ESD.33 -- Systems Engineering

Session #9

Critical Parameter Management & Error Budgeting

Dan Frey

Plan for the Session

- Follow up on session #8
- Critical Parameter Management
- Probability Preliminaries
- Error Budgeting
  - Tolerance
  - Process Capability
  - Building and using error budgets
- Next steps

S - Curves

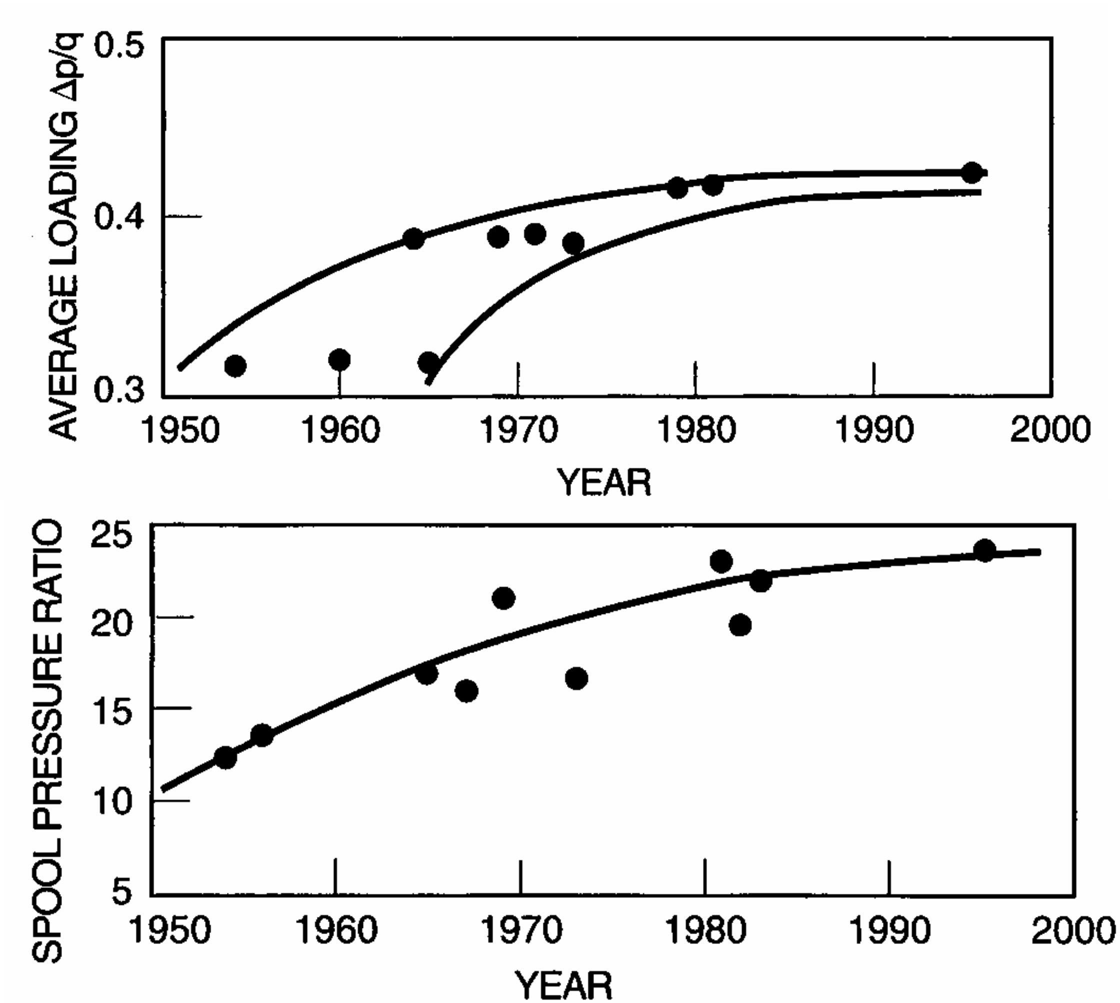
Atish Banergee –

We first studied S-curves in technology strategy…The question remained why the S-curve has the peculiar shape. Well I found the answer in system dynamics. It is a general phenomenon and not restricted to technology.

It can be thought of as two curves:

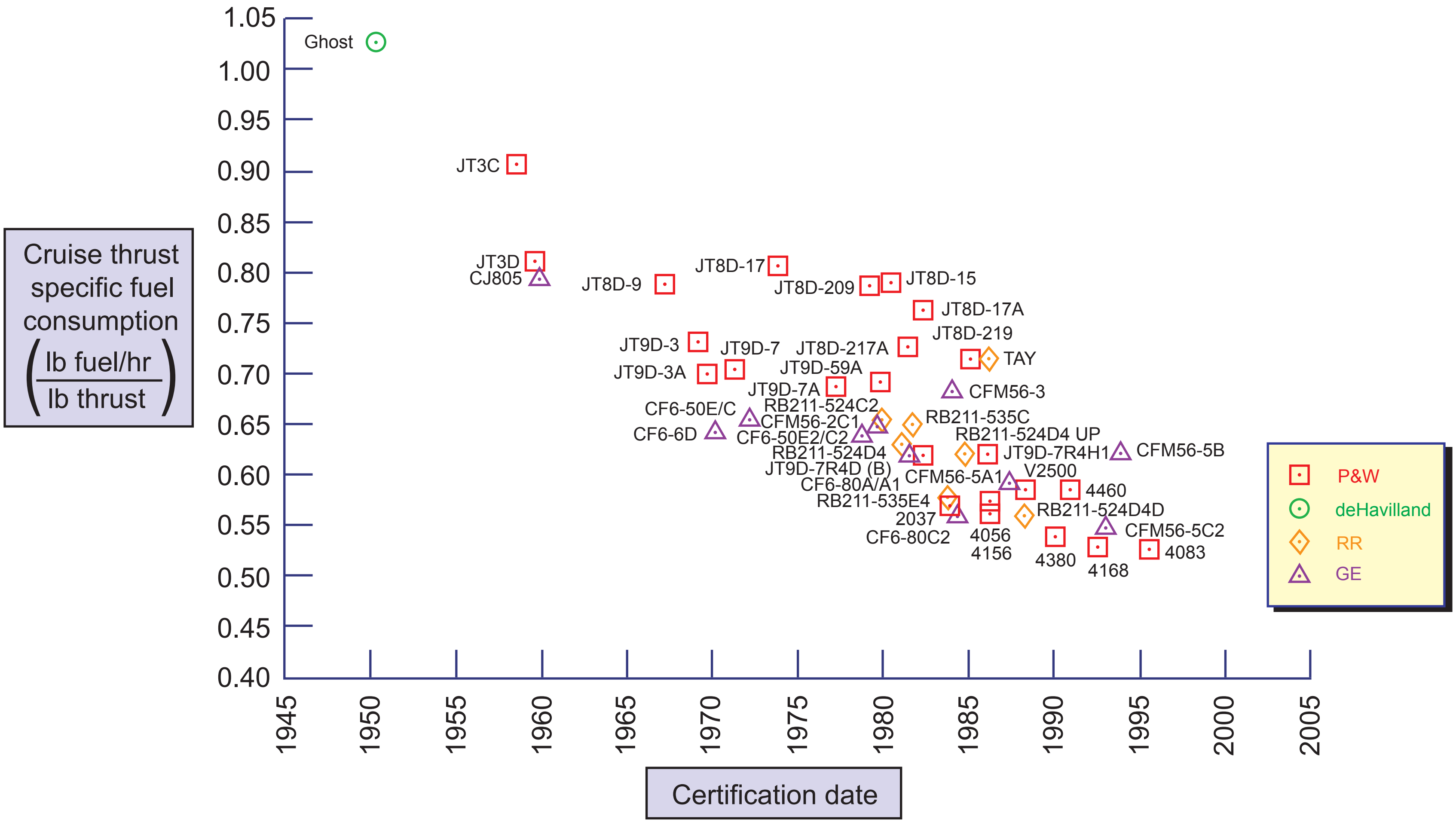
1. The lower part of the curve is growth with acceleration....
2. The upper part of the s-curve is called a goal-seeking curve and can be thought of as growth with deceleration…

Trends in Compressor Performance



Wisler, D. C., 1998, Axial Flow Compressor and Fan Aerodynamics", *Handbook of Fluid Dynamics*, CRC Press., ed. R. Johnson.

Evolution of Jet Engine Performance



Adapted from Koff, B. L. "Spanning the World Through Jet Propulsion." AIAA Littlewood Lecture. 1991.

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Critical Parameter Management

- CPM provides discipline and structure
- Produce critical parameter documentation
  - For example, a critical parameter drawing
- Traces critical parameters all the way through to manufacture and use
- Determines process capability (C_p or C_{pk})
- Therefore, requires probabilistic thinking

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Probability Definitions

- Sample space – a list of all possible outcomes of an experiment
  - Finest grained
  - Mutually exclusive
  - Collectively exhaustive
- Event - A collection of points in the sample space

Concept Question

- You roll 2 dice
- Give an example of a single point in the sample space?
- How might you depict the full sample space?
- What is an example of an “event”?

