Global Illumination and Monte Carlo

MIT EECS 6.837 Computer Graphics
Wojciech Matusik
with many slides from Fredo Durand and Jaakko Lehtinen

Today

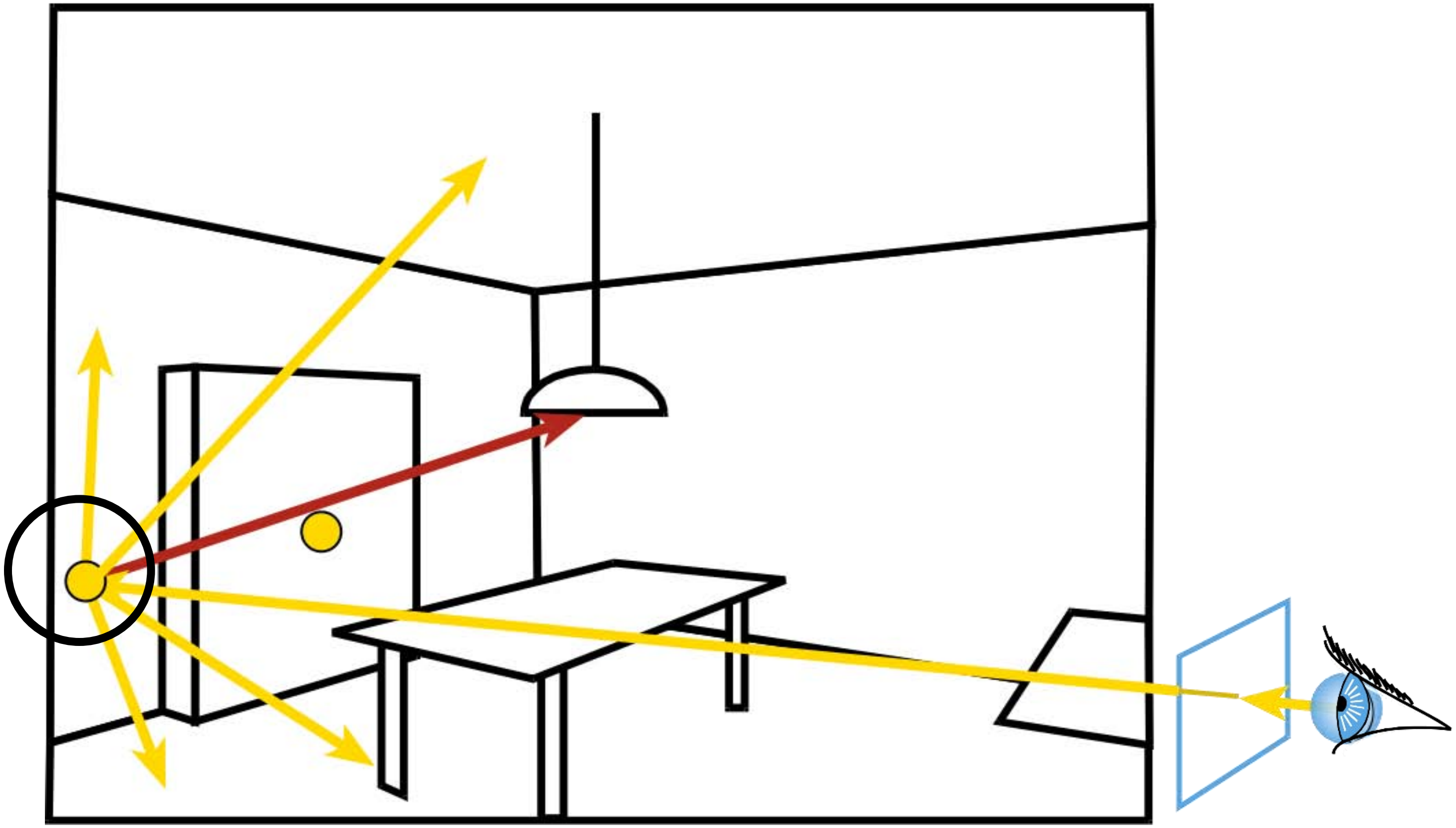
- Lots of randomness!

