your graph,

(i) in which quadrant or quadrants the curve is contained,
(ii) the endpoints of the curve,
(iii) the two slopes of the tangent lines at the endpoints of the curve,
(iv) and the angle or angles θ at which $r(\theta)$ is a maximum.

(b):(15 points) Compute the area of the region enclosed by the curve in (a). As a hint, recall the half-angle formulas,

$$\begin{cases} \cos^2\left(\frac{\alpha}{2}\right) &= \frac{1}{2}(1+\cos(\alpha)), \\ \sin^2\left(\frac{\alpha}{2}\right) &= \frac{1}{2}(1-\cos(\alpha)). \end{cases}$$

Name: _____ **Problem 4**: _____ /25

Problem 4(25 points)

(a):(15 points) A curve is given by parametrically by,

$$\begin{cases} x &= \cos(t) + t\sin(t), \\ y &= \sin(t) - t\cos(t), \end{cases} \quad 0 \leq t \leq \pi.$$

Compute the arclength of the parametrized curve.

(b):(10 points) Let $f(x)$ be a solution of the differential equation with side equation,

$$\frac{dy}{dx} = \sqrt{x^2 - 1},\ y(2) = 0,$$

on the interval $2 \leq x \leq 3$. Compute the arclength of the curve $y = f(x)$ on the interval $2 \leq x \leq 3$. DO NOT ATTEMPT TO SOLVE THE DIFFERENTIAL EQUATION!