Probability Measure

- Axioms
  - For any event A, $P(A) \geq 0$
  - $P(U)=1$
  - If $A \cap B=\phi$, then $P(A \cup B)=P(A)+P(B)$

For the case of rolling two dice:
A = rolling a 7 and
B = rolling a 1 on at least one die
Is it the case that $P(A+B)=P(A)+P(B)$?

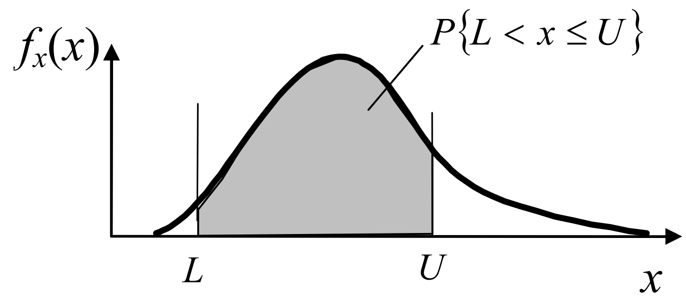
Discrete Random Variables

- A random variable that can assume any of a set of discrete values
- Probability mass function
  - $p_x(x_o)$ = probability that the random variable x will take the value x_o
- Let's build a pmf for rolling two dice
  - random variable x is the total

x=10

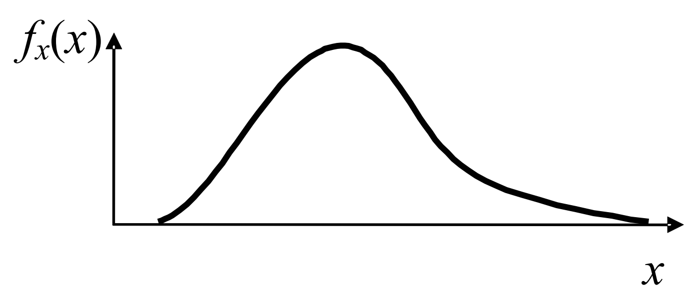
Continuous Random Variables

- Can take values anywhere within continuous ranges
- Probability density functions obey three rules
  - $P\{L < x \leq U\} = \int_L^U f_x(x)\mathrm{d}x$
  - $0 \leq f_x(x)$ for all x
  - $\int_{-\infty}^{\infty} f_x(x)\mathrm{d}x = 1$



Measures of Central Tendency

- Expected value $$E(g(x)) = \int_a^b g(x) f_x(x) \mathrm{d}x$$
- Mean $$\mu = E(x)$$
- Arithmetic average $$\frac{1}{n}\sum_{i=1}^{n} x_i$$
- Median
- Mode



Measures of Dispersion

- Variance $VAR(x) = \sigma^2 = E((x - E(x))^2)$
- Standard deviation $\sigma = \sqrt{E((x - E(x))^2)}$
- Sample variance $S^2 = \frac{1}{n-1}\sum_{i-1}^{n}(x_i - \overline{x})^2$
- n^{th} central moment $E((x - E(x))^n)$
- Covariance $E((x - E(x))(y - E(y)))$

Sums of Random Variables

- Average of the sum is the sum of the average (regardless of distribution and independence) $E(x+y)=E(x)+E(y)$

- Variance also sums iff independent

$$\sigma^2(x+y)=\sigma(x)^2+\sigma(y)^2$$

- This is the origin of the RSS rule
  - Beware of the independence restriction!

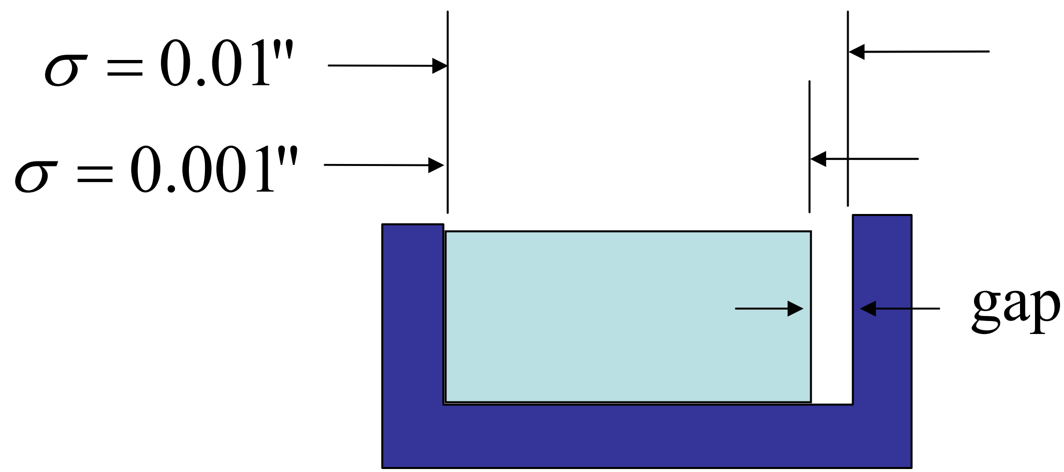
Concept Test

- A bracket holds a component as shown. The dimensions are independent random variables with standard deviations as noted. Approximately what is the standard deviation of the gap?

  A) 0.011″

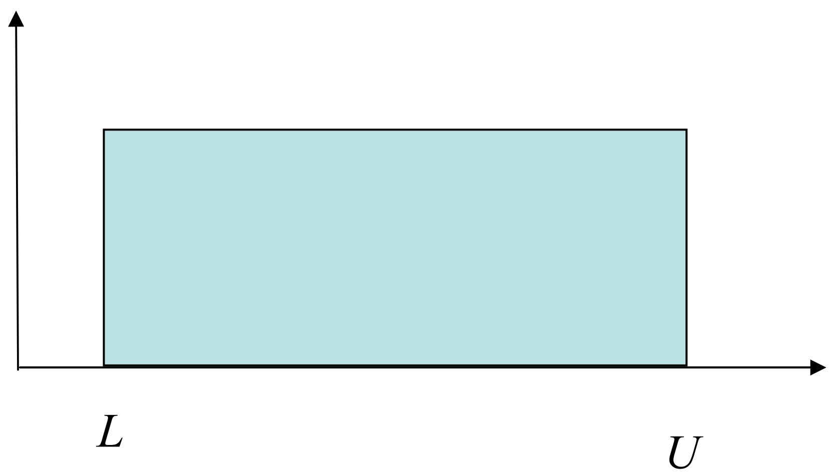
  B) 0.01″

  C) 0.001″



Uniform Distribution

- A reasonable (conservative) assumption when you know the limits of a variable but little else



$$\sigma = (U - L)\Big/ 2\sqrt{3}$$

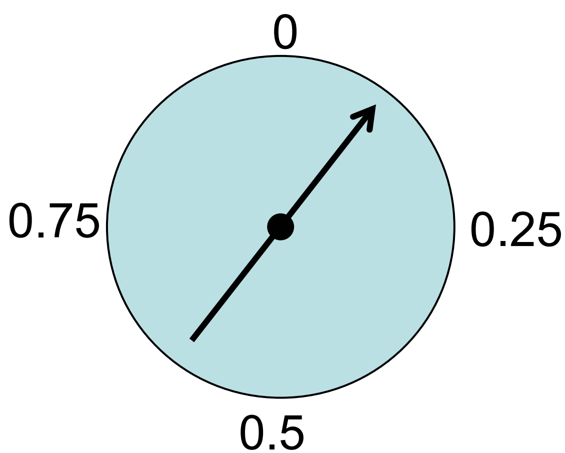
Basic Application

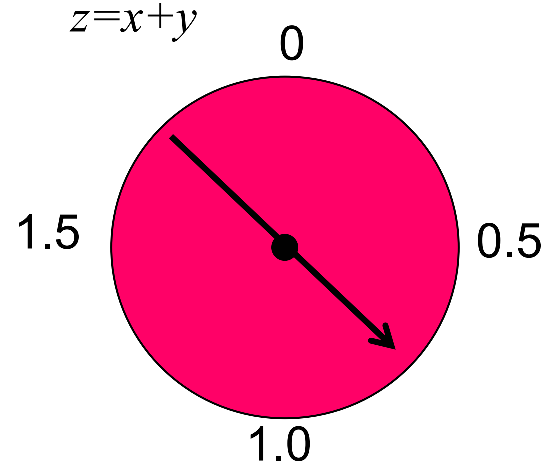
- I have two spinners

x=result of blue spinner

y=result of red spinner

$z=x+y$





- What are the pdfs for variables x, y, and z?