Today

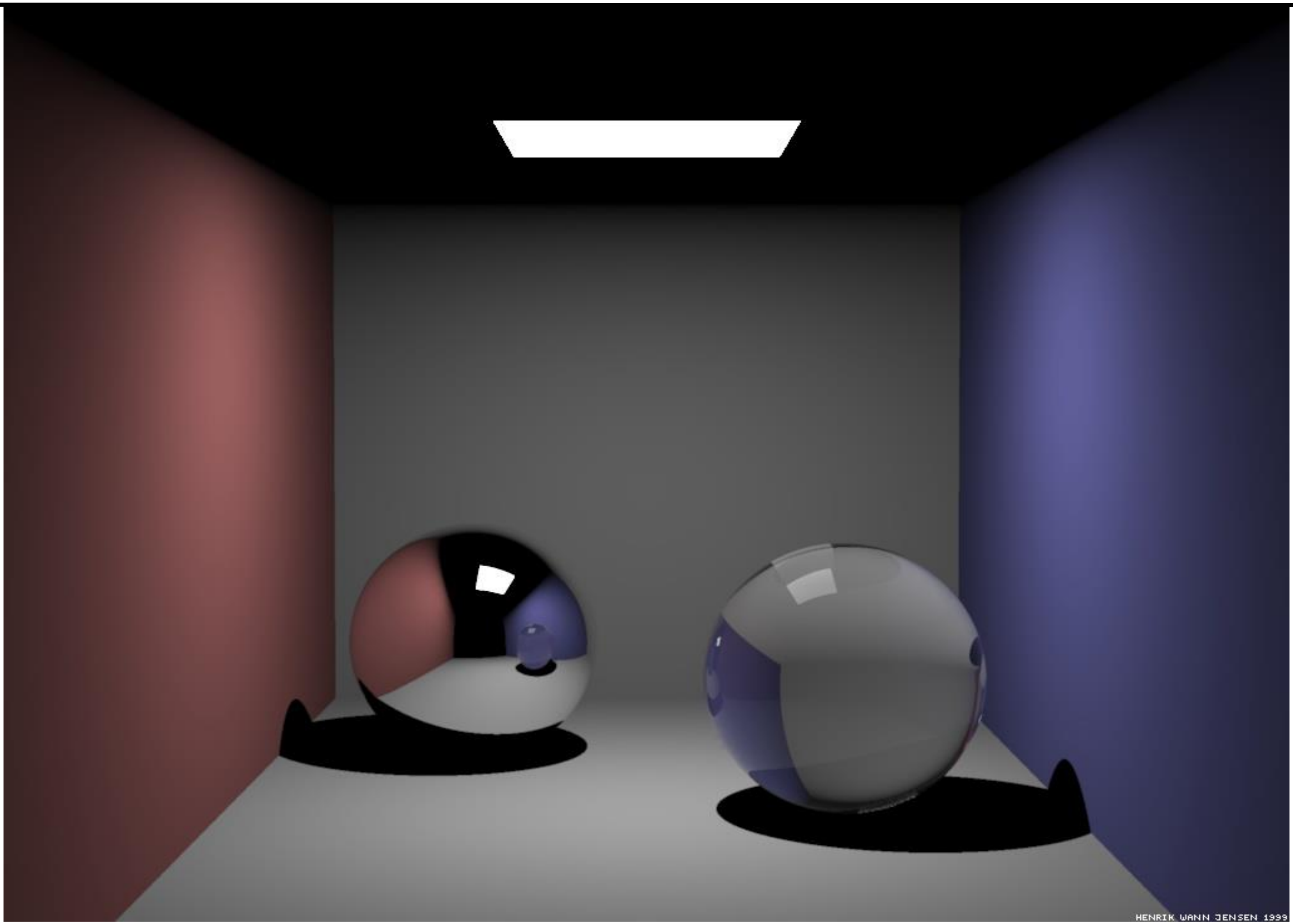
- Global Illumination
  - Rendering Equation
  - Path tracing
- Monte Carlo integration
- Better sampling
  - importance
  - stratification

Global Illumination

- So far, we've seen only direct lighting (red here)
- We also want indirect lighting
  - Full integral of all directions (multiplied by BRDF)
  - In practice, send tons of random rays

