HST.582J / 6.555J / 16.456J Biomedical Signal and Image Processing
Spring 2007

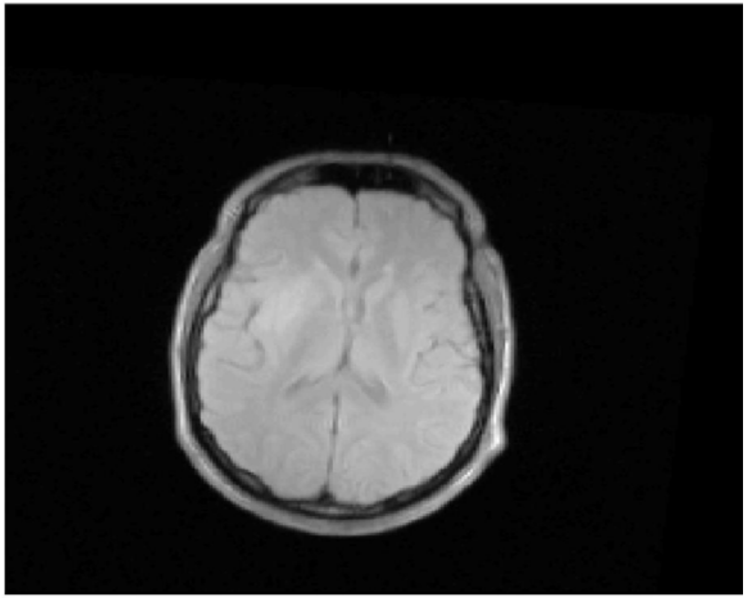
Harvard-MIT Division of Health Sciences and Technology
HST.582J: Biomedical Signal and Image Processing, Spring 2007
Course Director: Dr. Julie Greenberg