$$P\{a < x \le b\} = \int_a^b f_x(x)\mathrm{d}x \qquad \int_{-\infty}^{\infty} f_x(x)\mathrm{d}x = 1 \qquad 0 \le f_x(x) \text{ for all } x$$

Simulation Can Quickly Answer the Question

```
trials=10000;nbins=trials/1000;
x= random('Uniform',0,1,trials,1);
y= random('Uniform',0,2,trials,1);
z=x+y;
subplot(3,1,1); hist(x,nbins); xlim([0 3]);
subplot(3,1,2); hist(y,nbins); xlim([0 3]);
subplot(3,1,3); hist(z,nbins); xlim([0 3]);
```

Probability Distribution of Sums

- If *z* is the sum of two random variables *x* and *y*

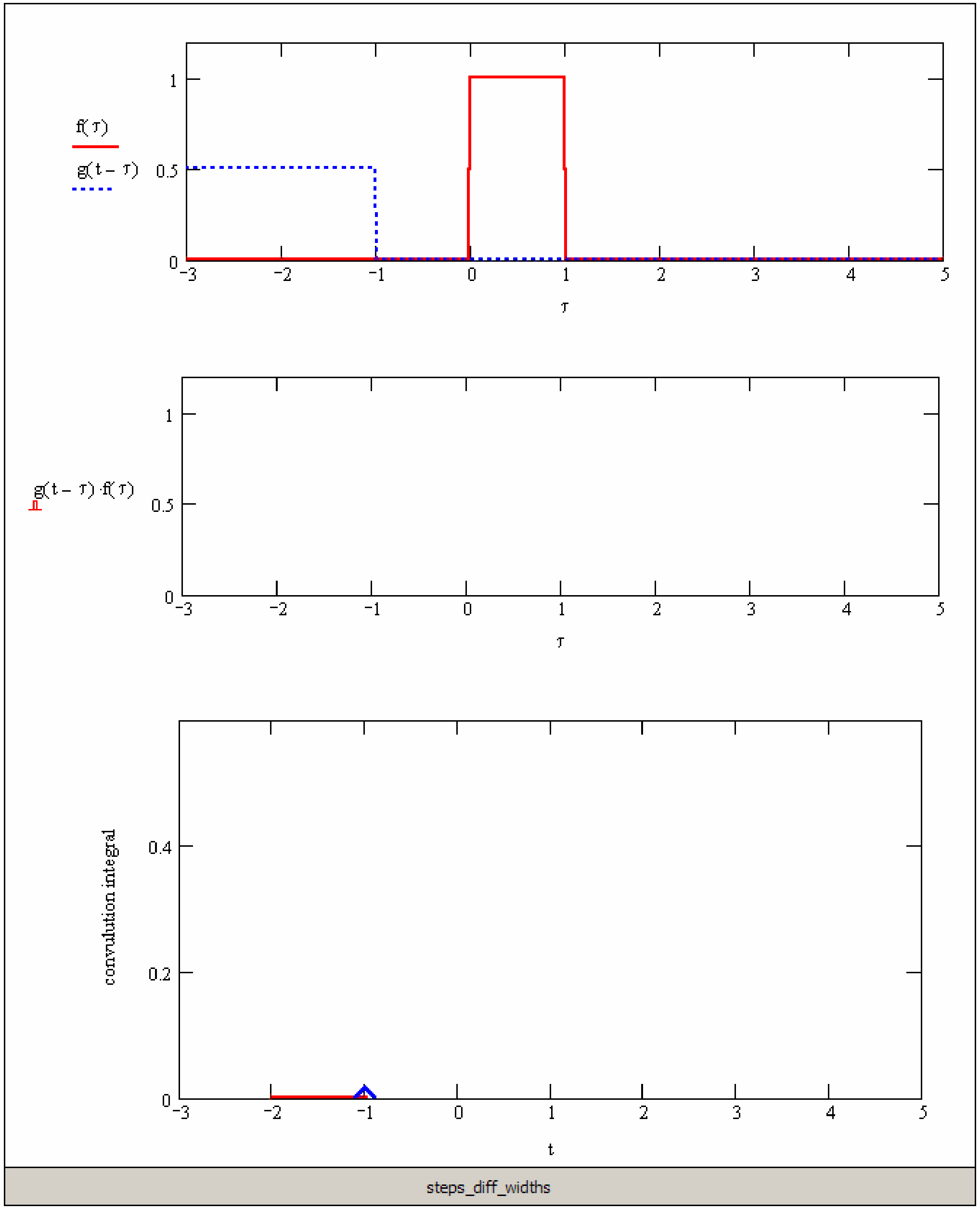
$$z = x + y$$

- Then the probability density function of z can be computed by convolution

$$p_z(z) = \int_{-\infty}^{z} x(z-\zeta)\,y(\zeta)\,\mathrm{d}\zeta$$

Convolution

$$p_z(z) = \int_{-\infty}^{z} x(z-\zeta)\,y(\zeta)\,\mathrm{d}\zeta$$



Convolution

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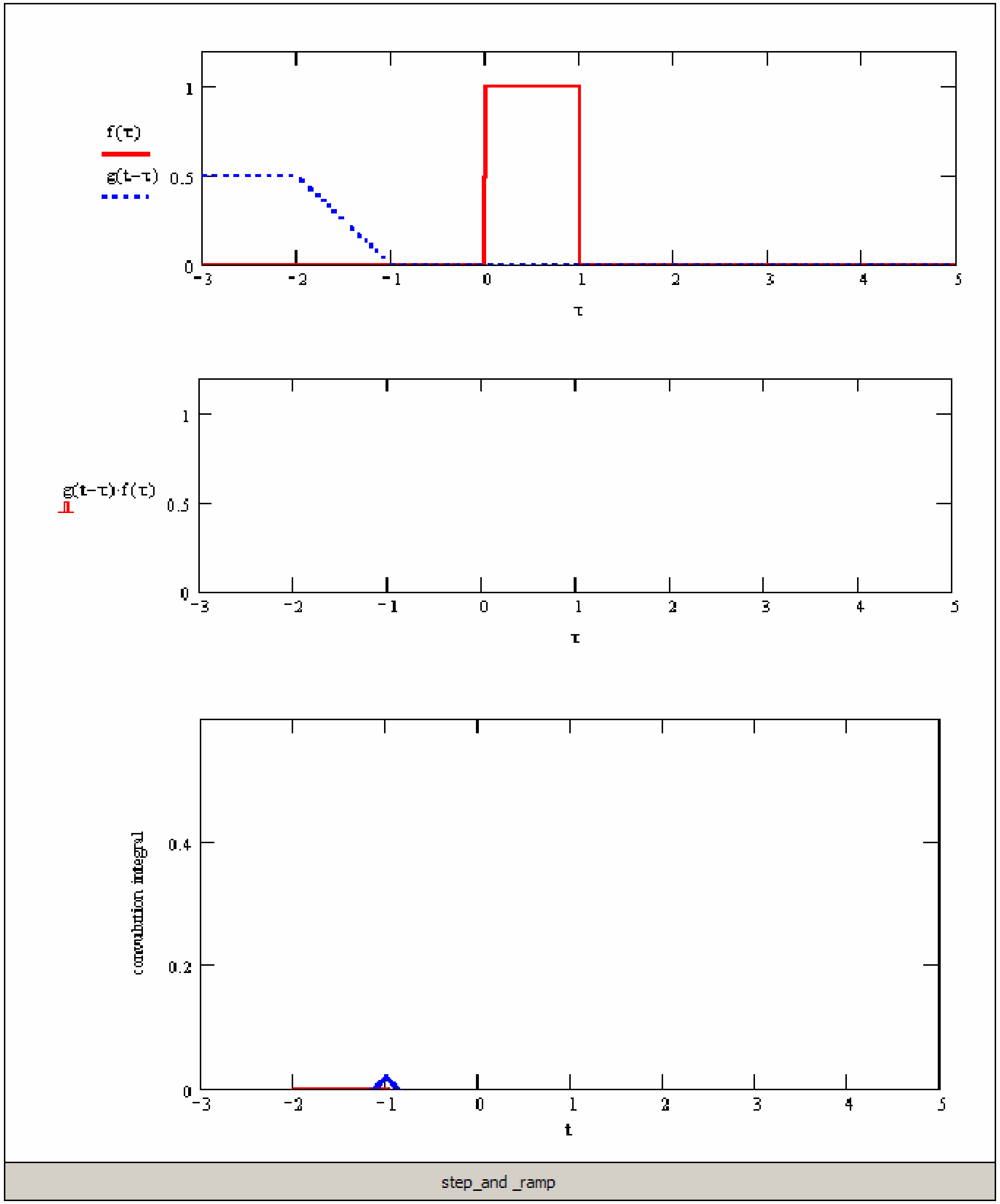