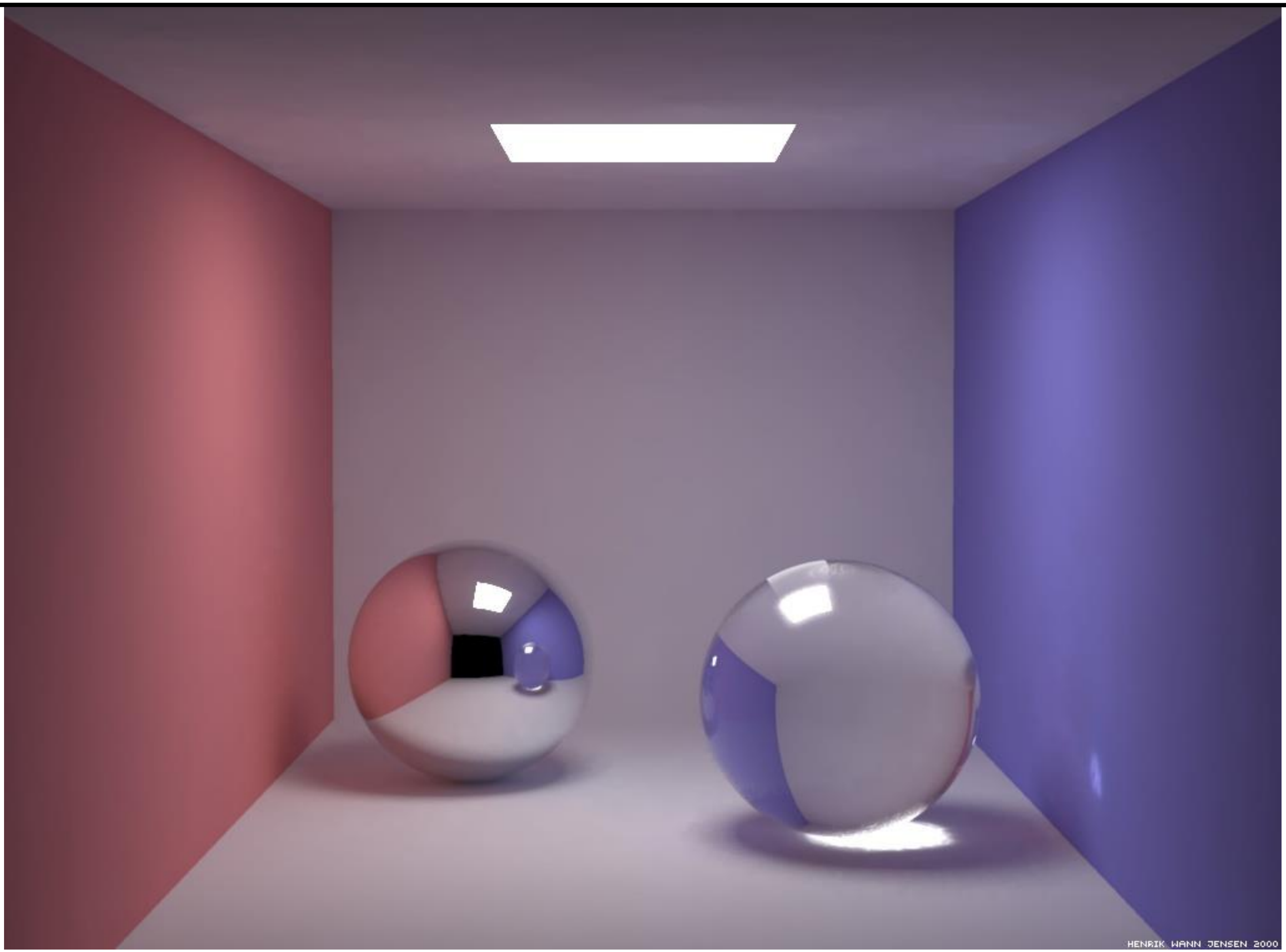


Direct Illumination



Courtesy of Henrik Wann Jensen. Used with permission.

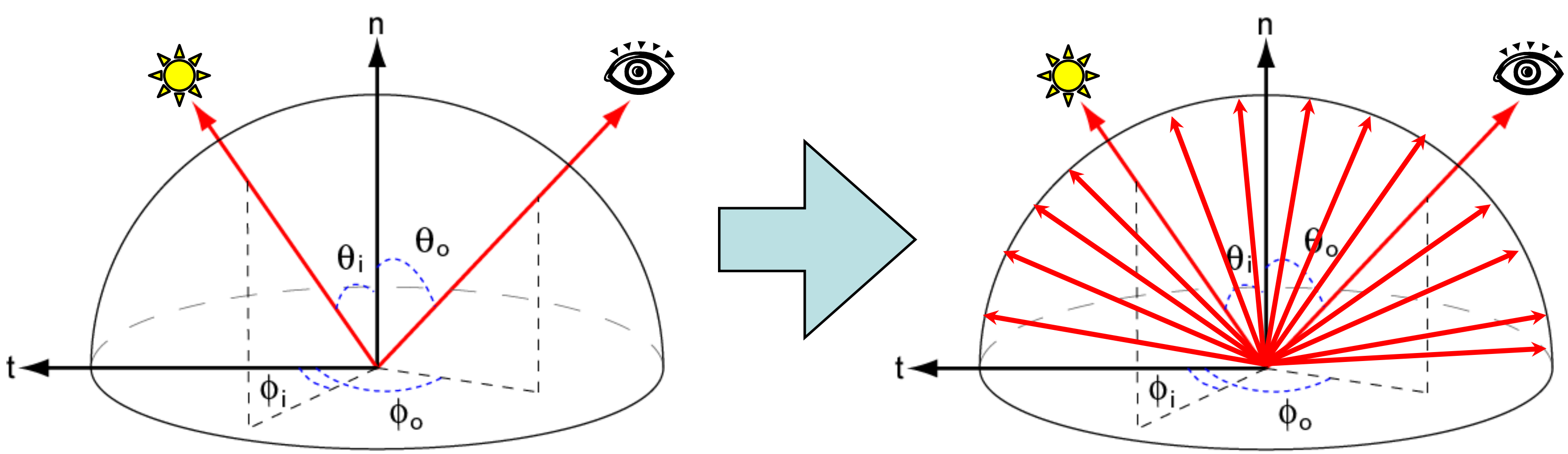
Global Illumination (with Indirect)



Courtesy of Henrik Wann Jensen. Used with permission.

Global Illumination

- So far, we only used the BRDF for point lights
  - We just summed over all the point light sources
- BRDF also describes how indirect illumination reflects off surfaces
  - Turns summation into integral over hemisphere
  - As if *every* direction had a light source



Reflectance Equation, Visually

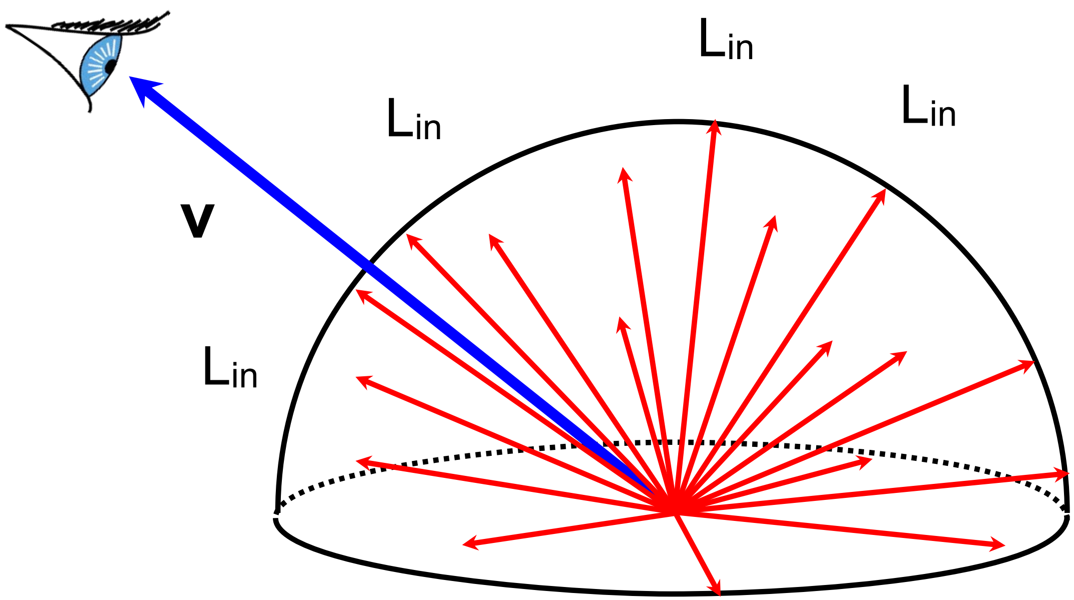
$$L_{\text{out}}(x, \boldsymbol{v}) = \int_{\Omega} L_{\text{in}}(\boldsymbol{l})\, f_r(x, \boldsymbol{l}, \boldsymbol{v})\, \cos\theta\, \mathrm{d}l$$