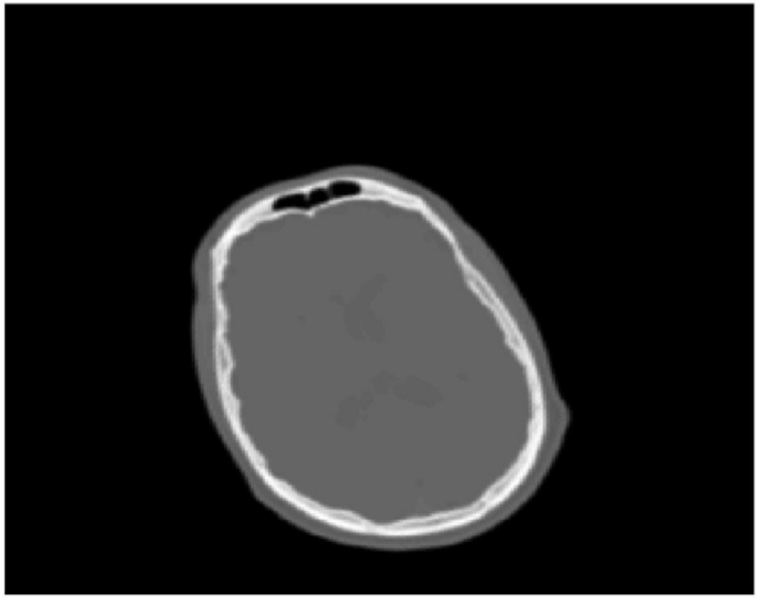
Medical Image Registration I

HST 6.555

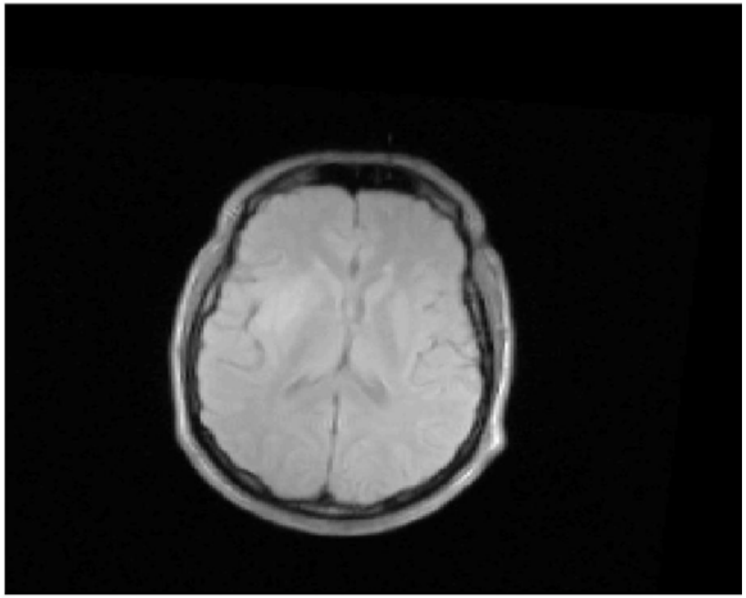
Lilla Zöllei and William Wells

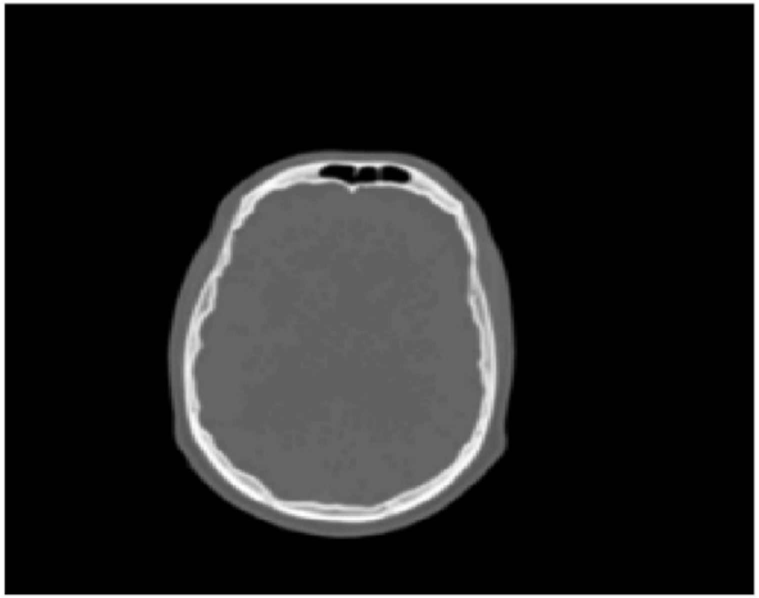
The Registration Problem



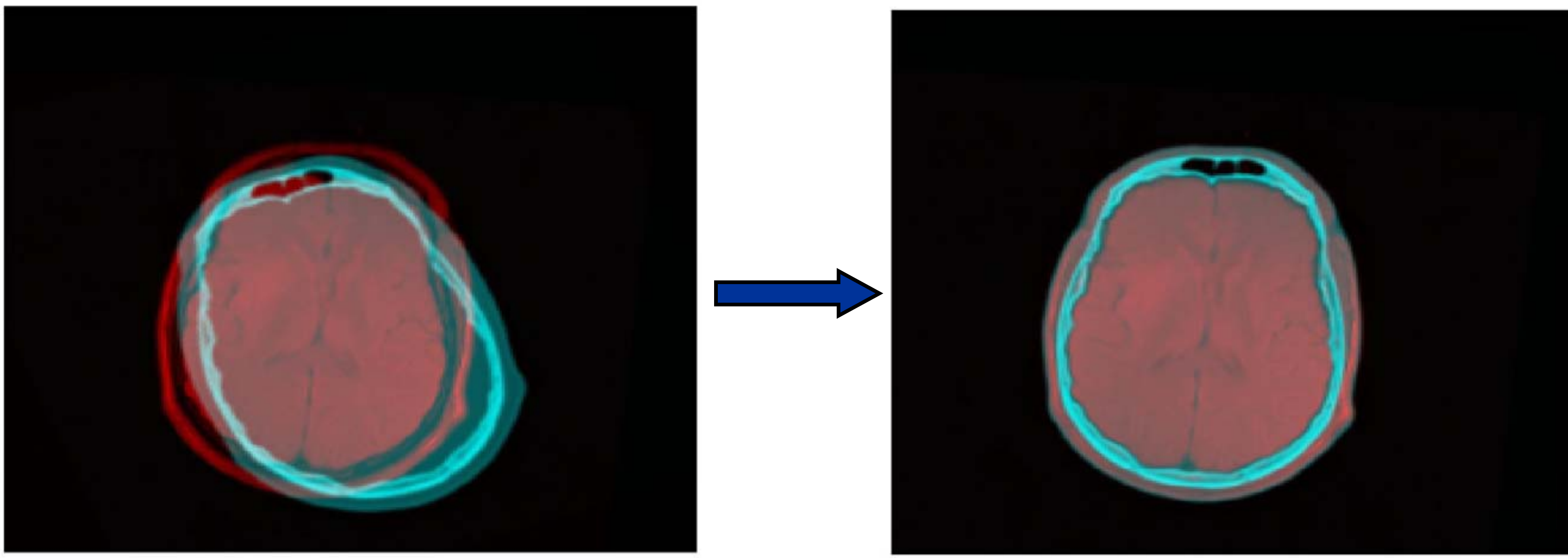


The Registration Problem





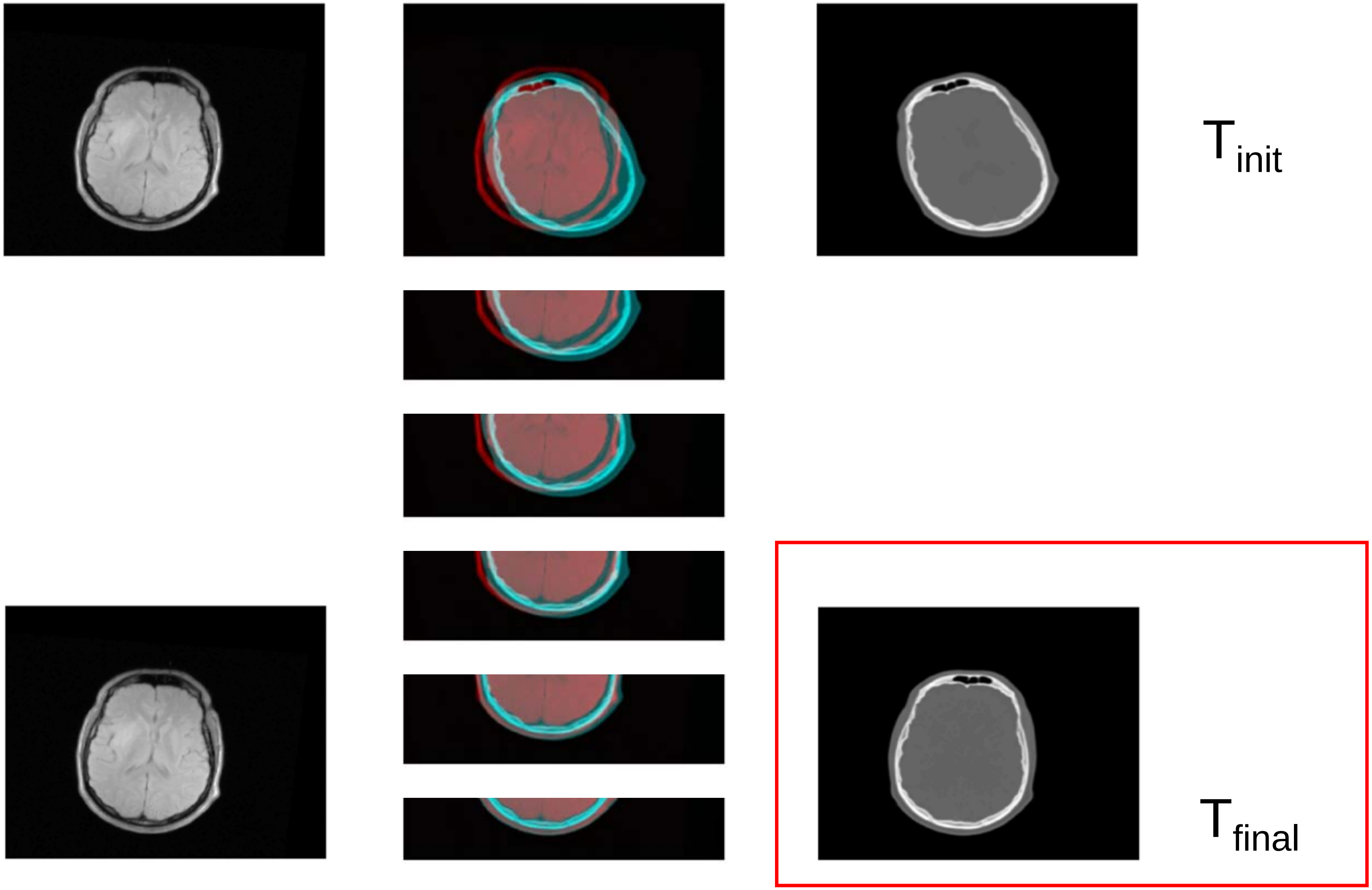
The Registration Problem



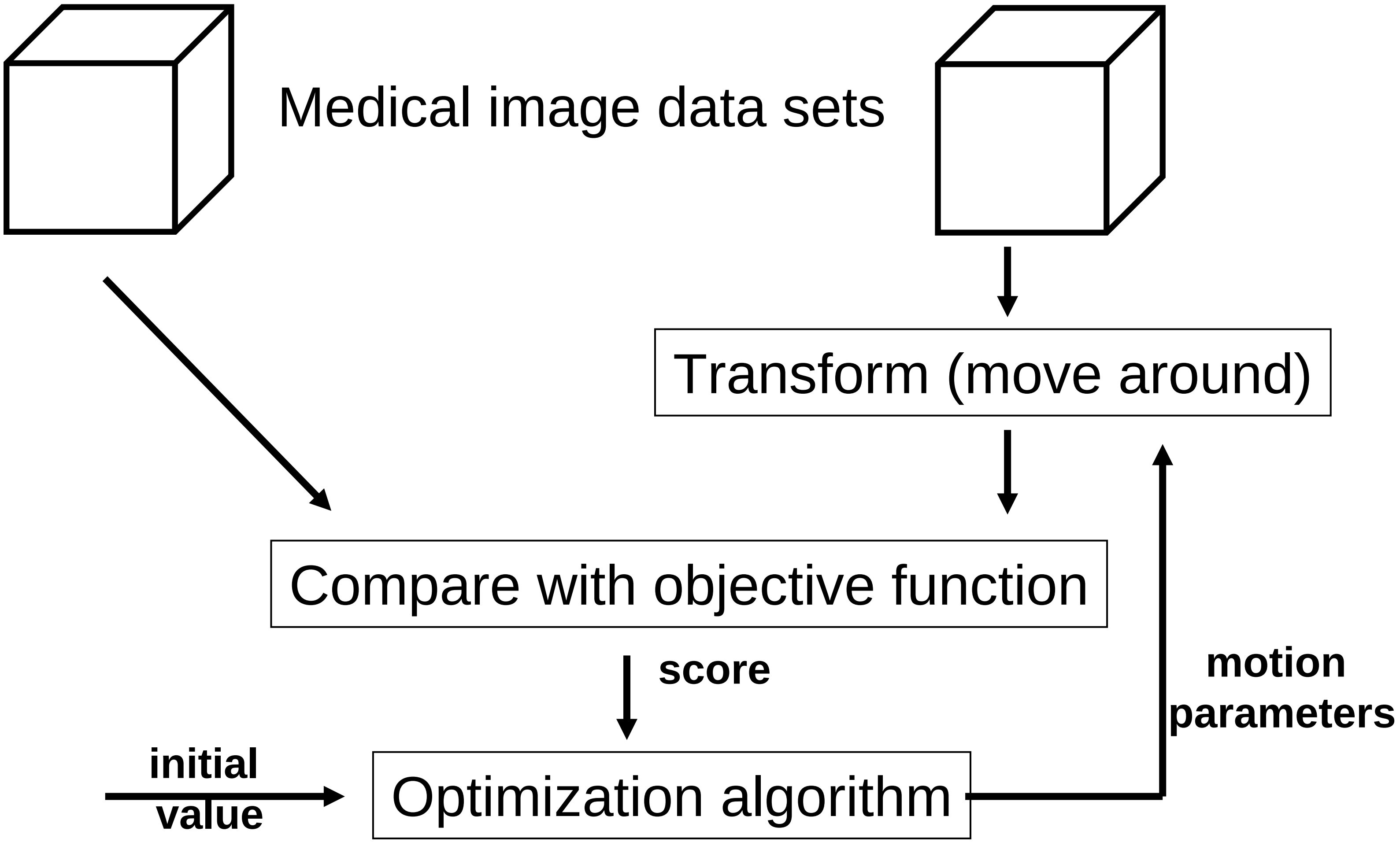
Applications

- multi-modality fusion (intra-subject)
- time-series processing
  - e.g.: *f*MRI experiments, cardiac ultrasound
- *warping* across patients (inter-subject, uni-modal)
- *warping* to / from atlas for anatomical labeling
- image-guided surgery:
  - modeling tissue deformation,
  - comparing pre- and intra-operative scans,…

The Registration Problem



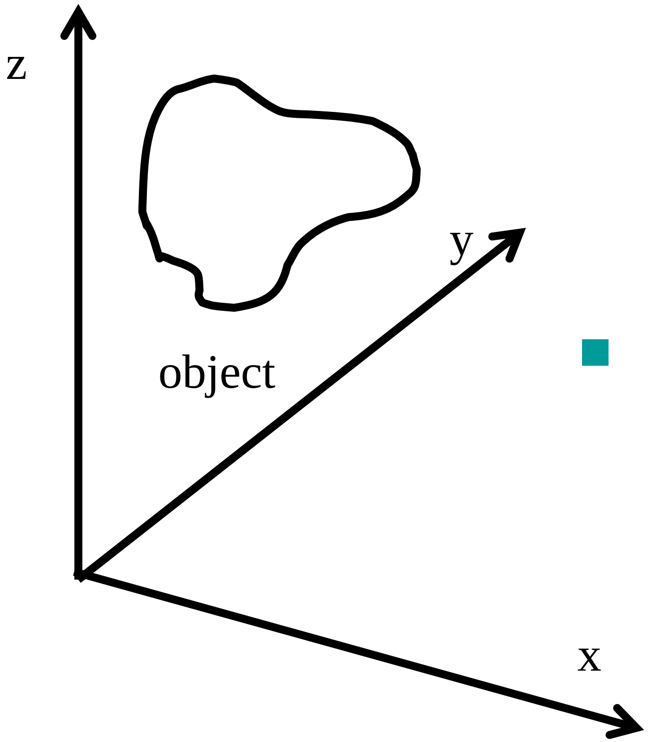
Medical Image Registration



Roadmap

- Data representation
- Transformation types
- Objective functions
  - Feature/surface-based
  - Intensity-based
- Optimization methods
- Current research topics

Data Representation

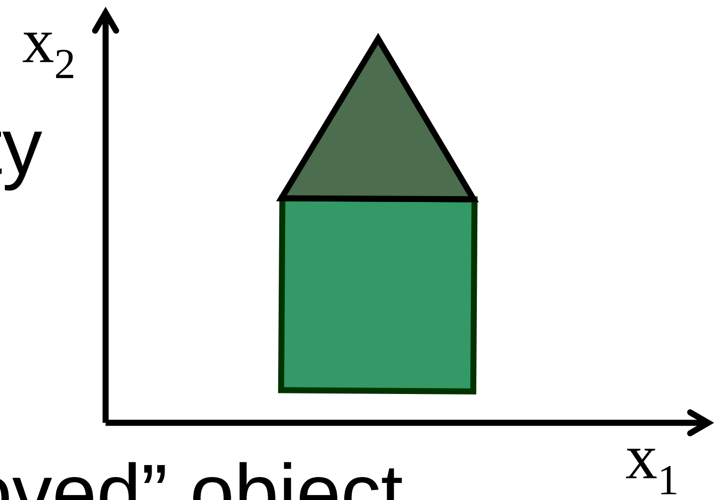


- Object represented as a function
  - example:

    coordinates → intensity

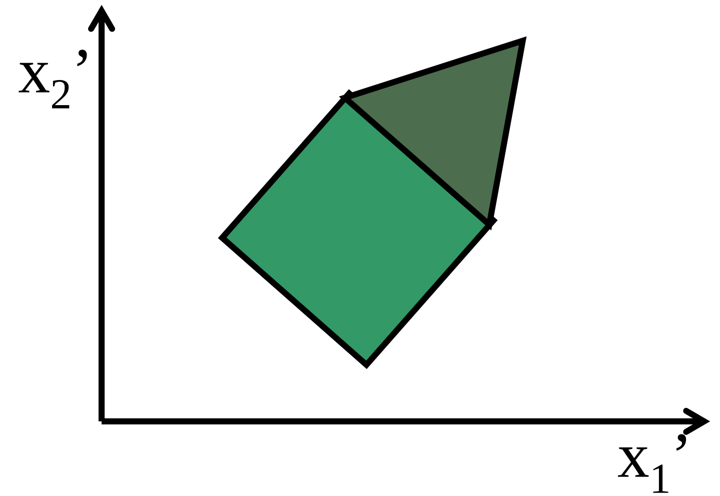
Data Representation

- $f(x)$: space → intensity



- representation of "moved" object

$$g(x') = f(T(x'))$$



Data Representation

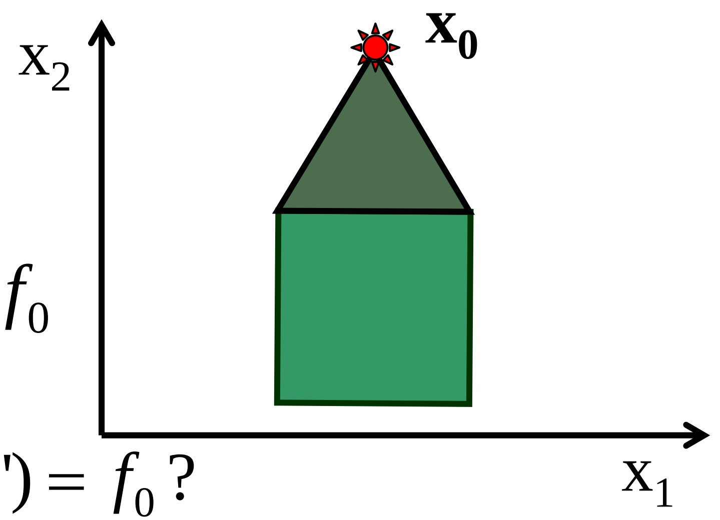
- focus on a feature
  - before motion: $f(x_0) = f_0$
  - after motion
    - What value of $\mathbf{x_0'}$: $g(x_0') = f_0$?

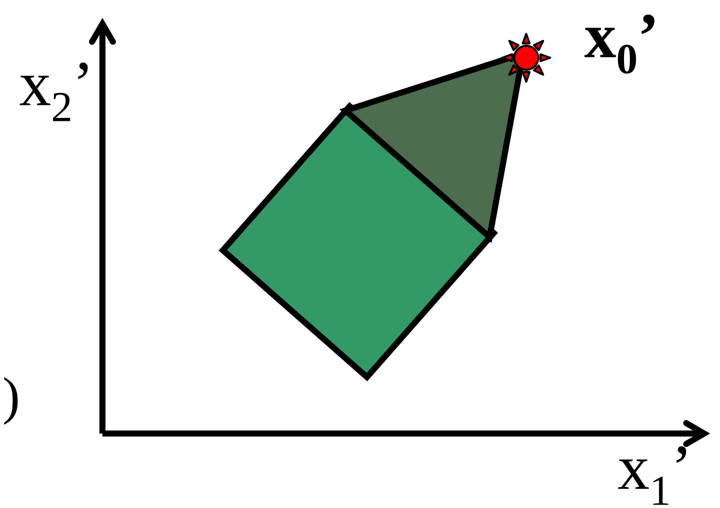
      $f(T(x_0')) = f_0$

      $T(x_0') = x_0 \quad x_0' = T^{-1}(x_0)$

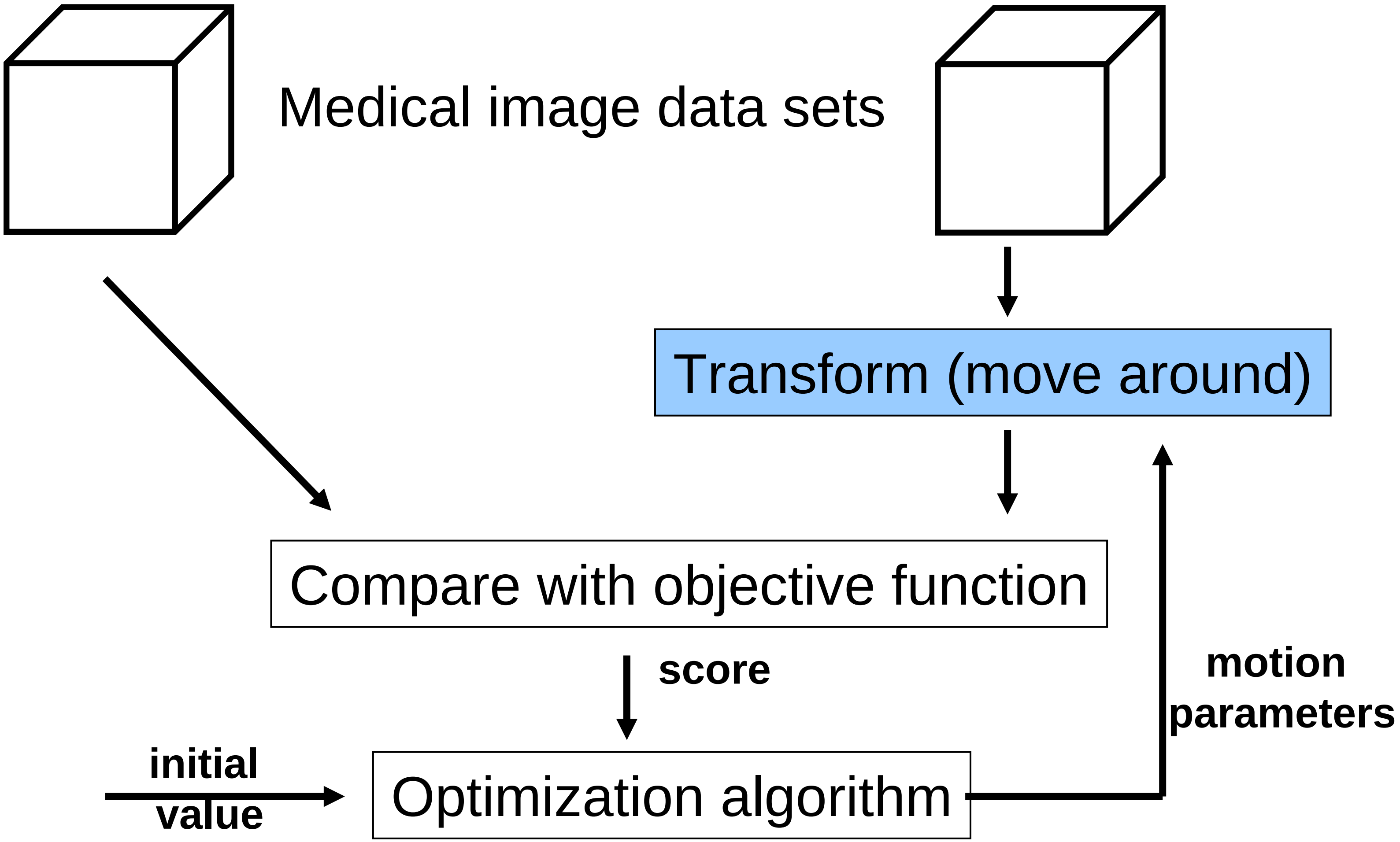
    - Feature of f_0 now is located at $T^{-1}(x_0)$

      $\Rightarrow$ motion of object is $T^{-1}(.)$





Medical Image Registration



Space of transformations

Data dimensions	Example
2D-2D	cardiac ultrasound, x-ray patient repositioning, histology – MRI, …
3D-3D	MR/MR, MR/CT, CT/CT, PET-MR, …
2D-3D	X-ray/CT, fluoroscopy-CT, surface model/video

Class of transformations

- What motions or distortions are allowed to merge datasets?
  - Rigid Transformations:
    - displacement
    - rotation & displacement
  - Non-Rigid Transformations:
    - parametric
      - affine
      - piecewise-affine
      - ….
    - non-parametric

Displacement only

$$T(x) \equiv x + D$$

- 2D: 2 parameters
- 3D: 3 parameters

- For example:

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

in 2D

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix}$$

in 3D

Rigid Motion

$$T(x) \equiv R(x) + D$$

- both rotations and displacements are allowed
- length-preserving transformation
- order of transformations does matter!