Central Limit Theorem

The mean of a sequence of *n* iid random variables with

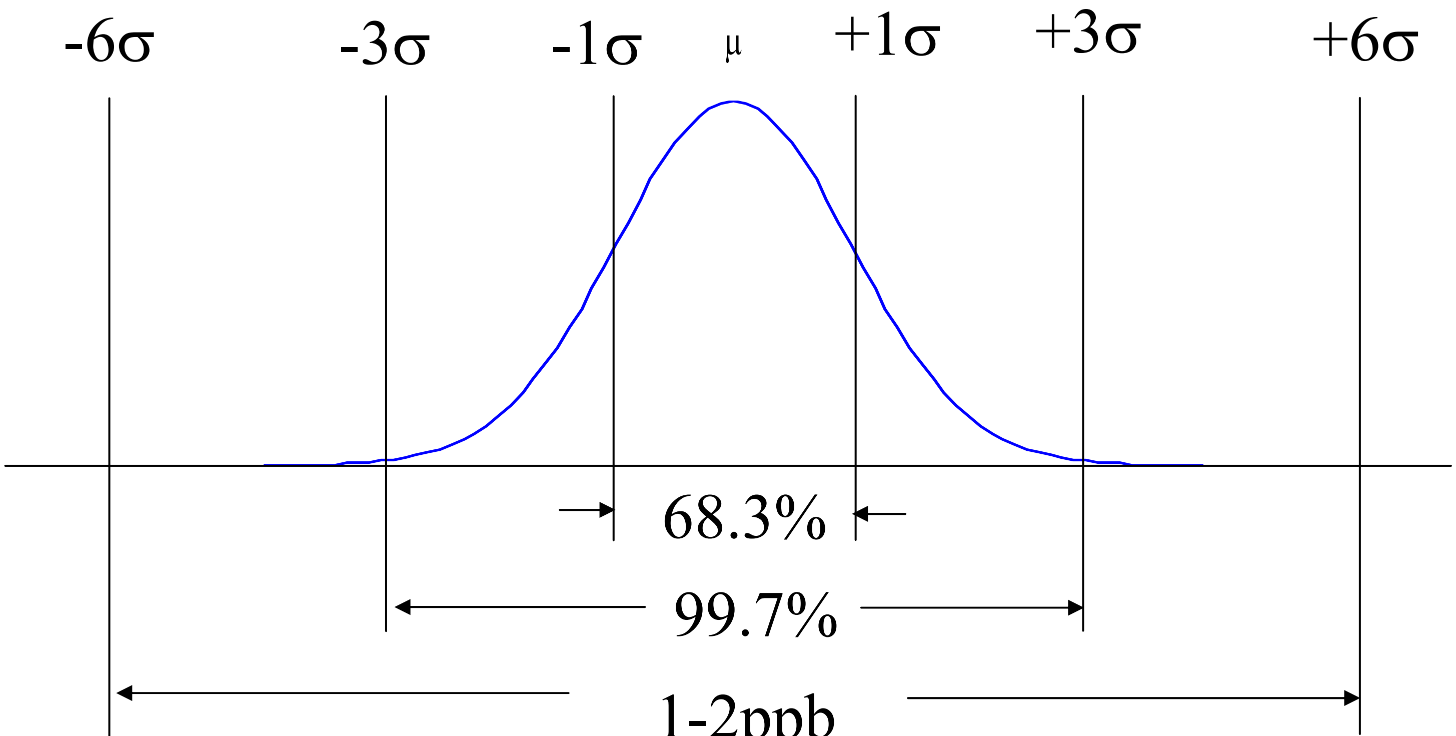
– Finite μ

– $E\left(\left|x_i - E(x_i)\right|^{2+\delta}\right) < \infty \quad \delta > 0$

approximates a normal distribution in the limit of a large *n*.

Normal Distribution

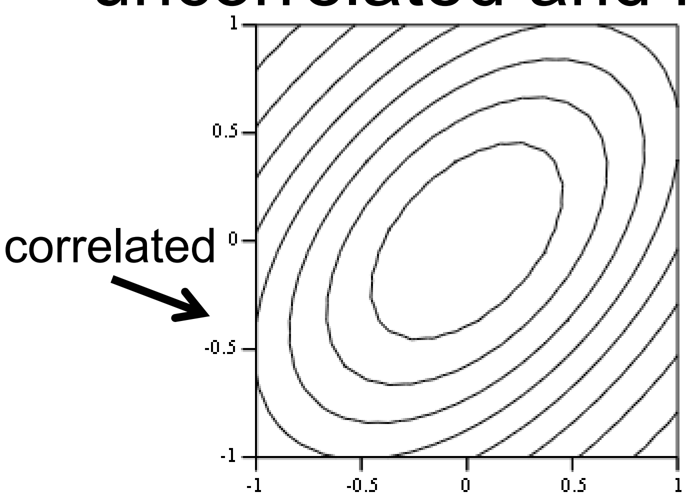
$$f_x(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

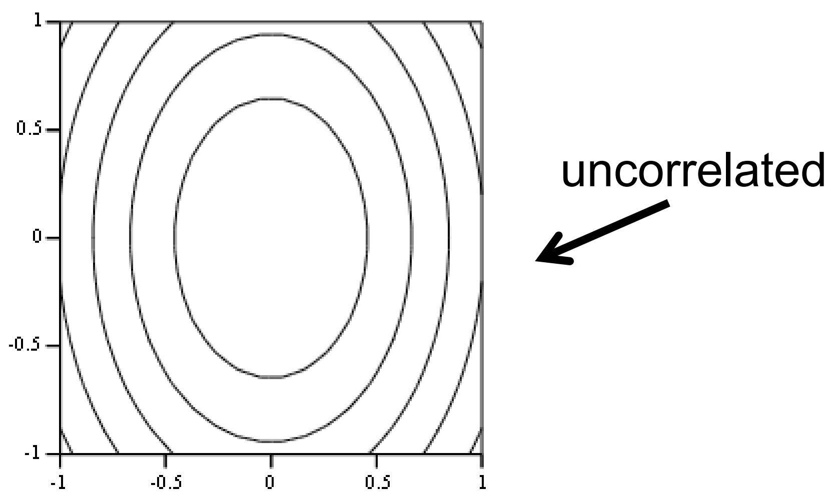


Joint Normal Distribution

$$p(\mathbf{x}) = \frac{1}{\left(\sqrt{2\pi}\right)^m \sqrt{|\mathbf{K}|}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \mathbf{K}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right\}$$

- The lines of constant probability density are ellipsoids
- If the matrix $\mathbf{K}$ is diagonal, then the variables are uncorrelated and independent





Independence

- Random variables x and y are said to be independent iff

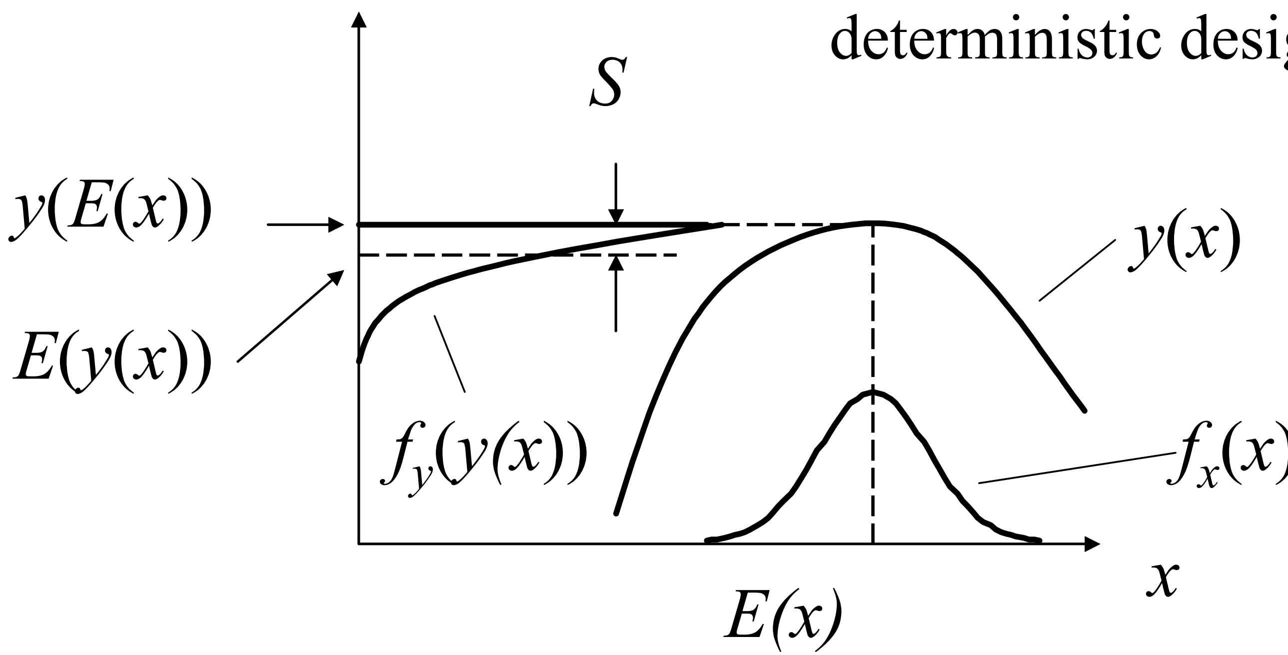
$$f_{xy}(x,y) = f_x(x) f_y(y)$$

- Or, knowledge of x provides no information to update the distribution of y

Expectation Shift

$S = E(y(\mathbf{x})) - y(E(\mathbf{x}))$

Under utility theory (DBD), *S* is a key difference between probabilistic and deterministic design



