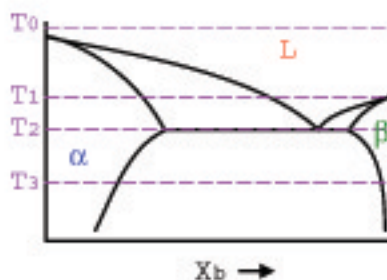


Thermodynamics of Materials 3.00  
**Example Problems for Week 13**

**Example Problem 13.1**

For the binary phase diagram given below draw schematics of plausible molar free energy curves showing the common tangent construction and chemical potentials of the two components,  $\mu_a$  and  $\mu_b$ , as a function of composition,  $X_b$ , at each marked temperature.



**Solution 13.1**

The following are molar Gibbs free energy curves on the left and chemical potential curves on the right for each of the four temperatures. The red curve is the liquid, blue  $\alpha$  phase and red  $\beta$  phase. Some things to notice are:

- The liquid curve moves up with respect to the two solid phases as temperature decreases.
- The  $T_2 = T_e$ , the eutectic temperature, all three curves share a common tangent line.
- The chemical potential of a component across a two-phase field is constant.
- The chemical potential of the minority component in a phase approaches  $-\infty$  as the composition of that component approaches zero which can be seen from the expression for the chemical potential of  $a$  in any phase:

$$\mu_a = \bar{G} - X_b \frac{d\bar{G}}{dX_b} \quad (1)$$

Consider this when  $X_b \rightarrow 1$ .

- The slope of the chemical potential curves is always negative with respect to the composition of the other component. This can be seen by examining:

$$\frac{d\mu_a}{dX_b} = -X_b \frac{d^2\bar{G}}{dX_b^2} < 0 \quad (2)$$

- At  $T_1 = T_m^\beta$  the  $\bar{G}_l$  and  $\bar{G}_\beta$  curves have the same tangent line.