- 2D: 3 parameters; 3D: 6 parameters
- for example: in 2D (non-linear in θ)

$$T(x) = Rx + D$$

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

R: valid rotation matrix

$R^T R = 1$ and $|R| = +1$

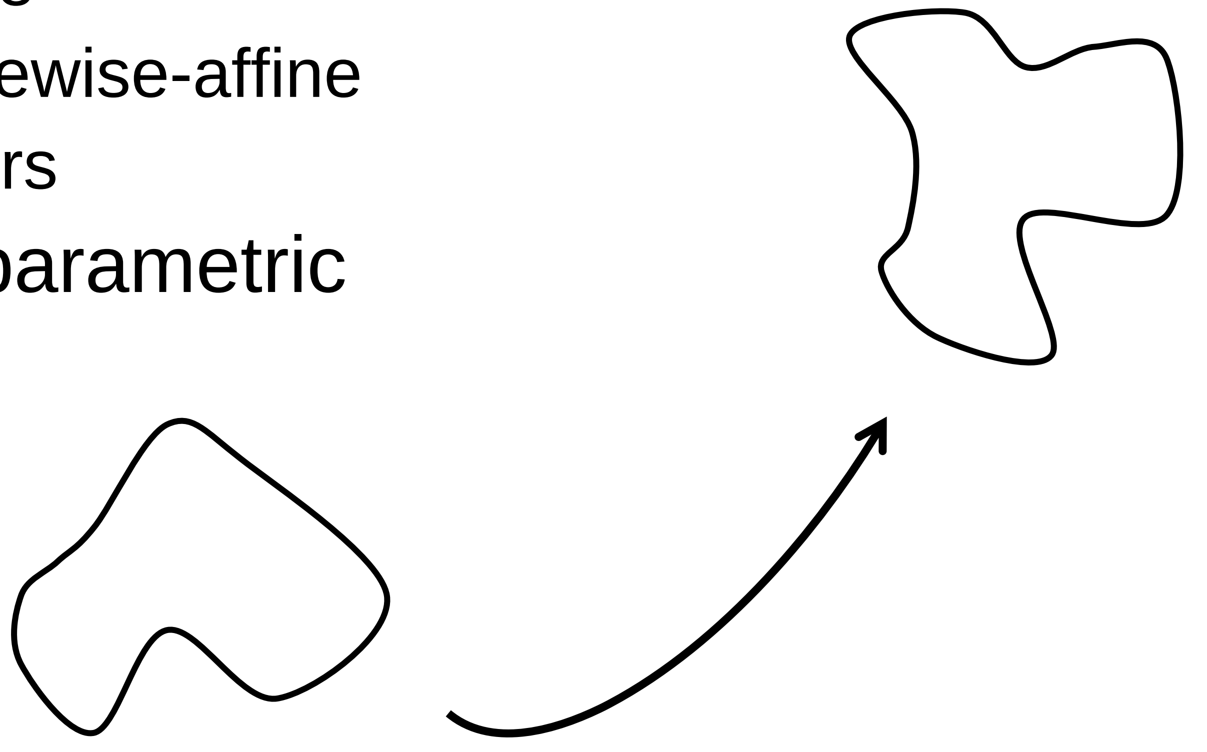
- in 3D: rotate by θ about axis $\hat{N}$
  - if q is a unit quaternion, such that $q = \left(\cos\frac{\theta}{2}; \hat{N} \sin\frac{\theta}{2} \right)$

$$R = \begin{pmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(-q_0 q_3 + q_1 q_2) & 2(q_0 q_2 + q_1 q_3) \\ 2\left(q_0 q_3 + q_2 q_1\right) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(-q_0 q_1 + q_2 q_3) \\ 2(-q_0 q_2 + q_3 q_1) & 2(q_0 q_1 + q_3 q_2) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{pmatrix}$$

Horn, B.K.P., *Closed Form Solution of Absolute Orientation using Unit Quaternions*, Journal of the Optical Society A, Vol. 4, No. 4, pp. 629--642, April 1987.

Non-rigid transformations

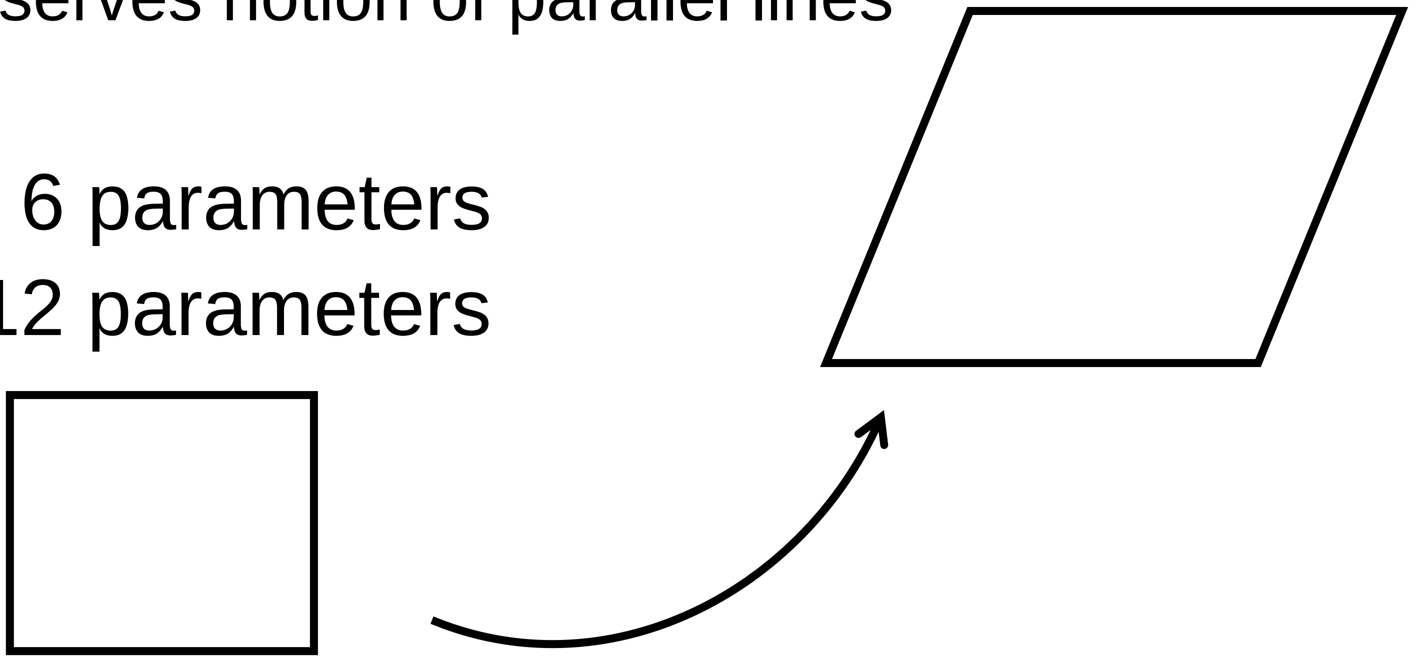
- Parametric
  - affine
  - piecewise-affine
  - others
- Non-parametric



Affine Transformation

$$T(x) \equiv M(x) + D$$

- *M:* square matrix
  - beyond rigid motion, allows shears and scaling
  - preserves notion of parallel lines
- 2D: 6 parameters
- 3D: 12 parameters



- 2D affine:

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

OR

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} m_{11} & m_{12} & D_1 \\ m_{21} & m_{22} & D_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$$

linear form

$$T(X) = QX$$

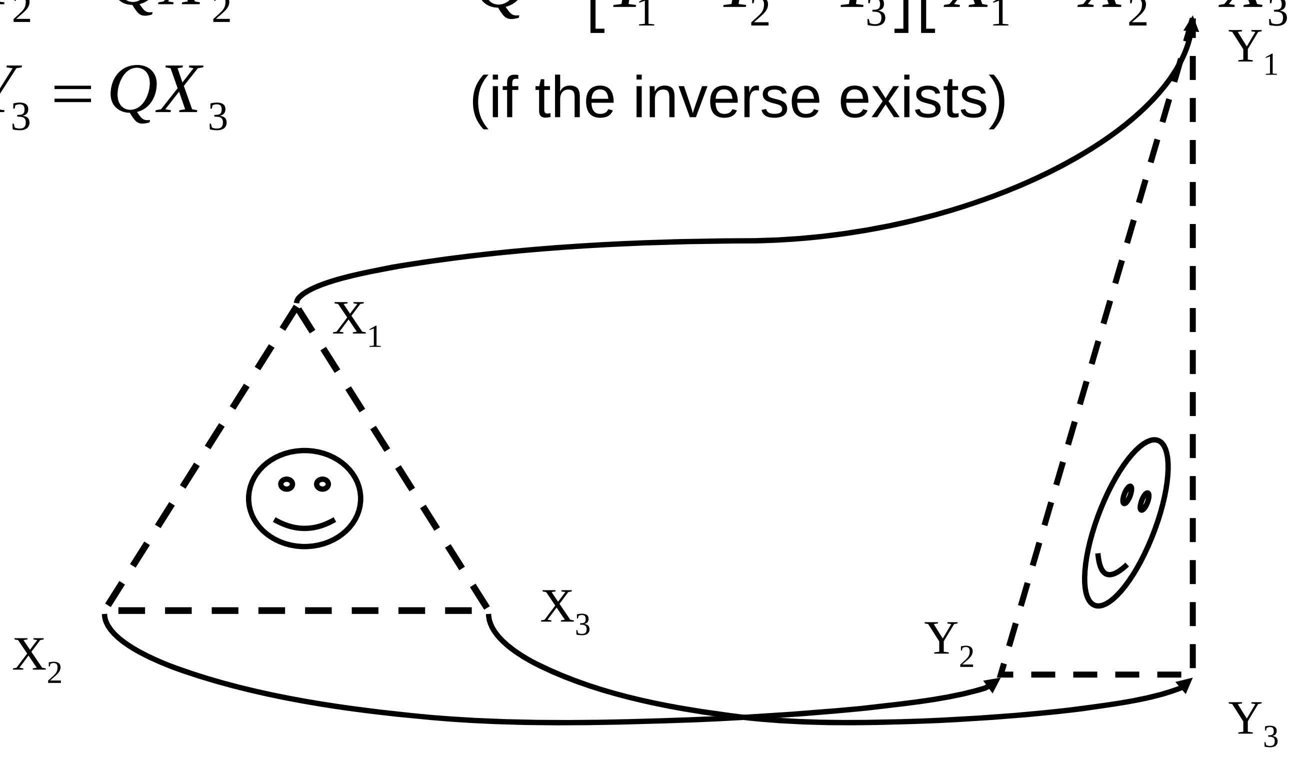
homogeneous coordinate

- 2D affine: determined by control points

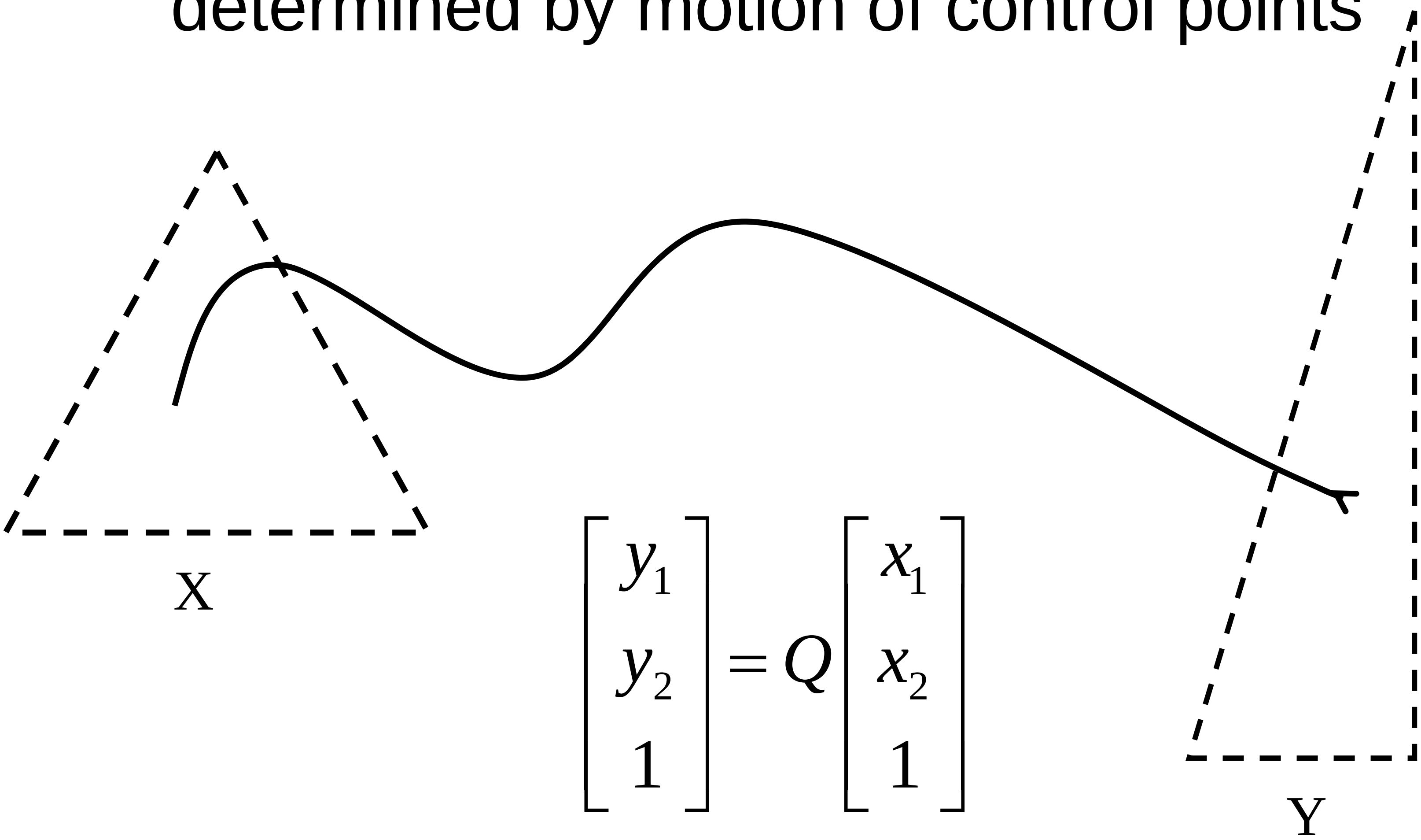
$$Y_1 = QX_1$$
$$Y_2 = QX_2$$
$$Y_3 = QX_3$$

$$\begin{bmatrix} Y_1 & Y_2 & Y_3 \end{bmatrix} = Q \begin{bmatrix} X_1 & X_2 & X_3 \end{bmatrix}$$
$$Q = \begin{bmatrix} Y_1 & Y_2 & Y_3 \end{bmatrix} \begin{bmatrix} X_1 & X_2 & X_3 \end{bmatrix}^{-1}$$