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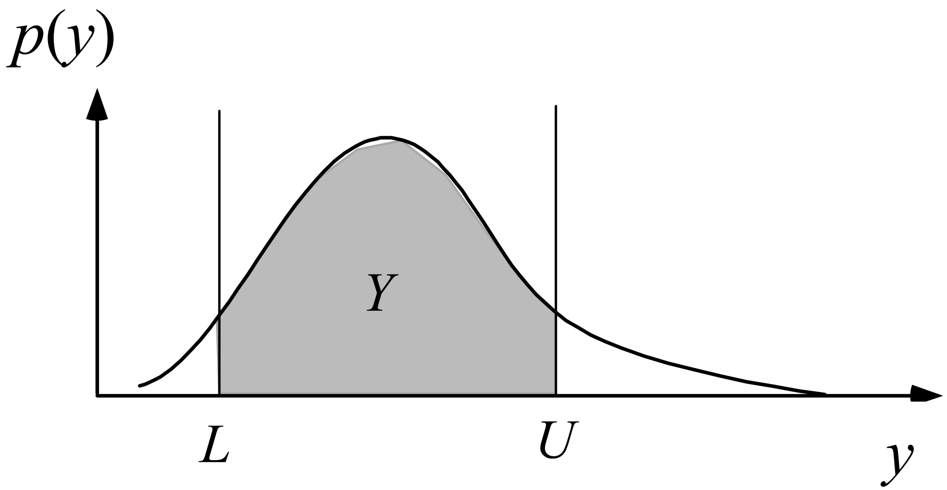
Error Budgets

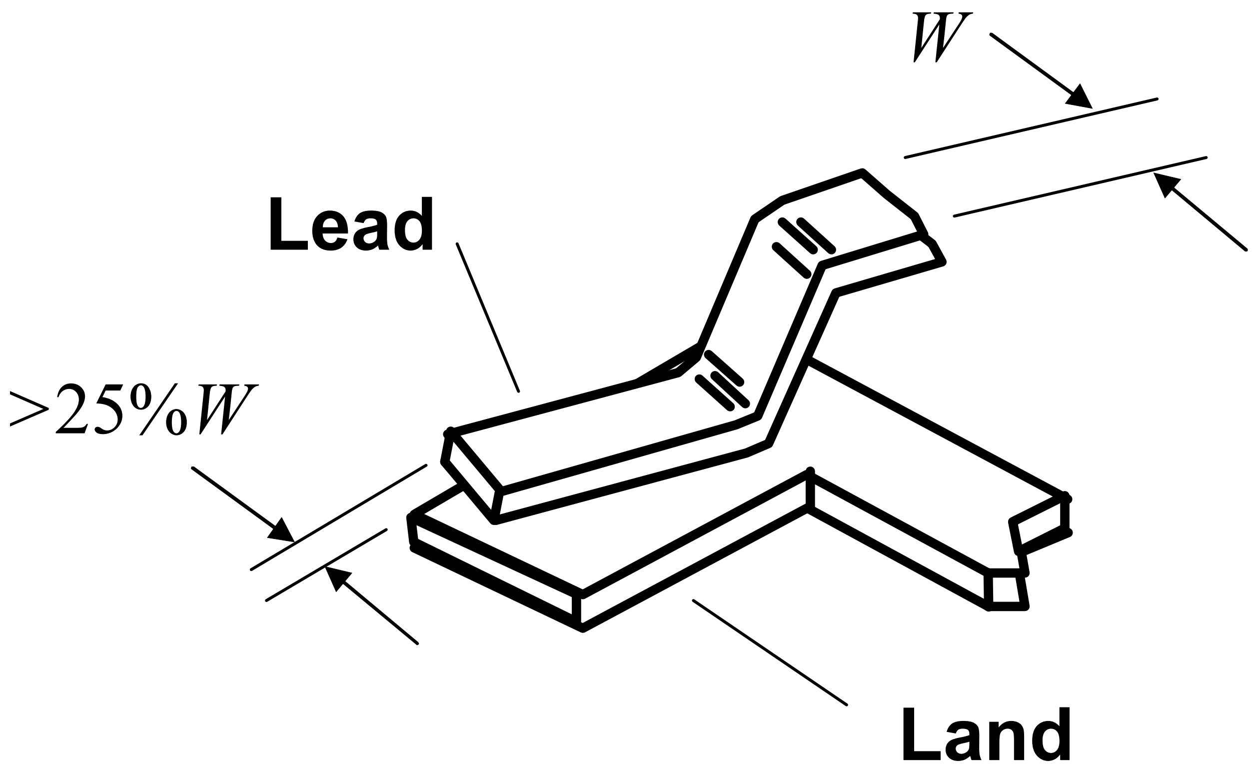
- A tool for predicting and managing variability in an engineering system
- A model that propagates errors through a system
- Links aspects of the design and its environment to tolerance and capability
- Used for tolerance design, robust design, diagnosis…

Engineering Tolerances

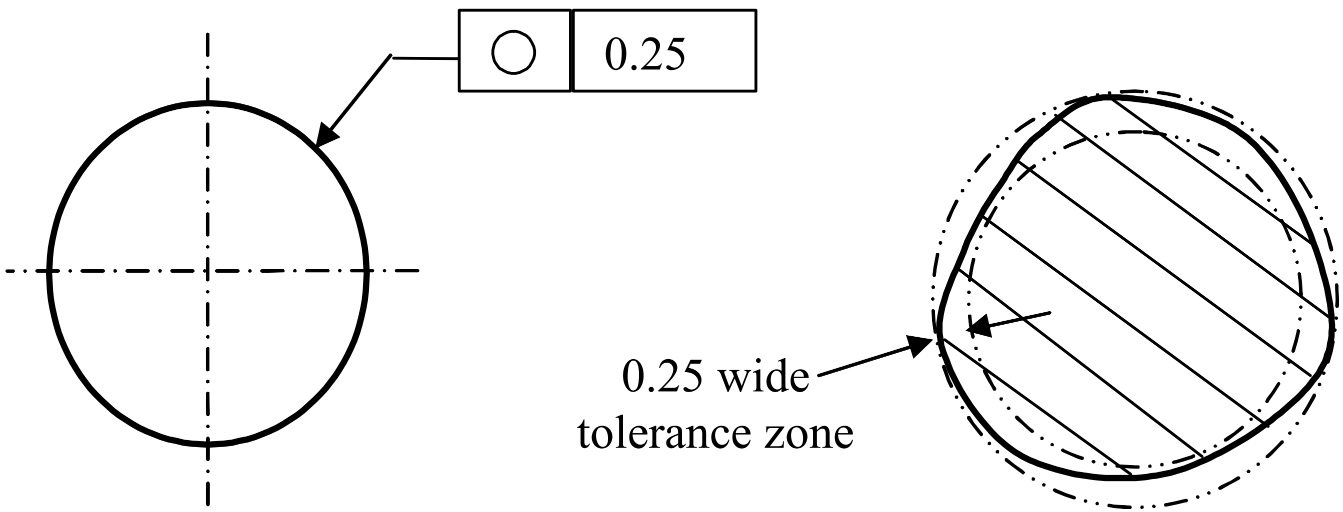
- Tolerance --The total amount by which a specified dimension is *permitted to vary* (ANSI Y14.5M)
- Every component within spec adds to the yield (*Y*)