outgoing light to direction v

incident light from direction omega

the BRDF

cosine term

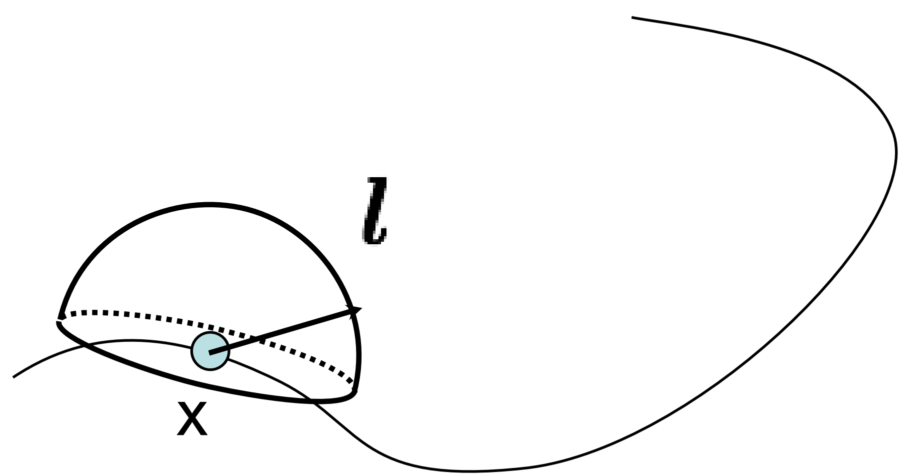


Sum (integrate) over every direction on the hemisphere, modulate incident illumination by BRDF

The Reflectance Equation

$$L_{\text{out}}(x, \boldsymbol{v}) = \int_{\Omega} L_{\text{in}}(\boldsymbol{l})\, f_r(x, \boldsymbol{l}, \boldsymbol{v})\, \cos\theta \,\mathrm{d}l$$

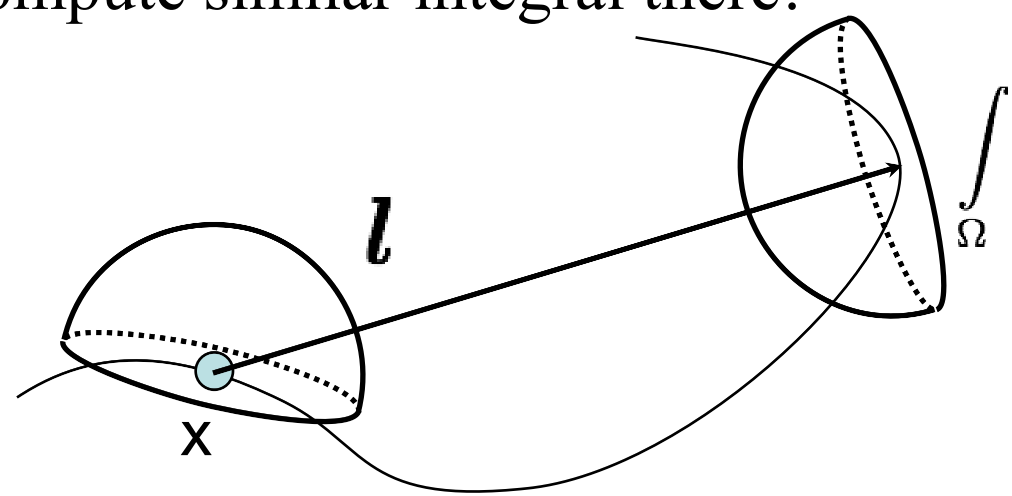
- Where does L_{in} come from?



The Reflectance Equation

$$L_{\text{out}}(x, \boldsymbol{v}) = \int_{\Omega} L_{\text{in}}(\boldsymbol{l})\, f_r(x, \boldsymbol{l}, \boldsymbol{v})\, \cos\theta \,\mathrm{d}l$$

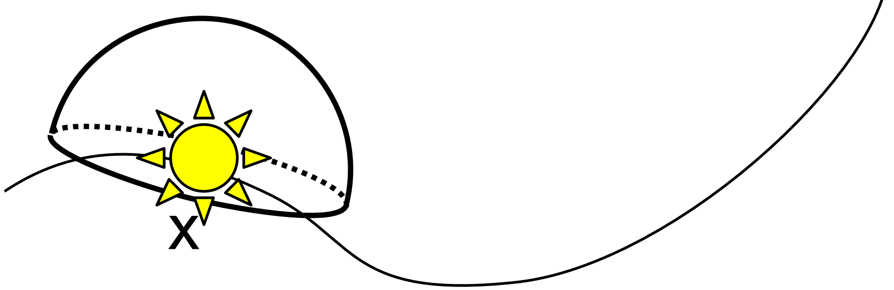
- Where does L_{in} come from?
  - It is the light reflected towards x from the surface point in direction $\boldsymbol{l}$ ==> must compute similar integral there!
    - Recursive!