(if the inverse exists)



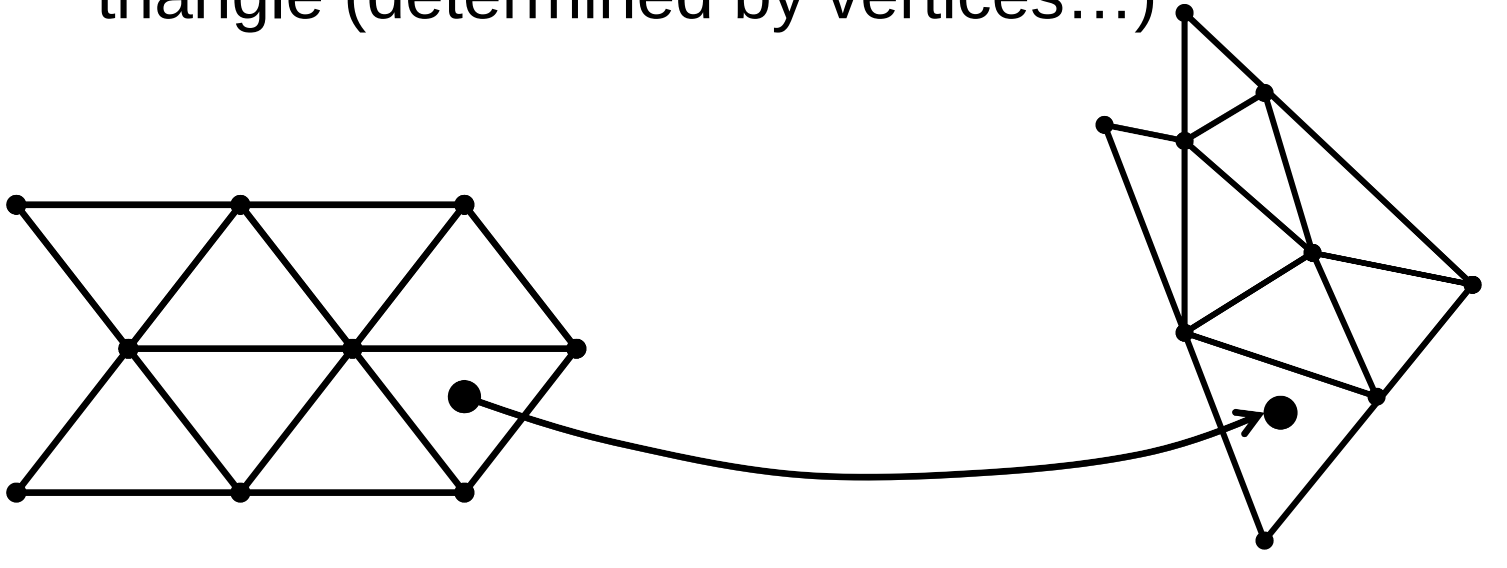
- Affine transformation for other points: determined by motion of control points



$$\begin{bmatrix} y_1 \\ y_2 \\ 1 \end{bmatrix} = Q \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$$

Piecewise affine

- 2D example:
  - subdivide space of *X* into triangles
  - use different affine transformation for each triangle (determined by vertices…)

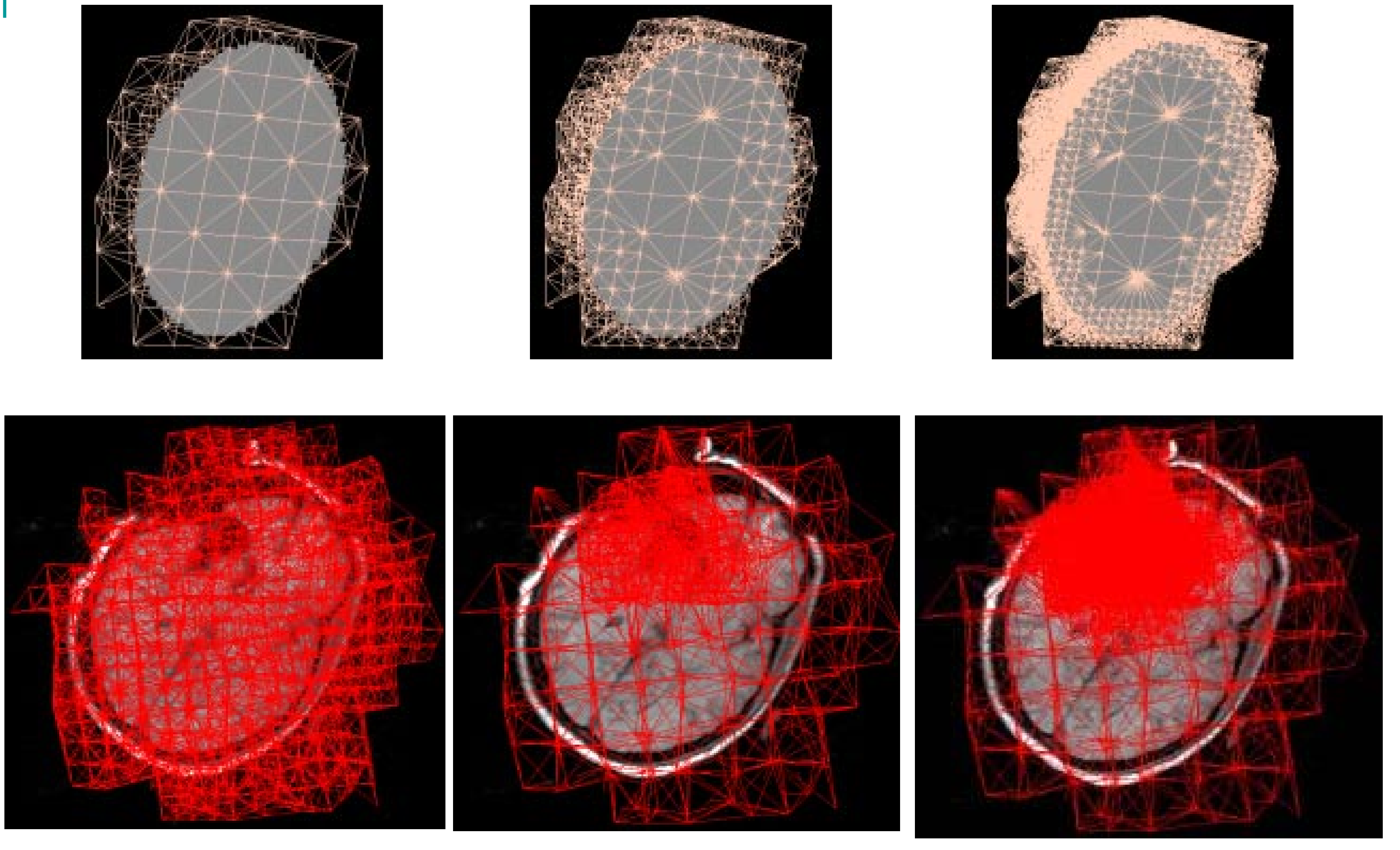


-e.g.: some use in breast registration

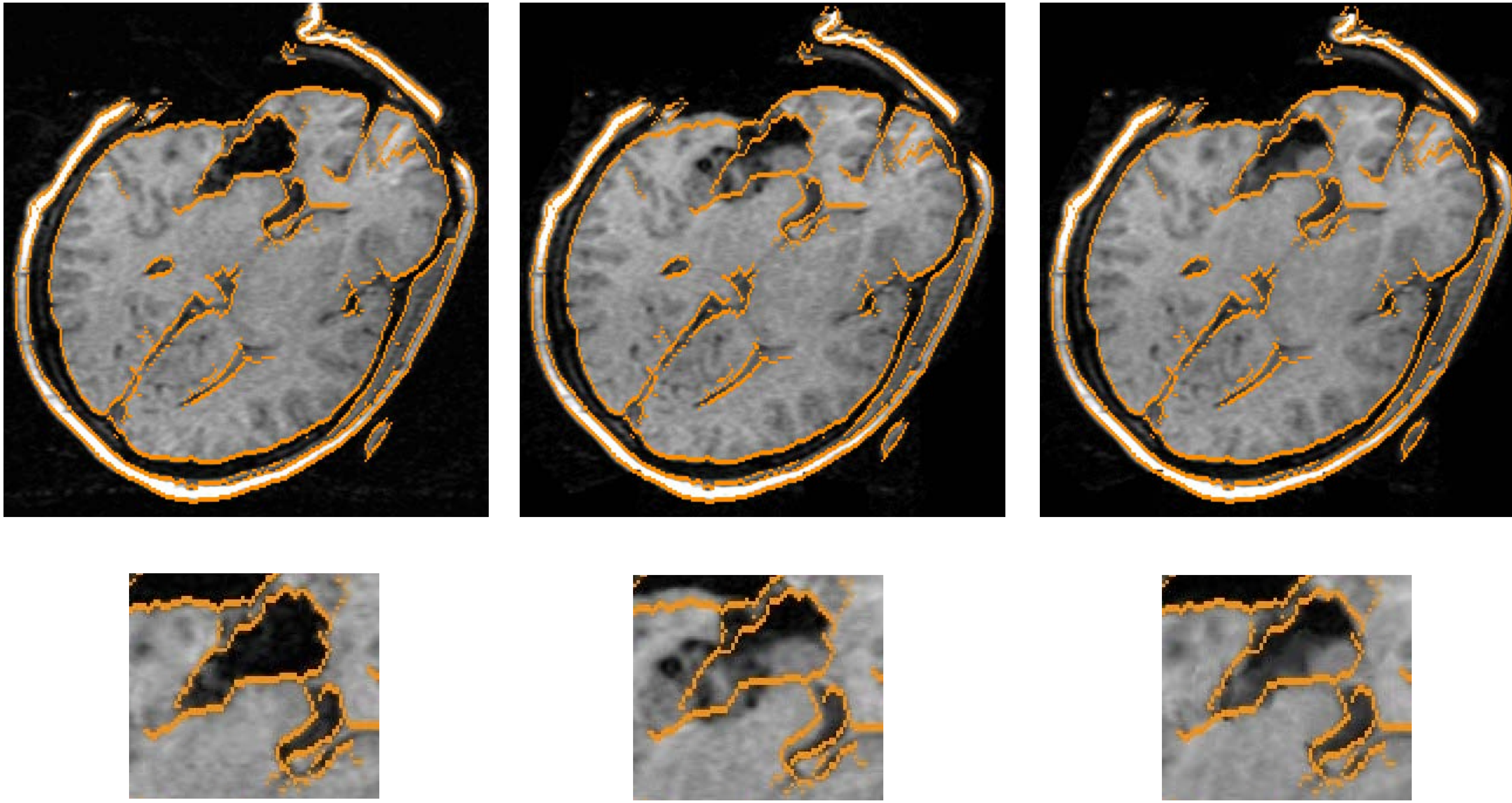
Other Parametric Transformations

- Finite element models
  - engineering methods to simulate mechanical / electrical systems (discretization of the space; integral problem formulation turned into system of linear equations)
- Spline models
  - Thin-plate splines, B-splines, Cubic splines

Fred Bookstein, *Morphometric Tools for Landmark Data : Geometry and Biology*
http://www.iog.umich.edu/faculty/bookstein.htm



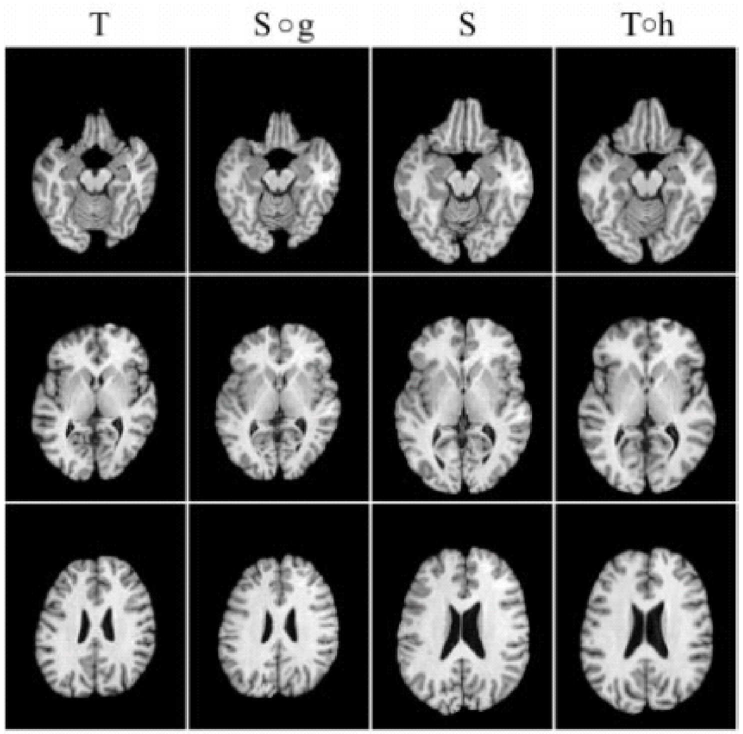
S. Timoner: Compact Representations for Fast Non-rigid Registration of Medical Images (MIT PhD`03)



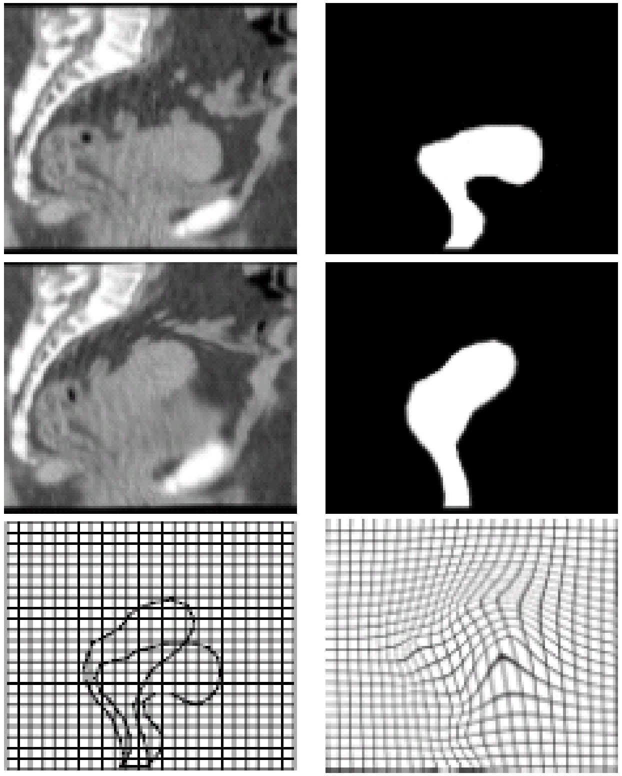
S. Timoner: Compact Representations for Fast Non-rigid Registration of Medical Images (PhD`03)