Tolerance on Position



Tolerance of Form



THIS ON A DRAWING

MEANS THIS

GD&T Symbols

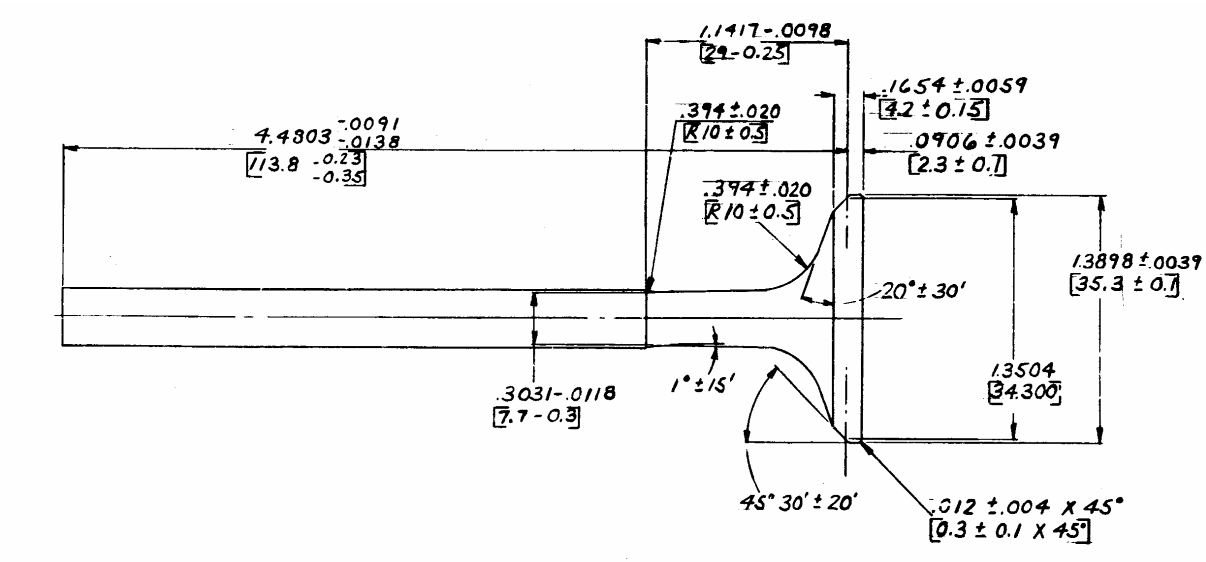
Geometric Characteristic Symbols

	Type of Tolerance	Characteristic	Symbol
For Individual Features	Form	Straightness	⏤
		Flatness	⏥
		Circularity (Roundness)	○
		Cylindricity	⌭
For Individual or Related Features	Profile	Profile of a Line	⌒
		Profile of a Surface	⌓
For Related Features	Orientation	Angularity	∠
		Perpendicularity	⊥
		Parallelism	∥
	Location	Position	⌖
		Concentricity	◎
	Runout	Circular Runout	↗ †
		Total Runout	⌰ †

† Arrowhead(s) may be filled in.

Multiple Tolerances

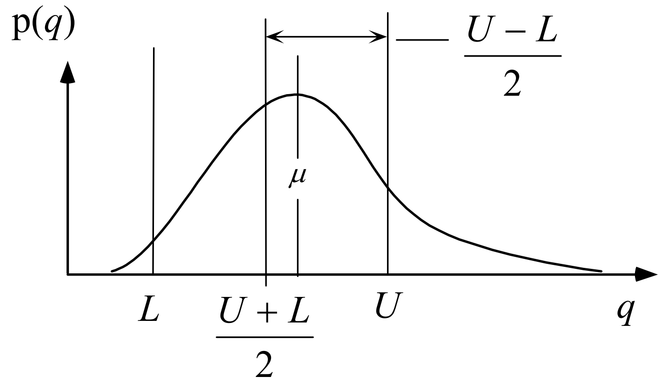
- Most products have many tolerances
- Tolerances are pass / fail
- *All* tolerances must be met (dominance)



Variation in Manufacture

- Many noise factors affect the system
- Some noise factors affect multiple dimensions (leads to correlation)

Process Capability Indices



- Process Capability Index $C_p \equiv \dfrac{(U-L)/2}{3\sigma}$
- Bias factor $k \equiv \dfrac{\left|\mu - \dfrac{U+L}{2}\right|}{(U-L)/2}$
- Performance Index $C_{pk} \equiv C_p(1-k)$

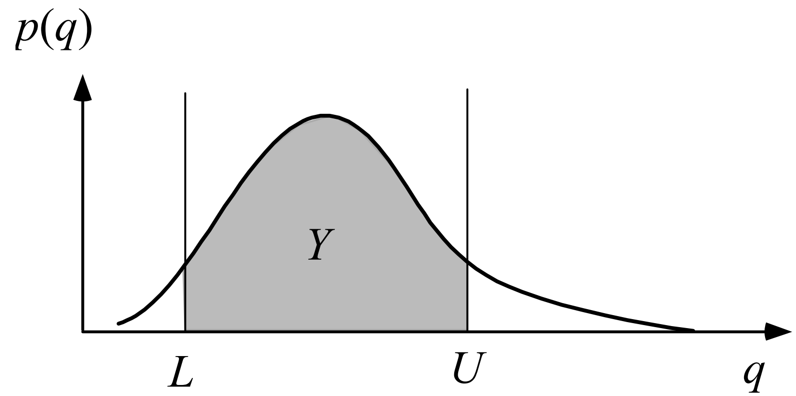
Concept Test

- Motorola's "6 sigma" programs suggest that we should strive for a C_p of 2.0. If this is achieved but the mean is off target so that k=0.5, estimate the process yield.

C_p and k Determine Yield

- By definition

$$Y_{FT} = \int_L^U p(q)\,\mathrm{d}q$$

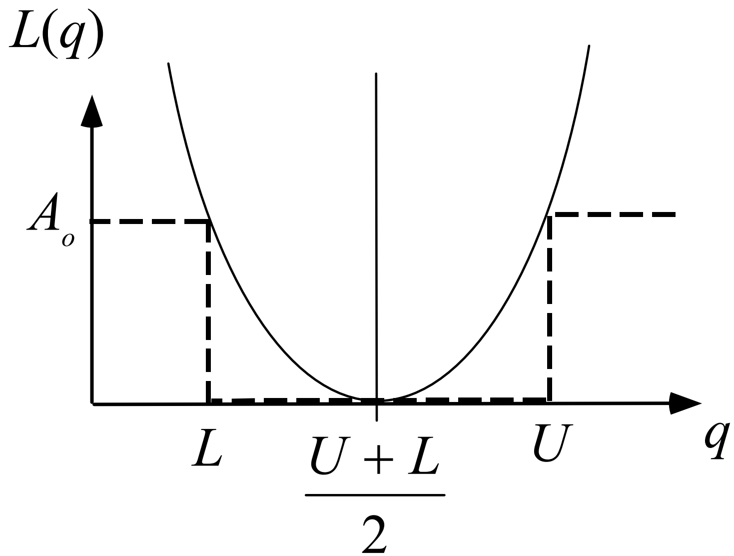


- If Gaussian

$$Y_{FT} = \frac{1}{2}\left[\mathrm{erf}\left(\frac{3\sqrt{2}}{2}C_p(1-k)\right) + \mathrm{erf}\left(\frac{3\sqrt{2}}{2}C_p(1+k)\right)\right]$$

This function to maps C_p and k to yield

C_p and k Determine Quality Loss



—— Taguchi's quality loss function

– – – ANSI's implied quality loss function

$$\text{Quality Loss} = \frac{A_o}{\left[(U-L)/2\right]^2}\left(d - \frac{U+L}{2}\right)^2$$

$$\text{E(Quality Loss)} = A_o\left(k^2 + \frac{1}{9C_p{}^2}\right)$$

Crankshafts

- What does a crankshaft do?
- How would you define the tolerances?
- How does variation affect performance?

Printed Wiring Boards

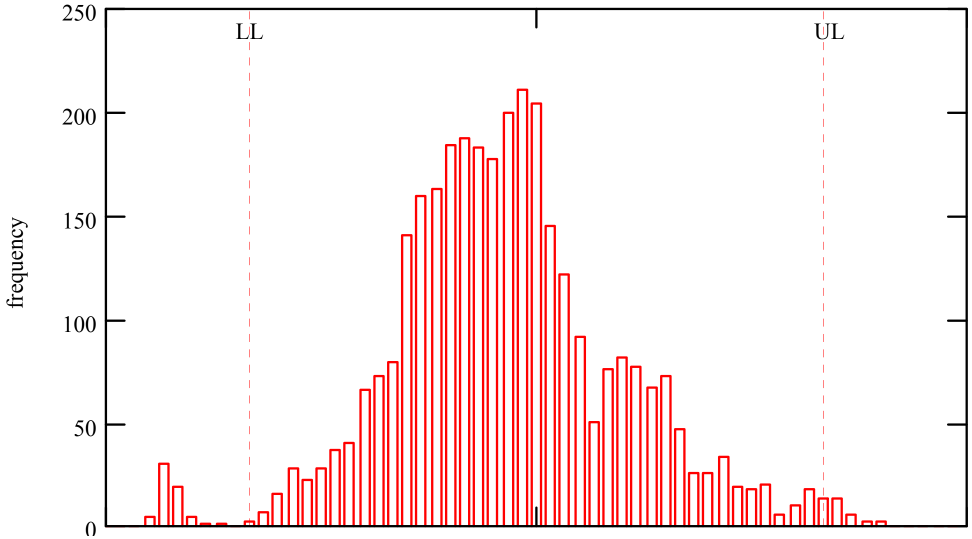
- What does the second level connection do?
- How would you define the tolerances?
- How does variation affect performance?