The Rendering Equation

$$L_{\mathrm{out}}(x, \boldsymbol{v}) = \int_{\Omega} L_{\mathrm{in}}(\boldsymbol{l})\, f_r(x, \boldsymbol{l}, \boldsymbol{v})\, \cos\theta\, \mathrm{d}l + E_{\mathrm{out}}(x, \boldsymbol{v})$$

- Where does L_{in} come from?
  - It is the light reflected towards x from the surface point in direction ***l*** ==> must compute similar integral there!
    - Recursive!
  - AND if x happens to be a light source, we add its contribution directly



The Rendering Equation

$$L_{\text{out}}(x, \boldsymbol{v}) = \int_{\Omega} L_{\text{in}}(\boldsymbol{l})\, f_r(x, \boldsymbol{l}, \boldsymbol{v})\, \cos\theta \, \mathrm{d}l + E_{\text{out}}(x, \boldsymbol{v})$$

- The rendering equation describes the appearance of the scene, including direct and indirect illumination
  - An “integral equation”, the unknown solution function L is both on the LHS and on the RHS inside the integral
    - Must either discretize or use Monte Carlo integration
  - Originally described by Kajiya and Immel et al. in 1986
  - More on 6.839
    - Also, see book references towards the end

The Rendering Equation

- Analytic solution is usually impossible
- Lots of ways to solve it approximately
- Monte Carlo techniques use random samples for evaluating the integrals
  - We'll look at some simple method in a bit...
- Finite element methods discretize the solution using basis functions (again!)
  - Radiosity, wavelets, precomputed radiance transfer, etc.

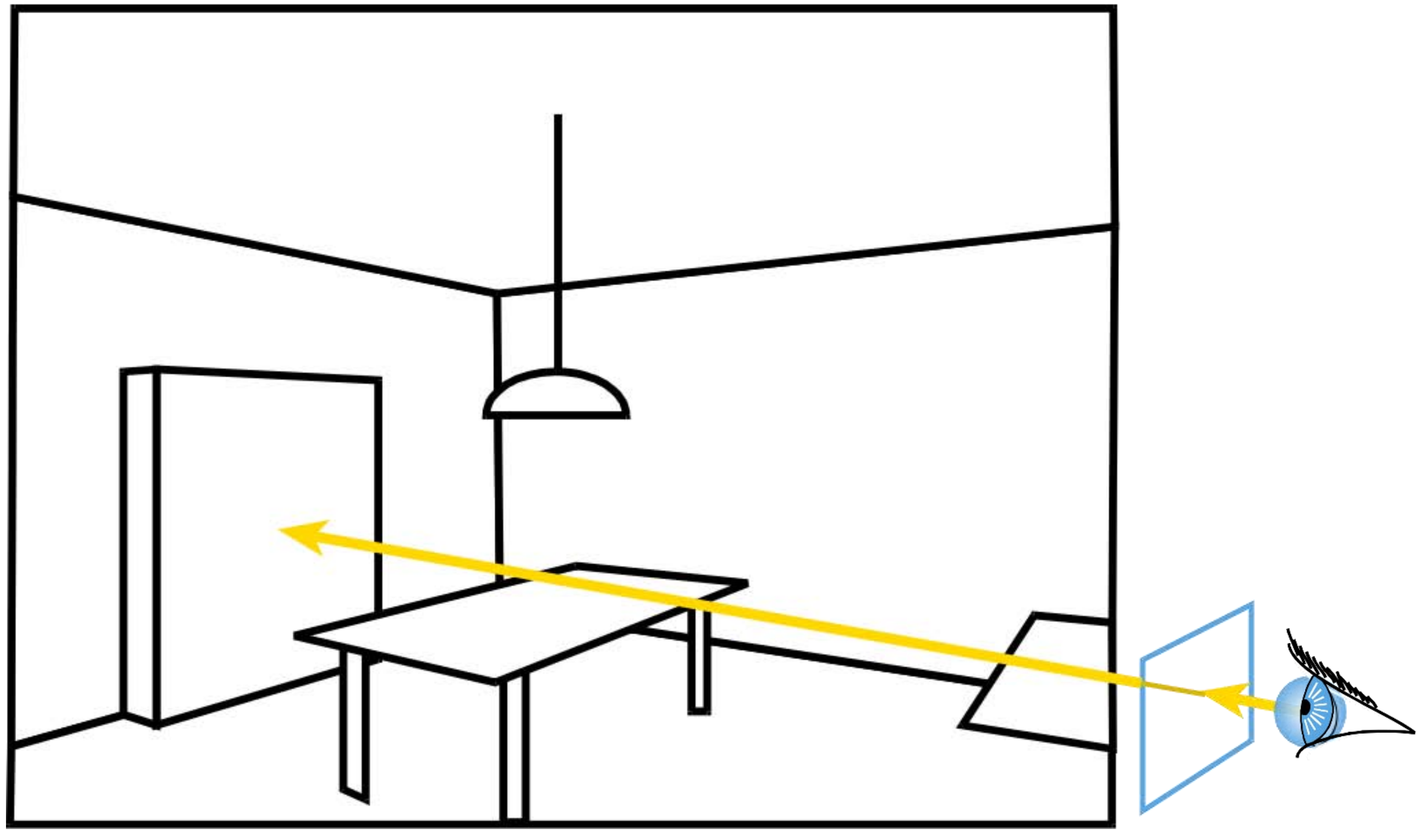
Questions?