Non-parametric models

- Continuum mechanics
  - elastic solid models, fluid transport, …

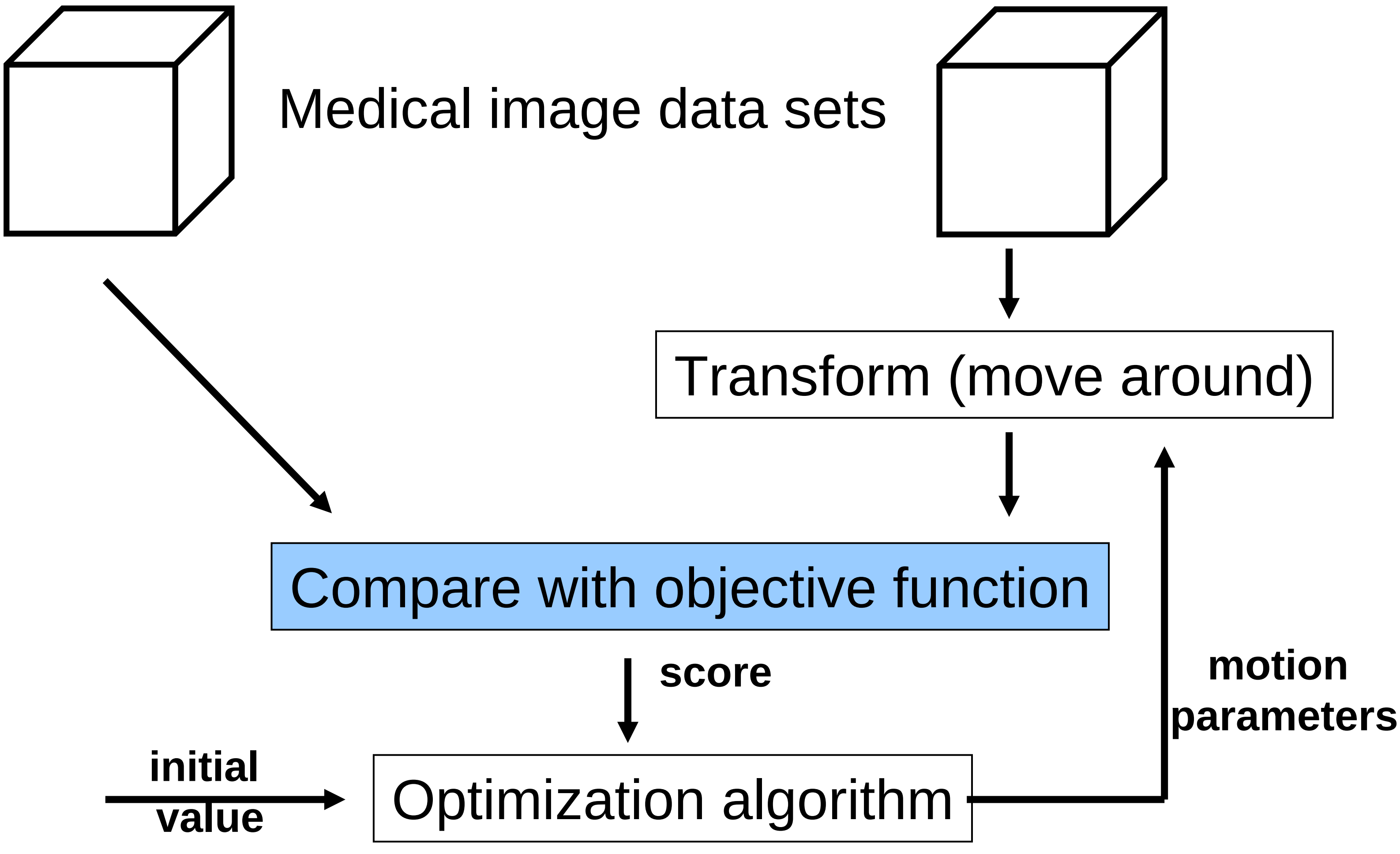


G. E. Christensen* and H. J. Johnson: "Consistent Image Registration." *IEEE TMI*, VOL. 20, NO. 7, JULY 2001



Christensen, G. E., et al. “Large-Deformation Image Registration using Fluid Landmarks.” *Image Analysis and Interpretation 2000, Proceedings of 4th IEEE Southwest Symposium*, pp. 269 -273

Medical Image Registration



Objective functions

- measure how well things are lined up
- assumption: the input datasets (U,V) are related to each other by some transformation T
- define: energy function to be optimized

$$E = f(U(x), V(T(x)))$$

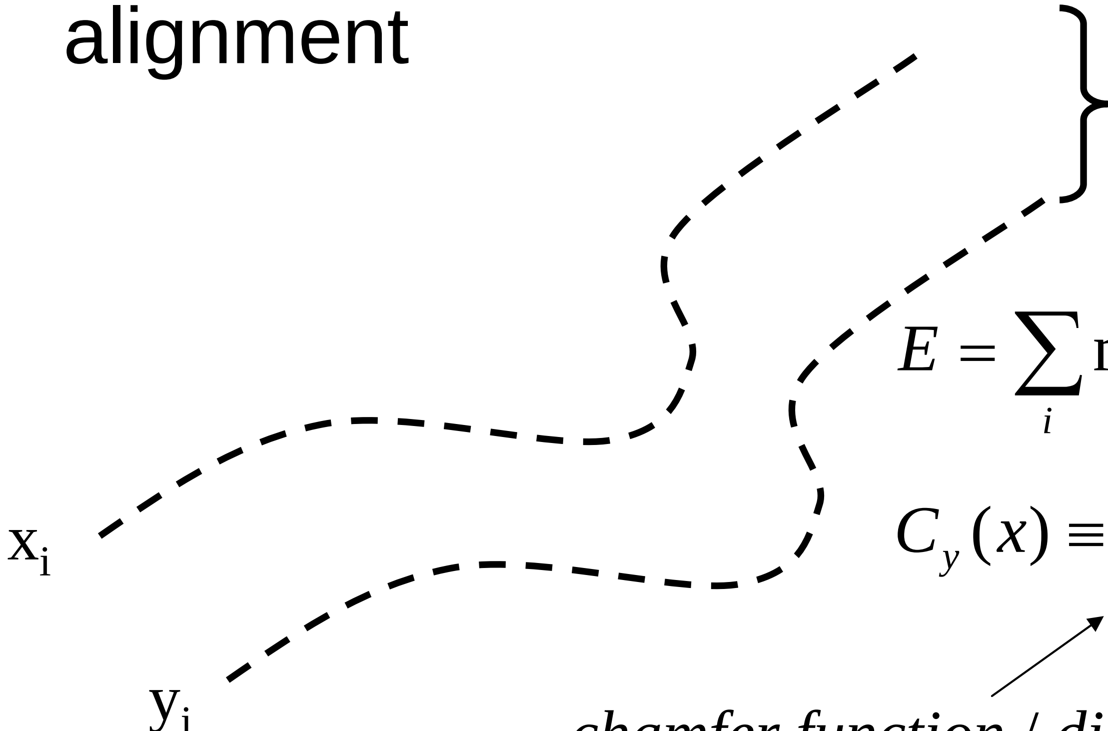
- 2 main styles:
  - feature- or surface-based
  - intensity-based

Feature-based measures

- compute alignment quality based upon the agreement of 2 sets of landmark features
- assumption:
  - landmarks visible in both images
  - they can be reliably located and
  - they can guide the alignment of the whole image

Surface-based measure

- a simple measure of alignment

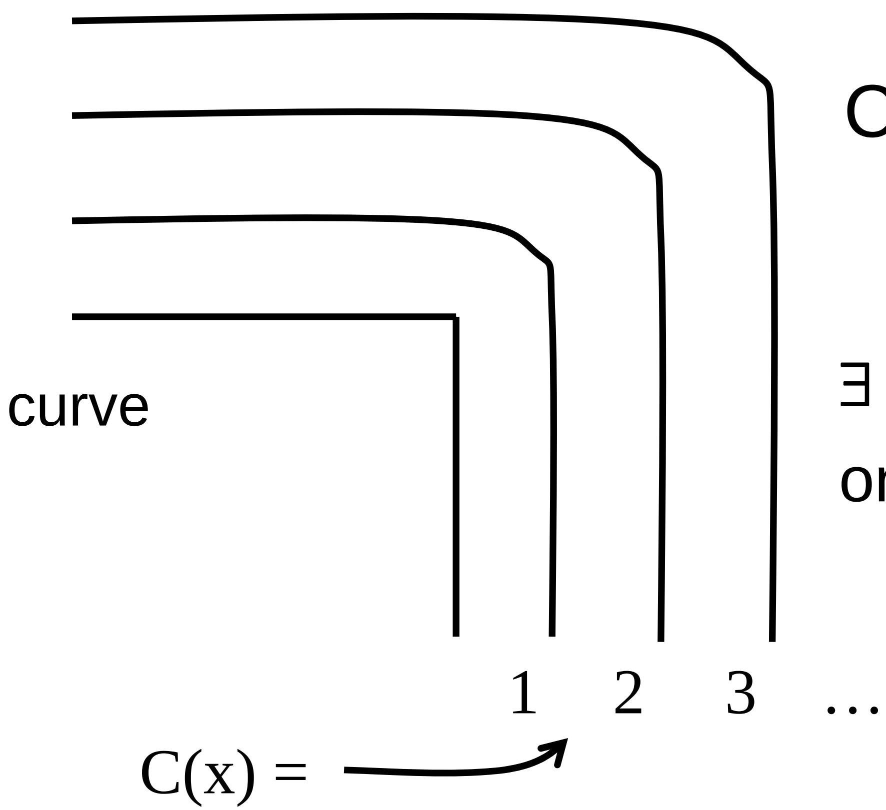


$$E = \sum_i \min_j \left| x_i - y_j \right|^2 = \sum_i C^2(x_i)$$

$$C_y(x) \equiv \min_j \left| x - y_j \right|$$

chamfer function / distance transform

Chamfer function

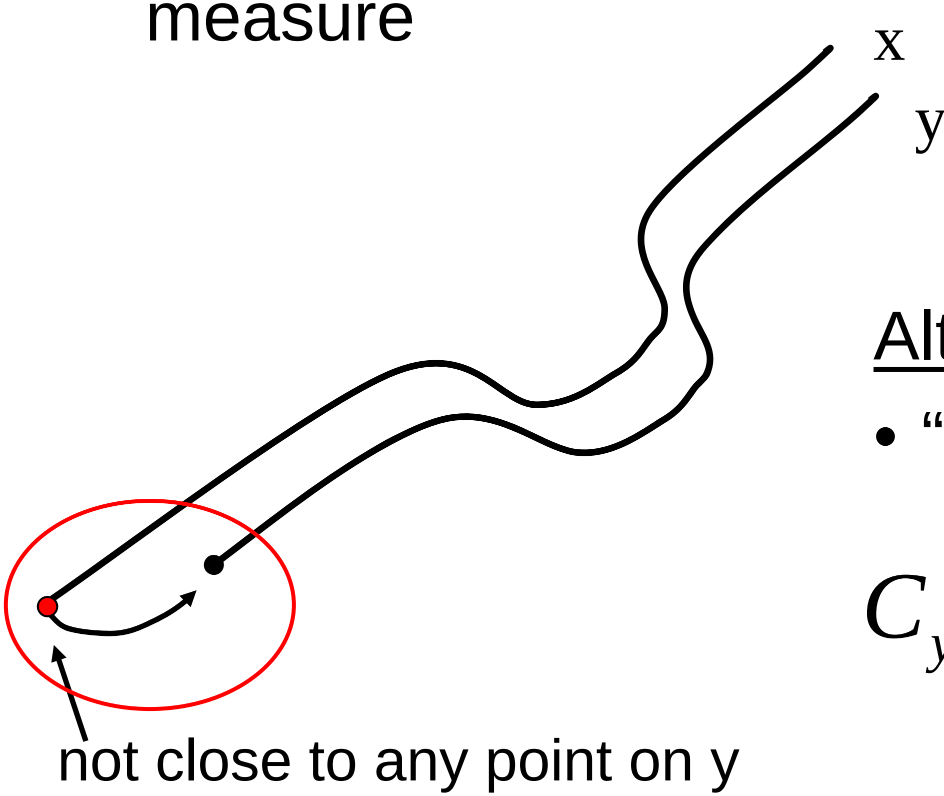


C(x): distance to nearest point on curve

∃ fast way to calculate C(x) on a grid

G. Borgefors. *Hierarchical chamfer matching: a parametric edge matching algorithm*. IEEE Trans. on Pattern Analysis and Machine Intelligence, PAMI- 10(6):849-865, November 1988

- if structure is missing, not a robust measure of alignment; large penalties may *swamp* the measure



Alternative solution:

- "robust chamfer matching"

$$C_y'(x) \equiv \min(d_0, C_y(x))$$

(penalty saturates)

- feature-based techniques used at the BWH:
  - laser data ⟷ patient skin in MR
  - locater data ⟷ patient skin in MR

- similar methods pioneered by
  - Charles Pelizzari (Dept Radiation Oncology, U. Chicago)
    - *"Head in Hat"* – MR-SPECT/PET registration; extract surfaces of heads; register the surfaces

Intensity-based measures

- compute alignment quality based upon the intensity profiles of the input images
- no landmark or feature selection is necessary

SSE and correlation (I)

- SSE: sum of squared errors

$$E = \sum_{x_i} \left| U(x_i) - V(T(x_i)) \right|^2$$

$$= \sum_{x_i} \left[U^2(x_i) - 2U(x_i)V(T(x_i)) + V^2(T(x_i)) \right]$$

- 1st term: no dependency over T; 3rd term contributes a constant* $\Rightarrow$ equivalent problem:

$$E' \equiv \sum_{x_i} U(x_i)V(T(x_i)) \qquad \Rightarrow \text{classical correlation}$$