C_p and k for the System

$$C_p = 0.82$$

$$k = 0.08$$

$$Y_{FT} = 98.3\%$$



Producibility Analysis

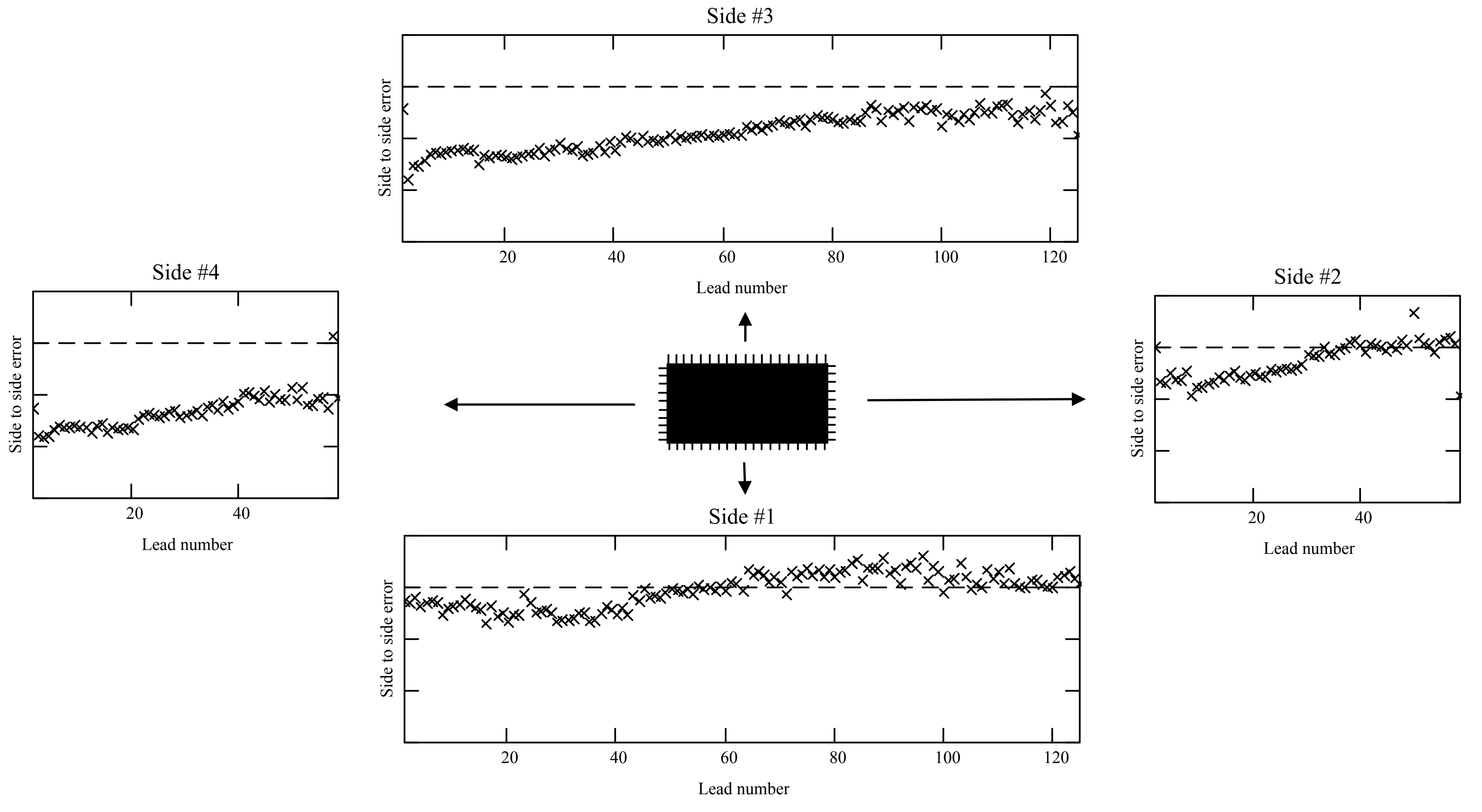
- Rolled throughput yield (Y_{RT})--
  The probability that *all* tolerances are met
- Motorola's approach

$$Y_{RT} = \prod_{i=1}^{m} Y_{FT\,i}$$

- Assumes probabilistic independence

Motorola's formula	Hughes' data
$Y_{RT} = 0.983^{368} = 0.2\%$	$Y_{RT} = 66.7\%$

Surface Mount Data



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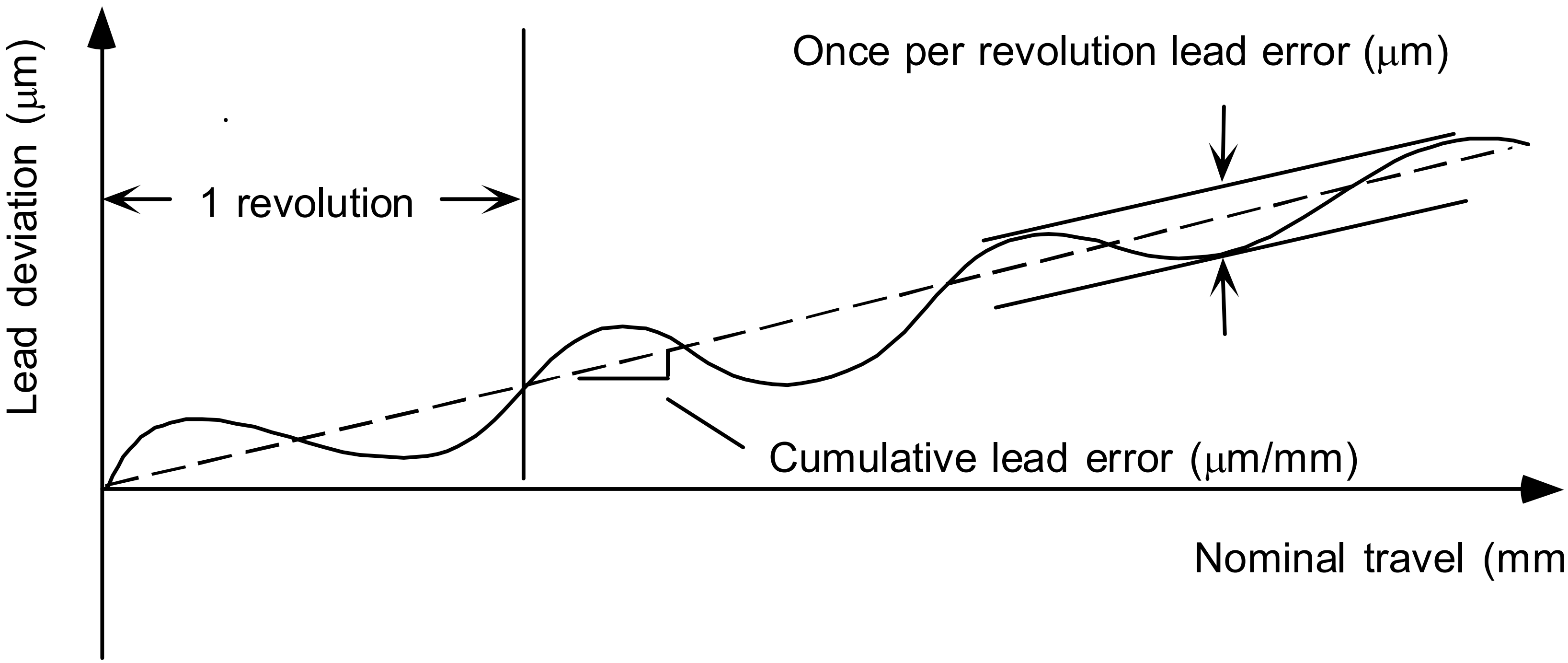
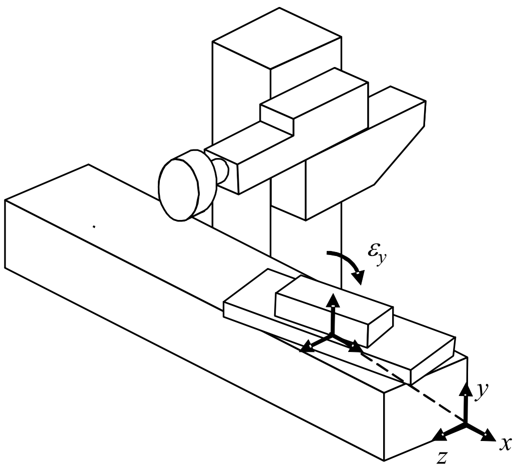
Error Sources