How To Render Global Illumination?

Lehtinen et al. 2008

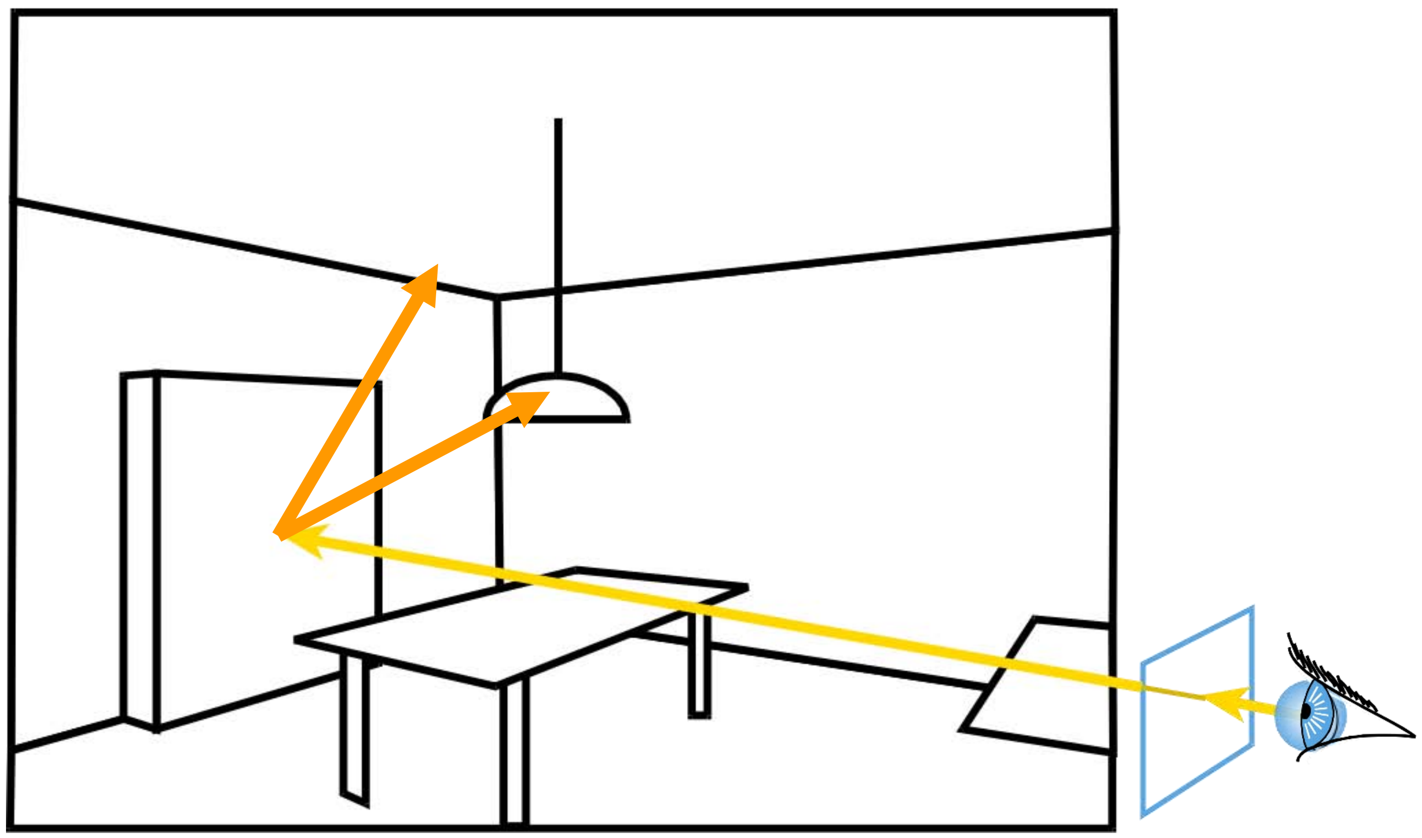
Ray Casting

- Cast a ray from the eye through each pixel



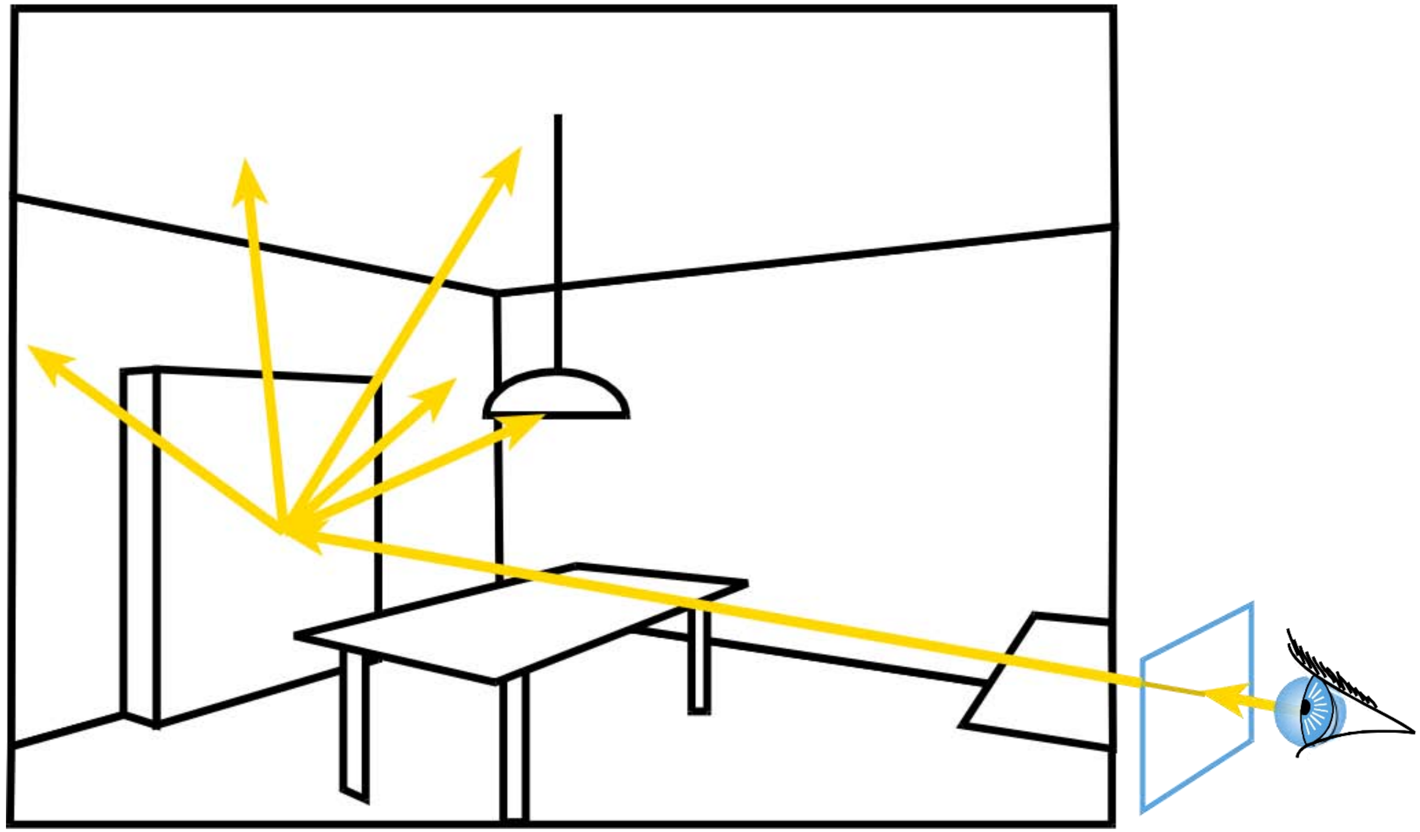