SSE and correlation (II)

- For translation only: T(x) = X + D

$$E' = \sum_{x_i} U(x_i) V(x_i + D)$$

Convolution $\Rightarrow$ can use Fourier methods…

SSE and correlation (III)

- CAVEAT:

  problems can surface if some structure is missing or extra-large quadratic penalties can swamp the measure

$$E = \sum_{x_i} \left| U(x_i) - V(T(x_i)) \right|^2$$

More robust measures (I)

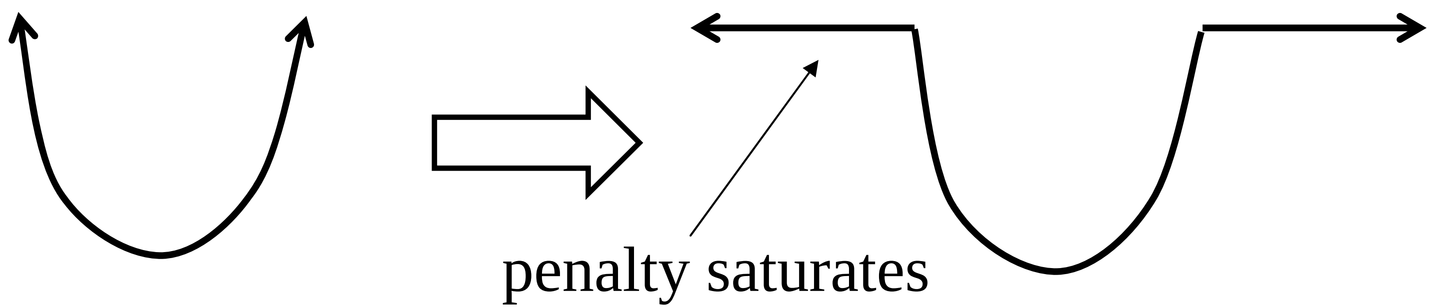
- SAV: *summed absolute value*

$$E(x) = \sum_{x_i} \left| U(x_i) - V(T(x_i)) \right|$$

  - Amy Gieffers, 6A HP Andover*: cardiac ultrasound registration
    - *later Agilent, later Phillips

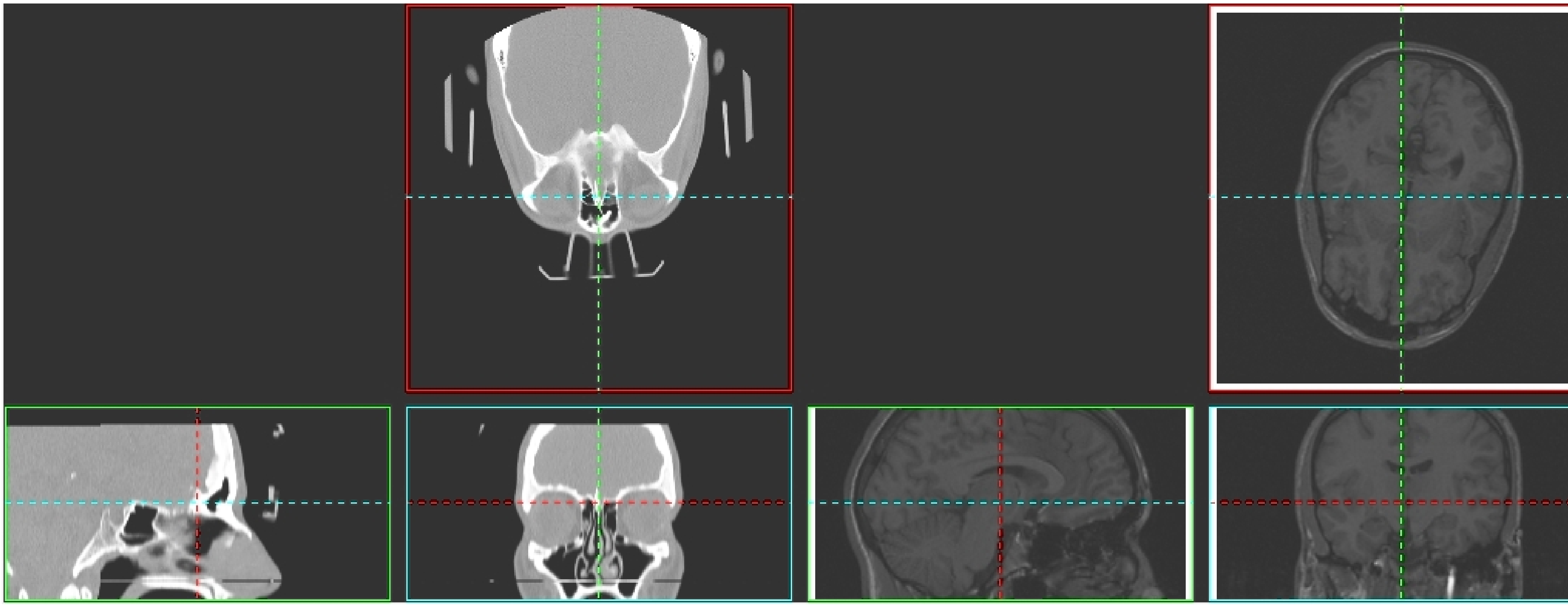
More robust measures (II)

- Saturated SSE



$$E(x) = \sum_{x_i} \min\left(d, \left|U(x_i) - V(T(x_i))\right|^2\right)$$

Multimodal inputs…



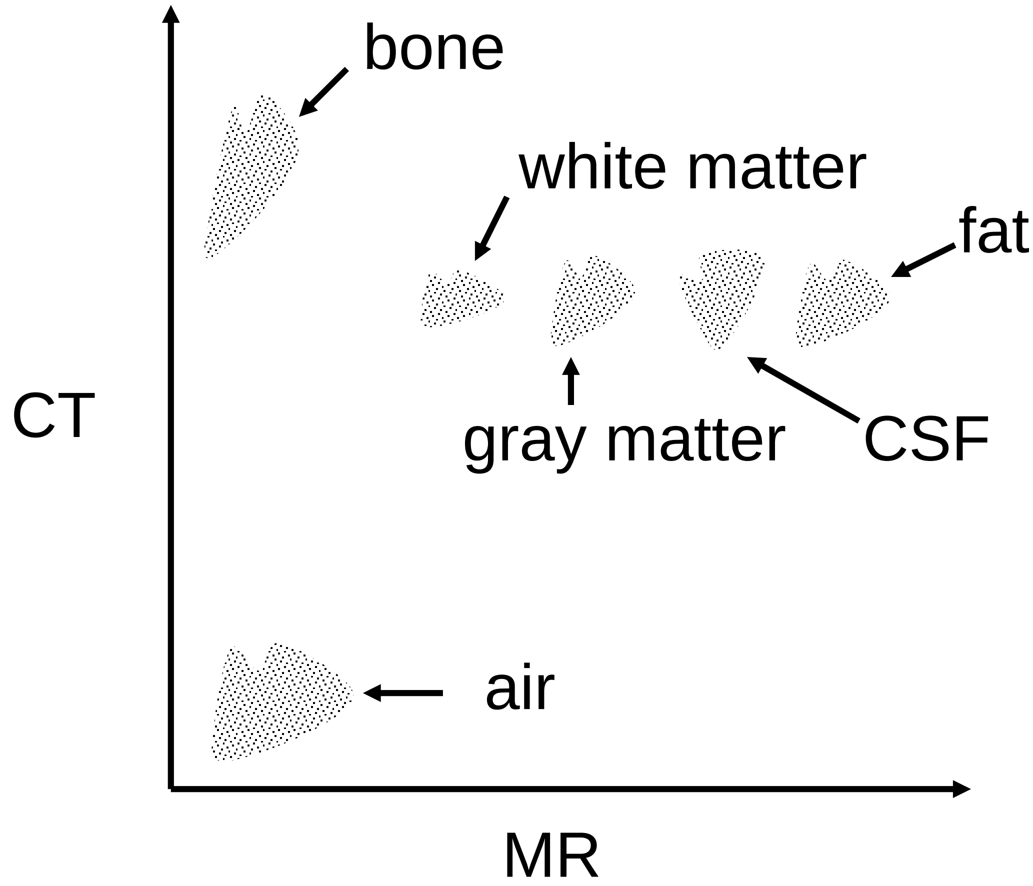
Multimodal Inputs

- *correlation* fails for multi-modal registration when intensities are different; e.g.: MR-CT
- one solution:

  $\Rightarrow$ apply a special intensity transform to the MRI to make it look more like CT; then compute the correlation measure

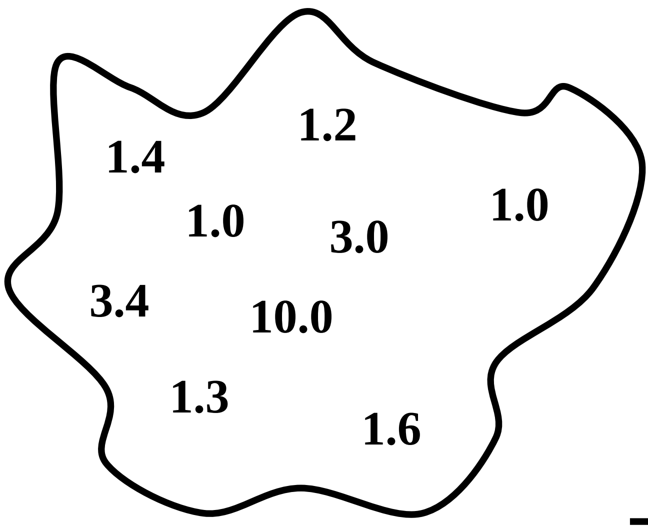
  e.g.: *Petra Van Den Elsen*

MR-CT situation

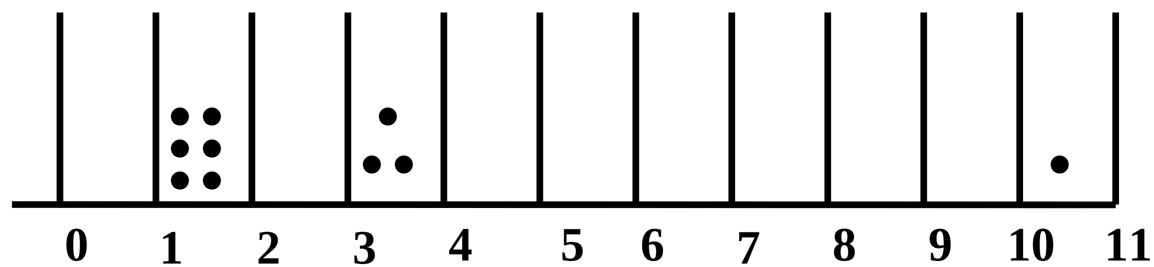


Histograms

Data



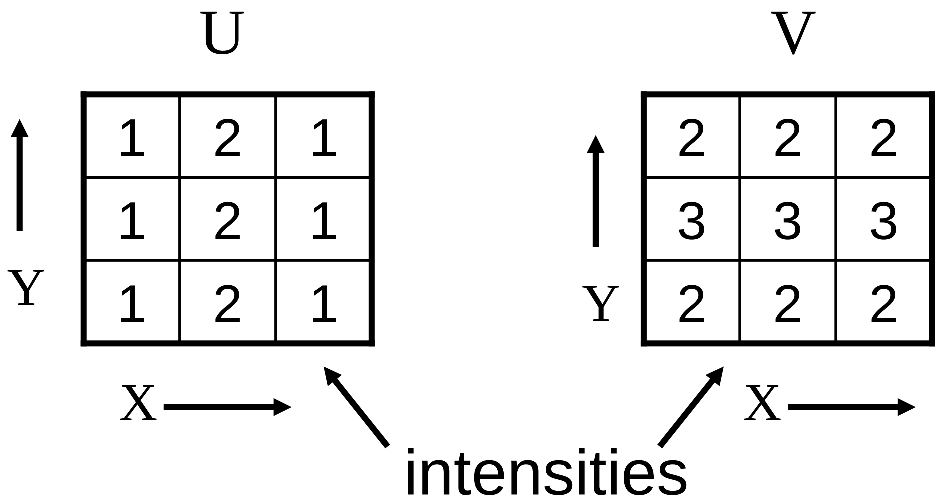
Bins or buckets



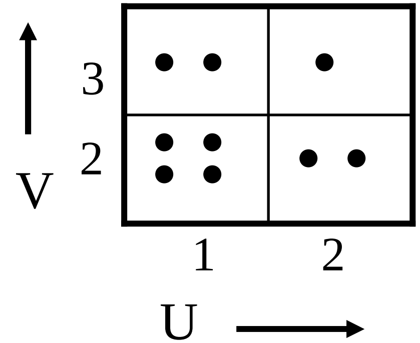
Counts:	0	6	0	3	0	0	0	0	0	0	1
Rel. frequency:	0	$\frac{6}{10}$	0	$\frac{3}{10}$	0	0	0	0	0	0	$\frac{1}{10}$

Histogram Joint Intensity of Images

Images:



histogram



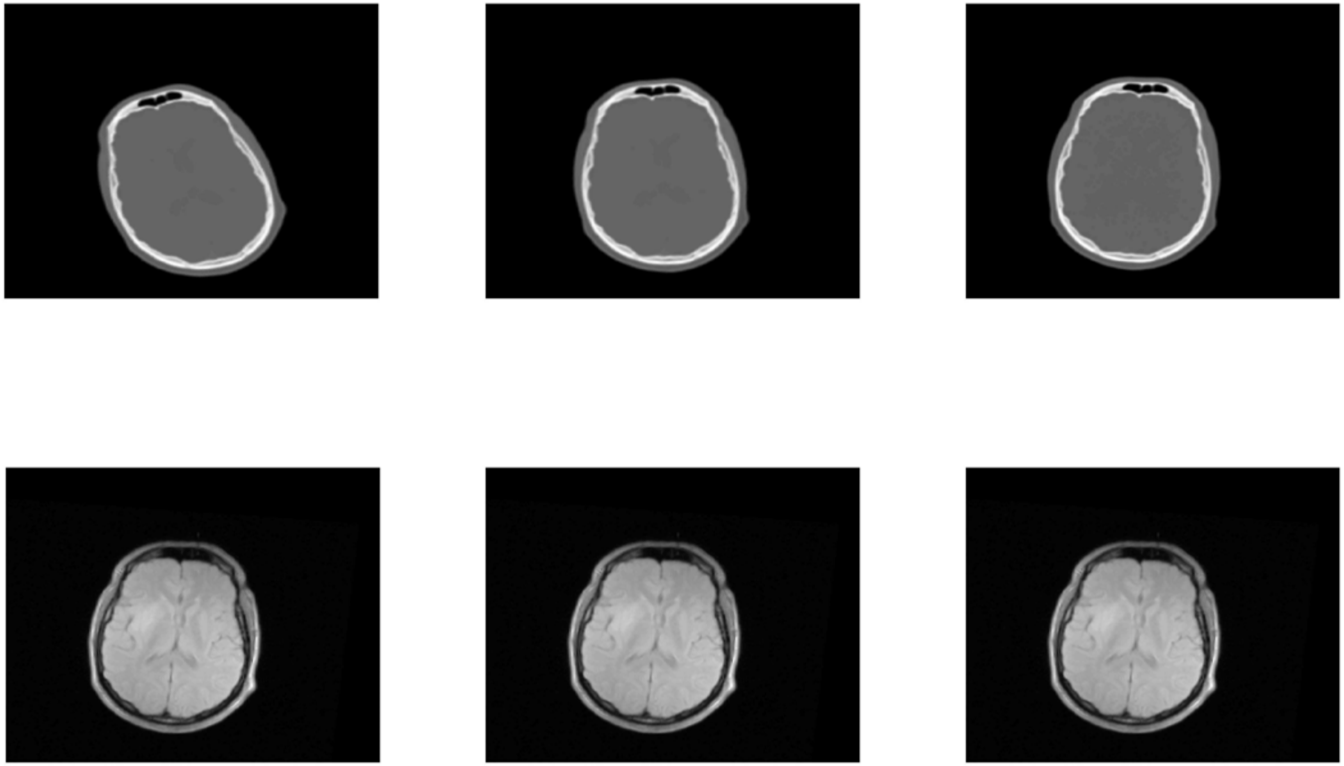
relative freq.