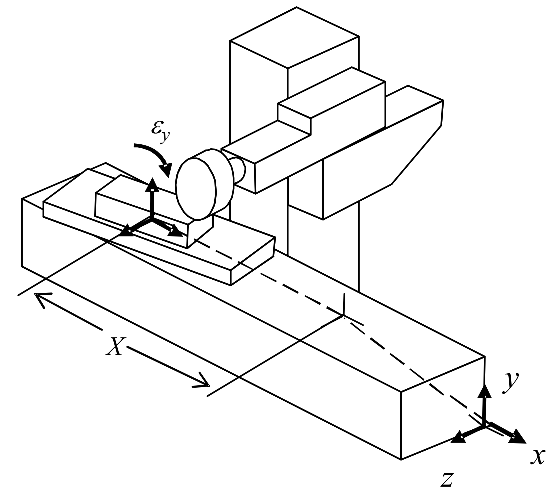
- Kinematic errors
  - Straightness
  - Squareness
  - Bearings
- Drive related errors
- Thermal errors
- Static loading
- Dynamics

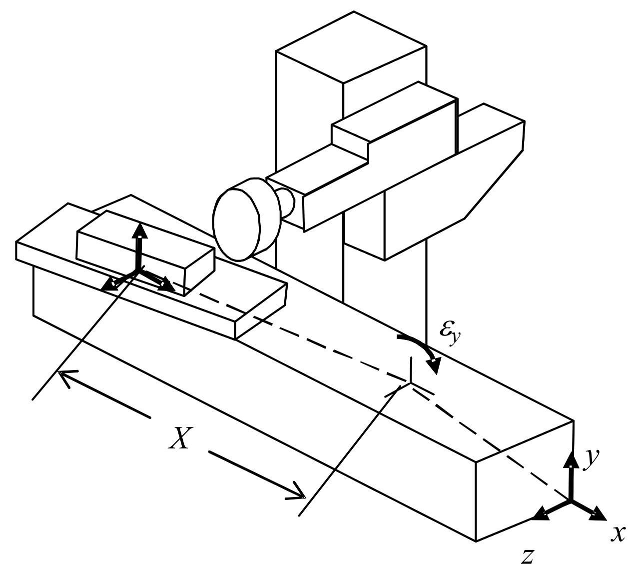
Errors in a Linear Drive



Angular Errors

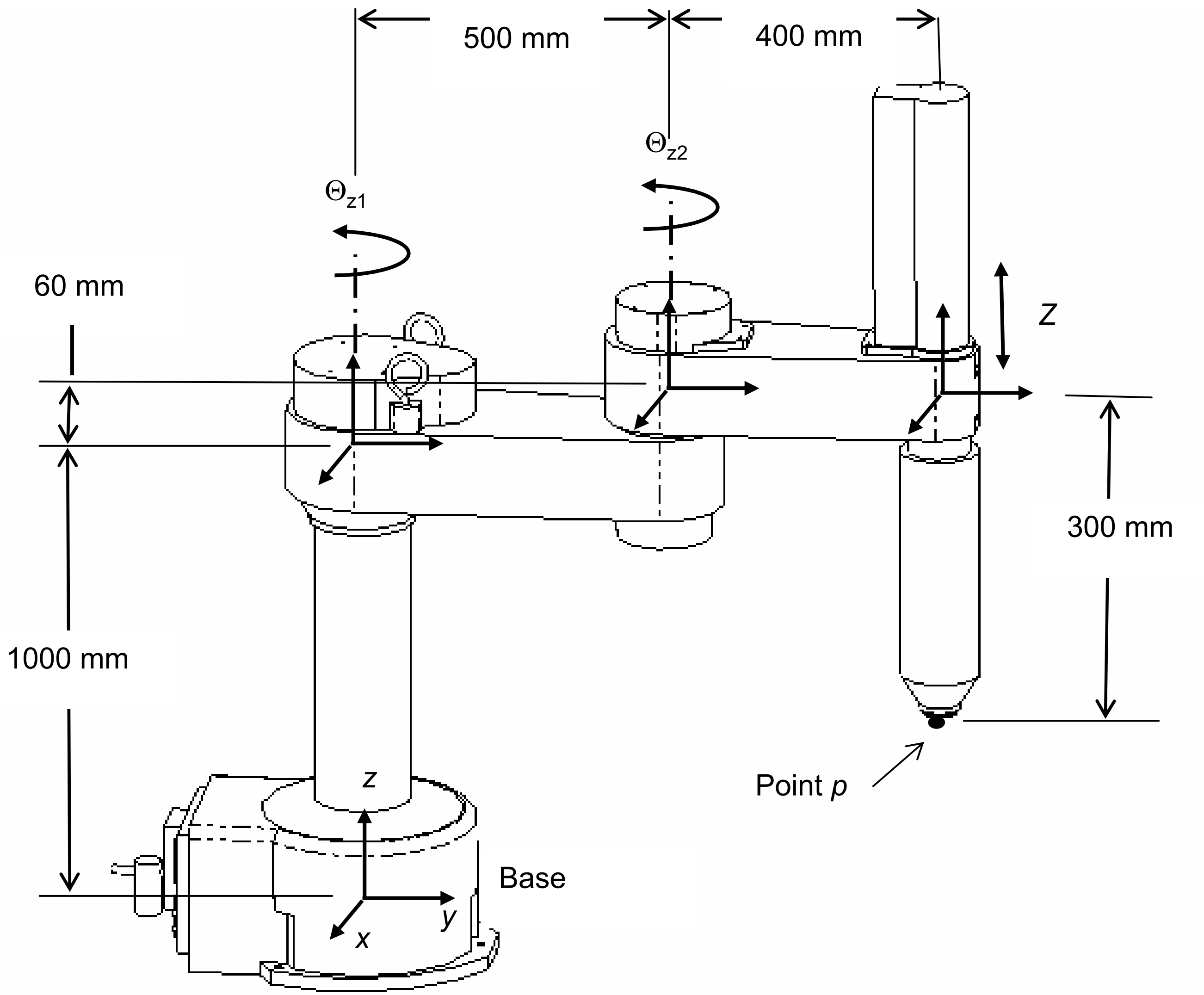






OK, so you put the error in the model. Now what will happen when the machine moves?

A Model of a Robot



Errors in the Robot

Error	Description	μ	σ
ε_{z1}	Drive error of joint #1	0 rad	0.0001 rad
ε_{z2}	Drive error of joint #2	0 rad	0.0001 rad
δ_{z3}	Drive error of joint #3	$Z \cdot 0.0001$	0.01mm
ε_{x3}	Pitch of joint #3	0 rad	0.00005 rad
ε_{y3}	Yaw of joint #3	0 rad	0.00005 rad
xp_2	Parallelism of joint 2 in the x direction	0.0002 rad	0.0001 rad

A Model of a Robot

- The matrices describe the intended motions and the errors

NOTE: These two should be swapped

$$
{}^{0}\mathbf{T}_3 = \begin{bmatrix} 1 & 0 & 0 & 1000mm \\ 0 & 1 & 0 & 0mm \\ 0 & 0 & 1 & 0mm \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(\Theta_{z1}+\varepsilon_{z1}) & -\sin\Theta_{z1} & 0 & 0mm \\ \sin\Theta_{z1} & \cos(\Theta_{z1}+\varepsilon_{z1}) & 0 & 0mm \\ 0 & 0 & 1 & 0mm \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 500mm \\ 0 & 1 & -xp_2 & 0mm \\ 0 & xp_2 & 1 & 60mm \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\cdot \begin{bmatrix} \cos(\Theta_{z2}+\varepsilon_{z2}) & -\sin\Theta_{z2} & 0 & 0mm \\ \sin\Theta_{z2} & \cos(\Theta_{z2}+\varepsilon_{z2}) & 0 & 0mm \\ 0 & 0 & 1 & 0mm \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 400mm \\ 0 & 1 & 0 & 0mm \\ 0 & 0 & 1 & 0mm \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & \varepsilon_{y3} & 0mm \\ 0 & 1 & -\varepsilon_{x3} & 0mm \\ -\varepsilon_{y3} & \varepsilon_{x3} & 1 & -Z-\delta_{z3} \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

- Can be applied to any point on the end effector

$$
\begin{Bmatrix} p'_x \\ p'_y \\ p'_z \\ 1 \end{Bmatrix} = {}^{0}\mathbf{T}_3 \begin{Bmatrix} 0 \\ 0 \\ -300 \\ 1 \end{Bmatrix}
$$

Homework #5

- Short answers on TRIZ and probability
- Error budgeting
  - Two tasks are to be done with the robot
  - Analyze the tasks
  - Discuss changes to the system
- A Matlab file is available in the HW folder just so you don't have to re-type the matrices

Next Steps

- You can download HW #5 Error Budgetting
  - Due 8:30AM Tues 13 July
- See you at Thursday's session
  - On the topic "Design of Experiments"
  - 8:30AM Thursday, 8 July
- Reading assignment for Thursday
  - All of Thomke
  - Skim Box
  - Skim Frey