Ray Tracing

- Cast a ray from the eye through each pixel
- Trace secondary rays (shadow, reflection, refraction)



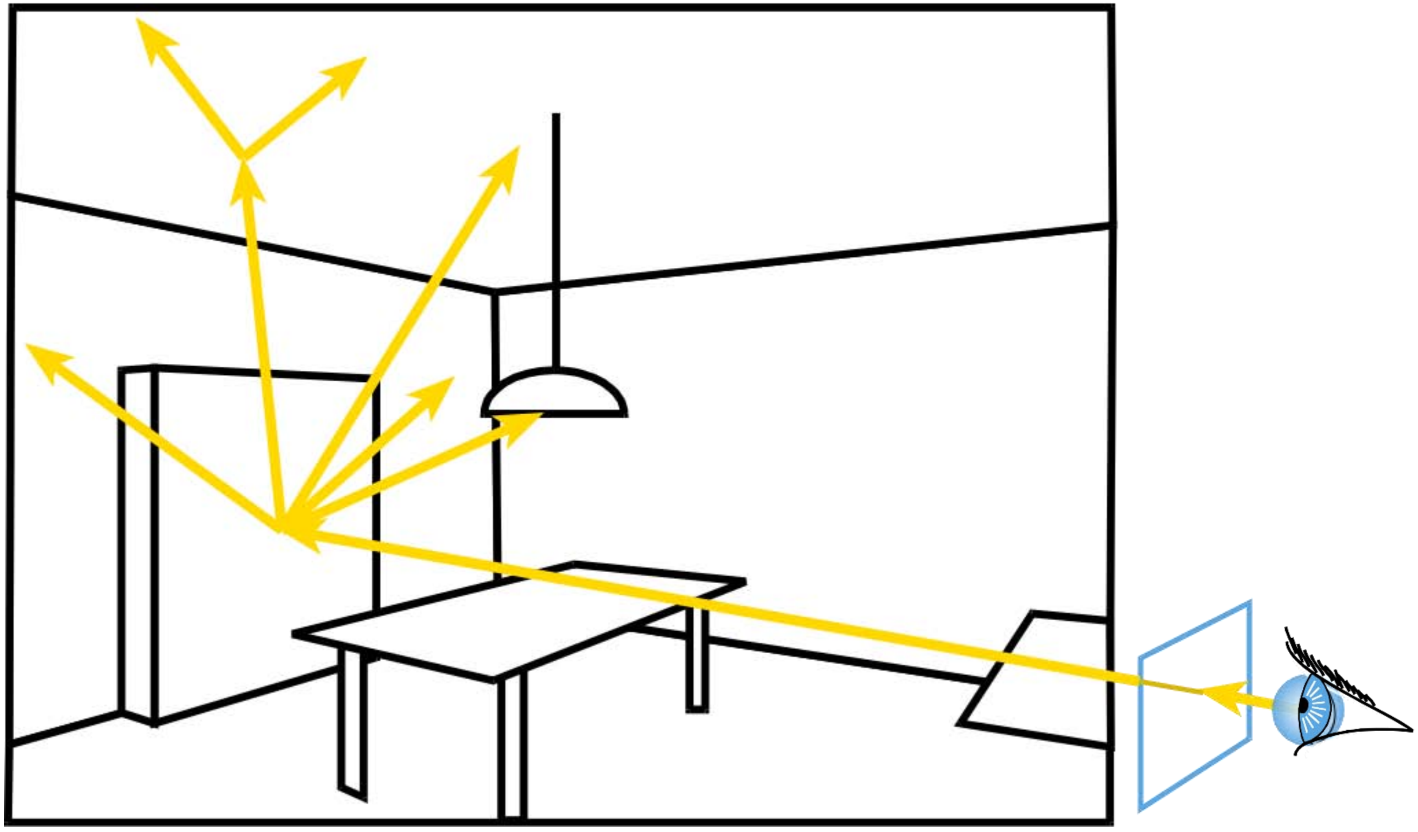
"Monte-Carlo Ray Tracing"