$\frac{2}{9}$	$\frac{1}{9}$
$\frac{4}{9}$	$\frac{2}{9}$

Joint intensities:

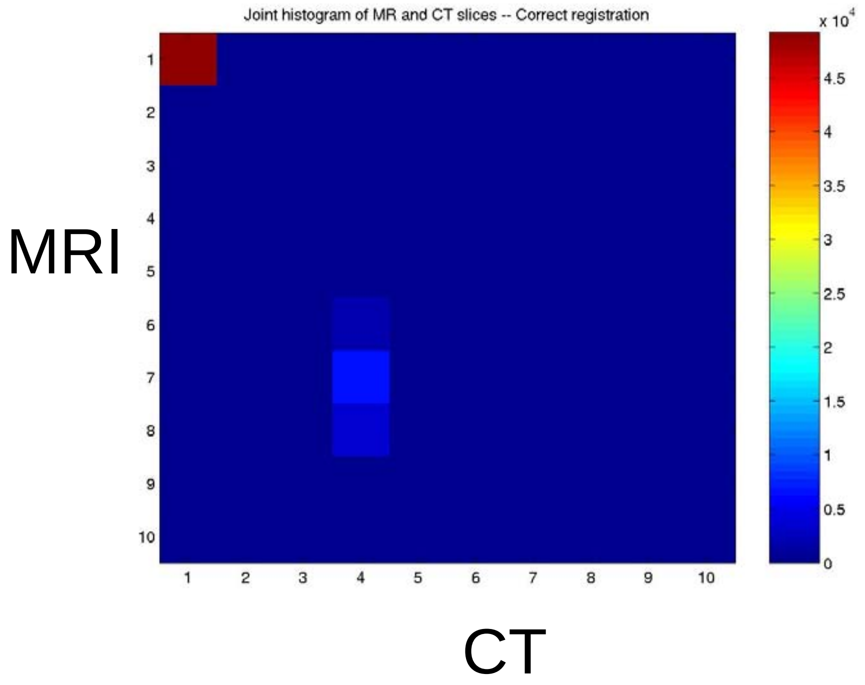
(1,2)(2,2)(1,2)(1,3)(2,3)(1,3)(1,2)(2,2)(1,2)

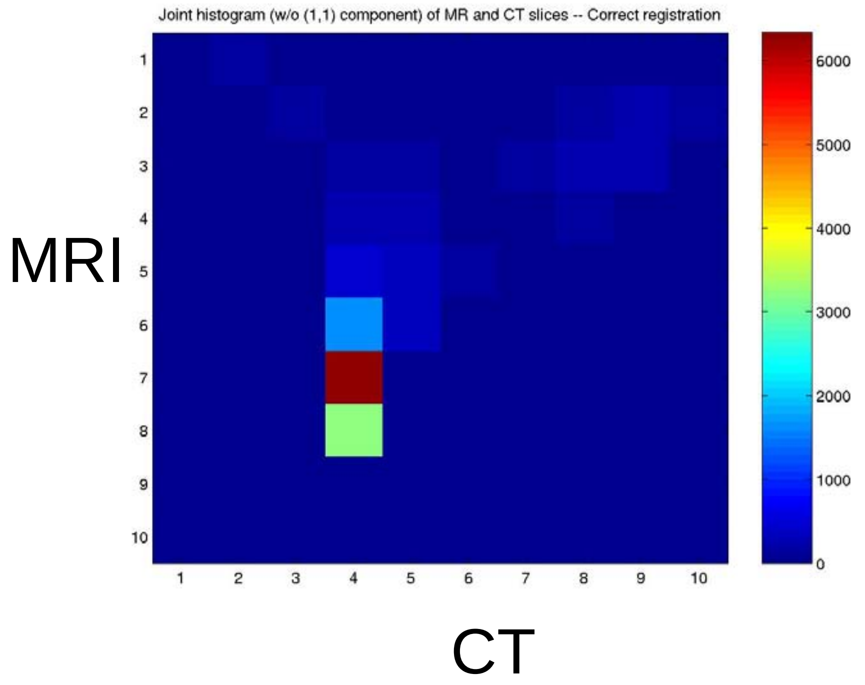
MRI & CT pairs



misaligned | slightly misaligned | aligned

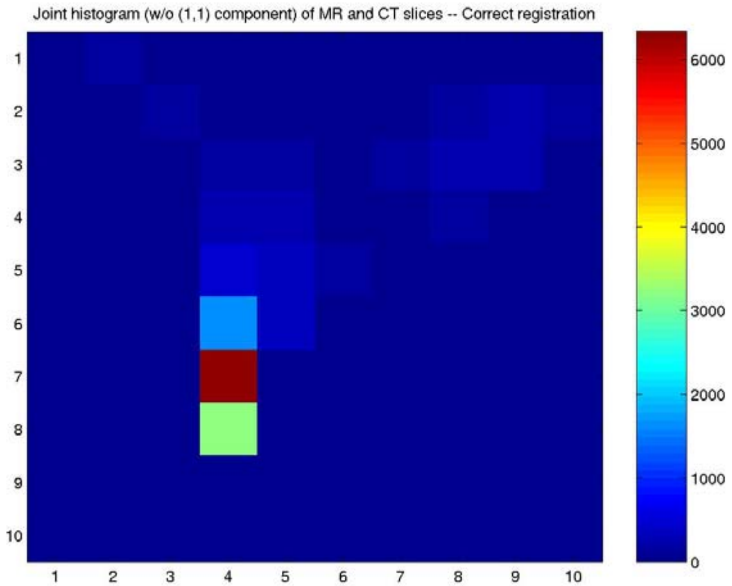
Joint histogram: MRI & CT registered





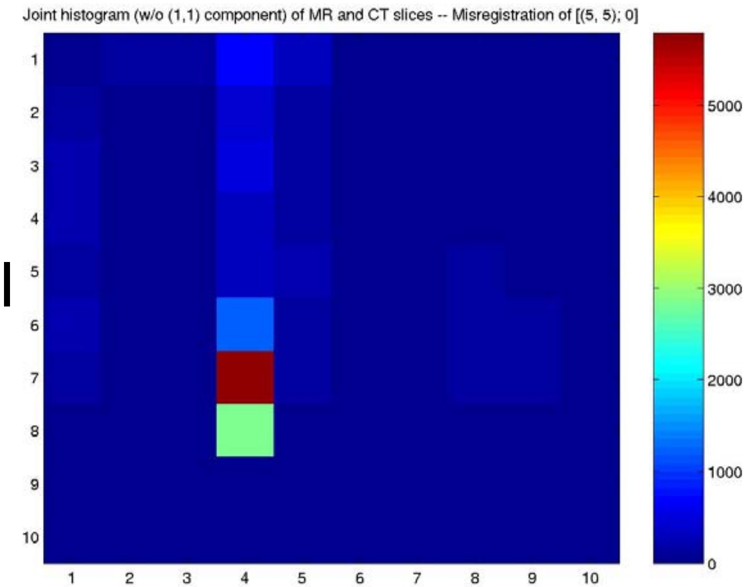
Joint histogram:
MRI & CT; slightly off

MRI



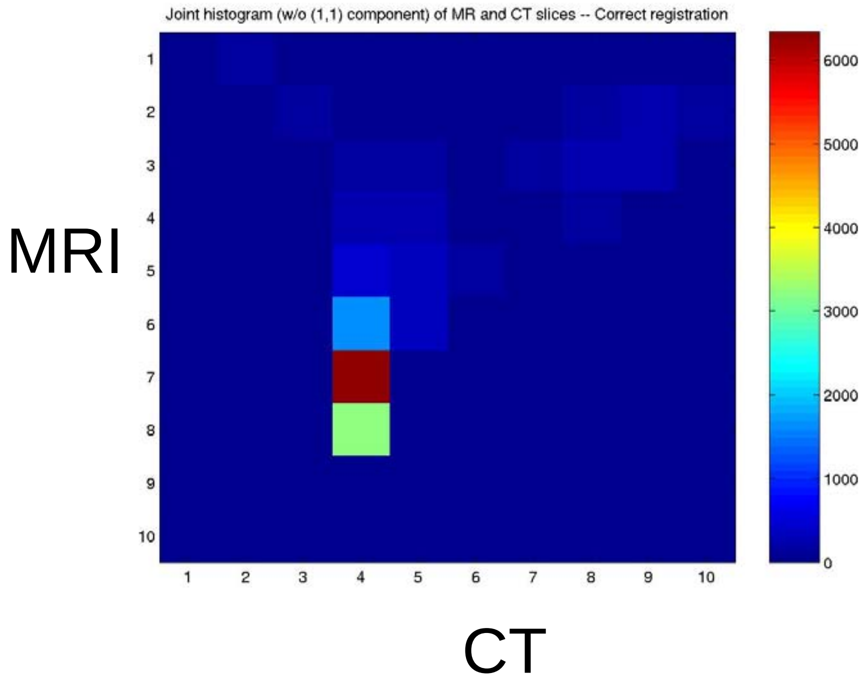
CT
Correct registration

MRI

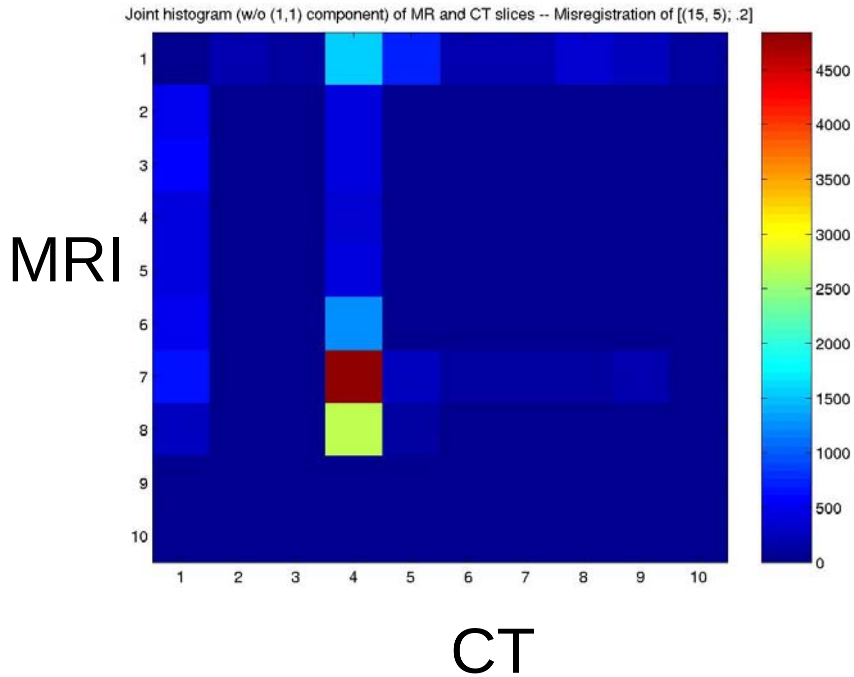


CT
Slight mis-registration

Joint histogram: MRI & CT; significantly off



Correct registration



Significant mis-registration

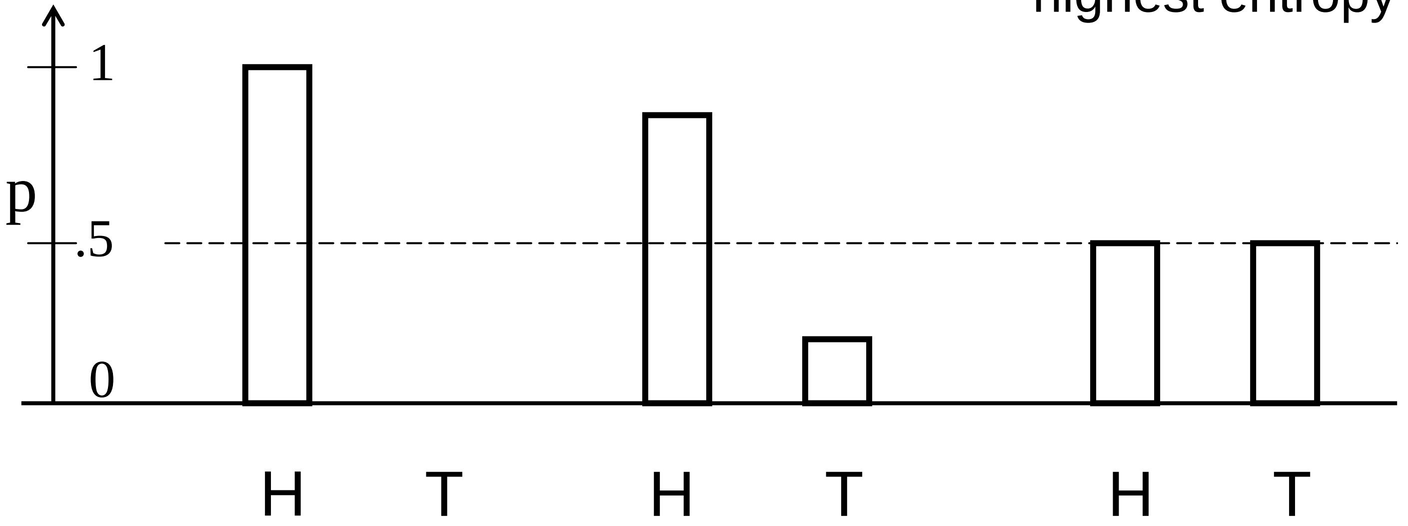
entropy: $$H(p(x)) \equiv -\sum_x p(x)\log_2(p(x))$$

e.g.:

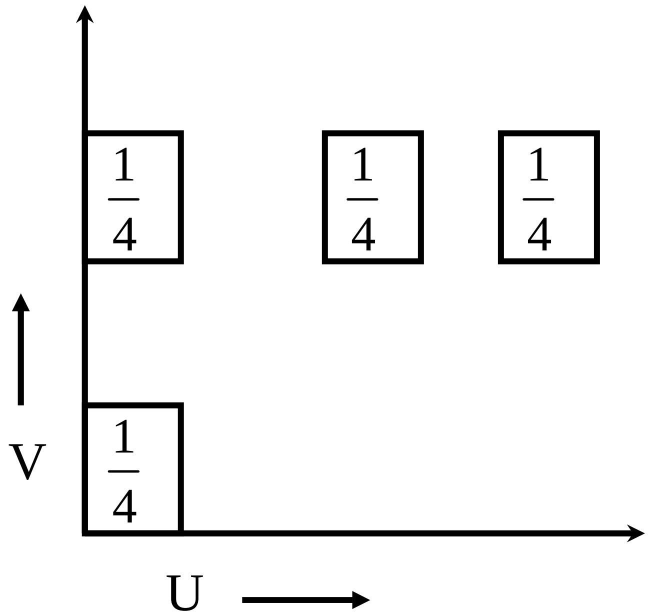
<u>predictable coin</u> -- no uncertainty, lowest entropy

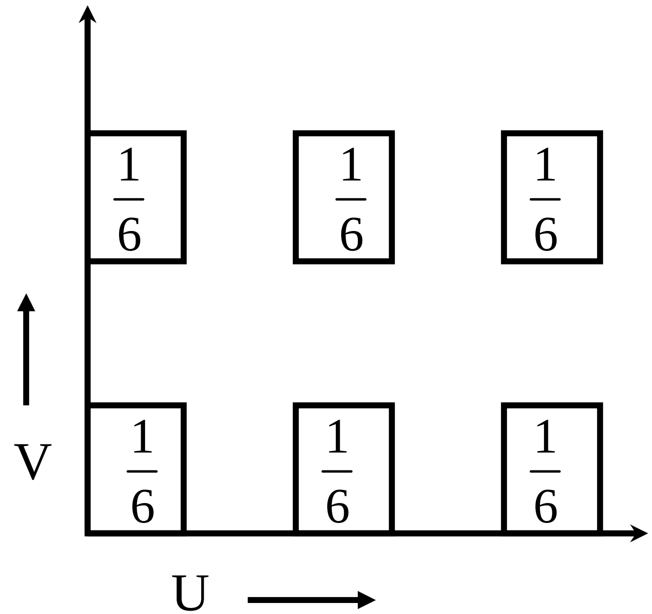
<u>biased coin</u> -- moderate uncertainty, moderate entropy

<u>fair coin</u> -- most uncertain, highest entropy



Examples of joint intensity distributions

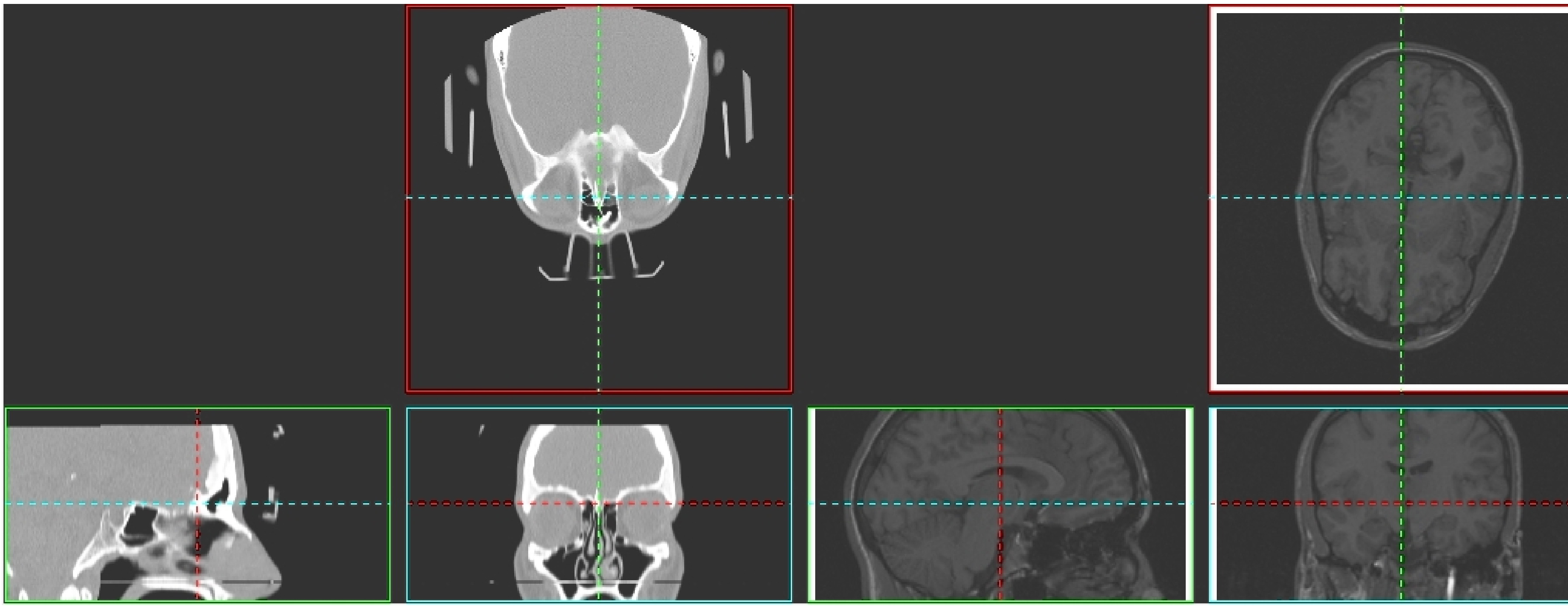




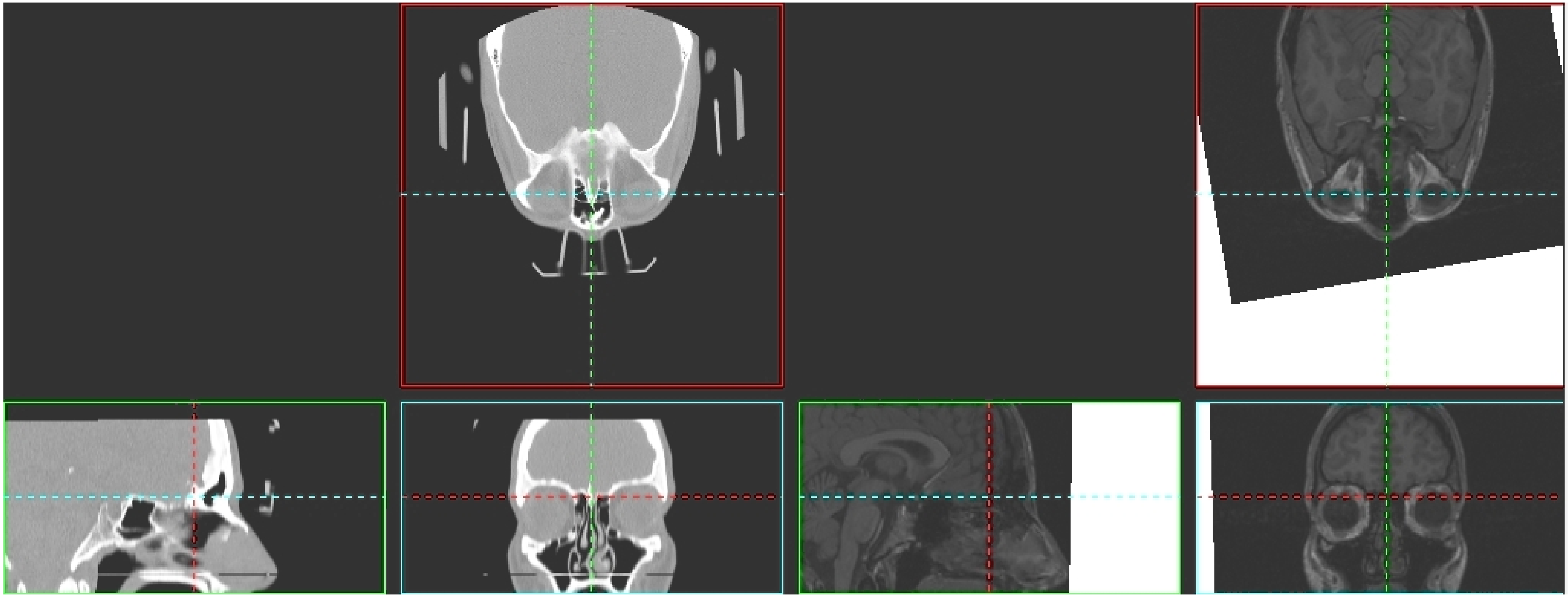
$$H = \log_2(4)$$

$$H = \log_2(6)$$

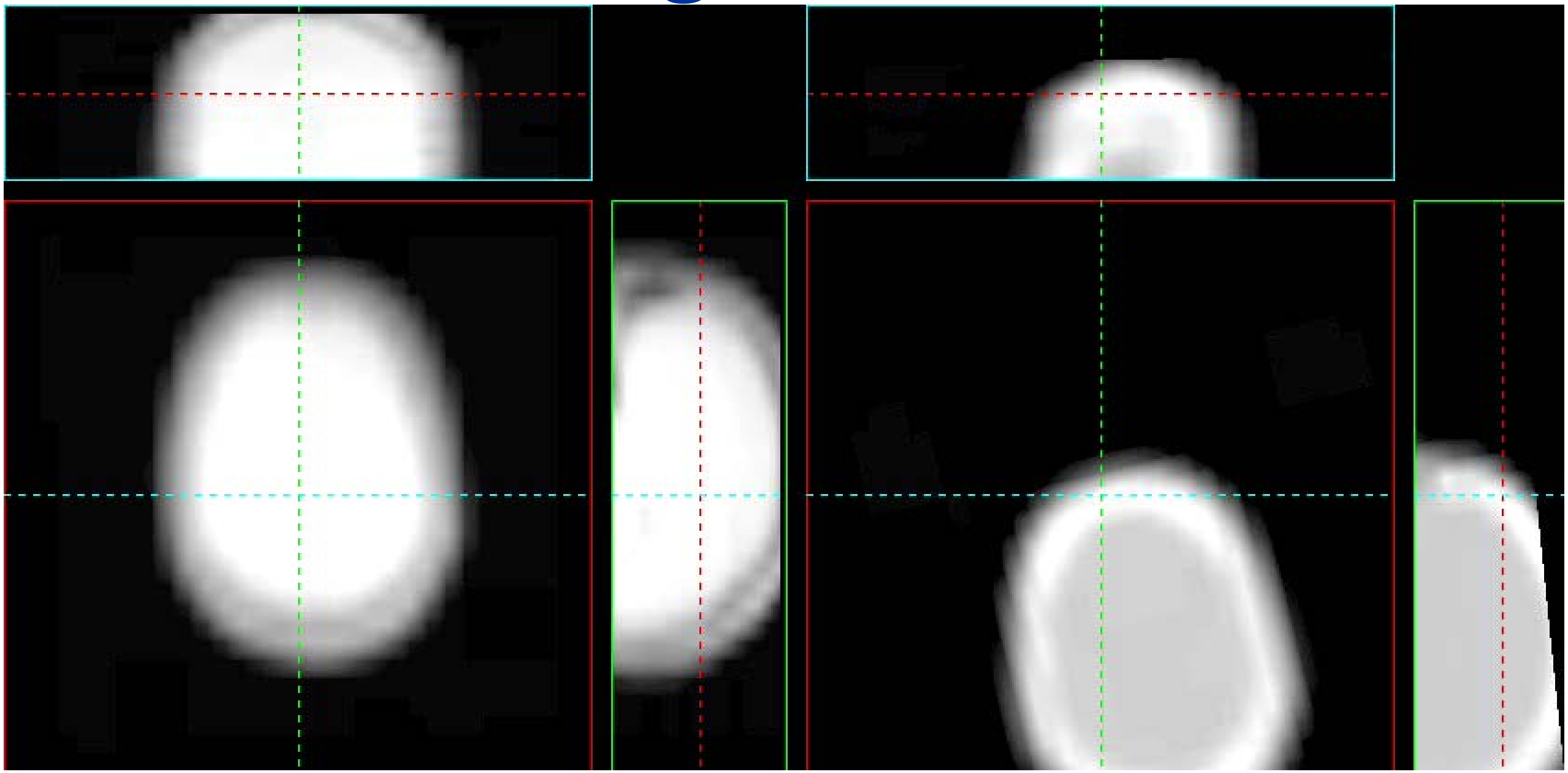
"Real" CT-MR registration: 3D starting position



CT-MR registration final result



CT-MR registration movie



From: Wells, W. M., et al. "Multi-modal Volume Registration by Maximization of Mutual Information." *Medical Image Analysis* 1, no. 1 (March 1996): 35-51.
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Registration by minimization of joint entropy

$$\cong$$

Alignment by maximization of mutual information

- Viola, PhD 1995
- Pluim, PhD 2001
- many more papers (2002: more than 100)
- West Fitzpatrick et al 1998 JCAT
  - …clear winner MR/CT…
  - …also good for MR/PET…

- Chuck Meyer et al. (Radiology Dept, U Mich.)
  - Non-rigid mutual information registration
  - Thin-plate spline warp model
  - Downhill simplex optimizer
    - rat brain auto-radiograph ↔ CT
    - breast MRI ↔ breast MRI

END