- Cast a ray from the eye through each pixel
- Cast random rays from the hit point to evaluate hemispherical integral using random sampling



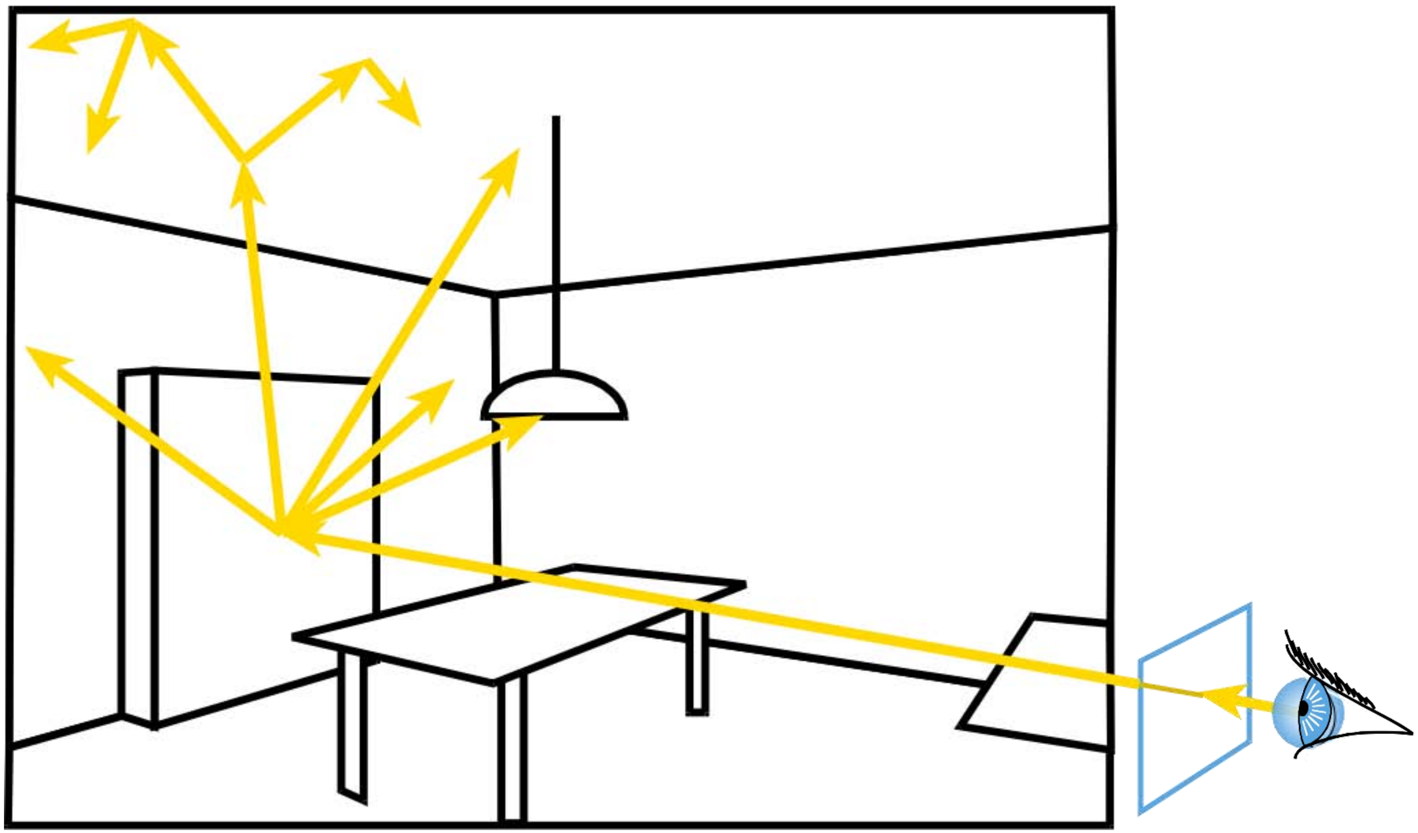
"Monte-Carlo Ray Tracing"

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
- Recurse



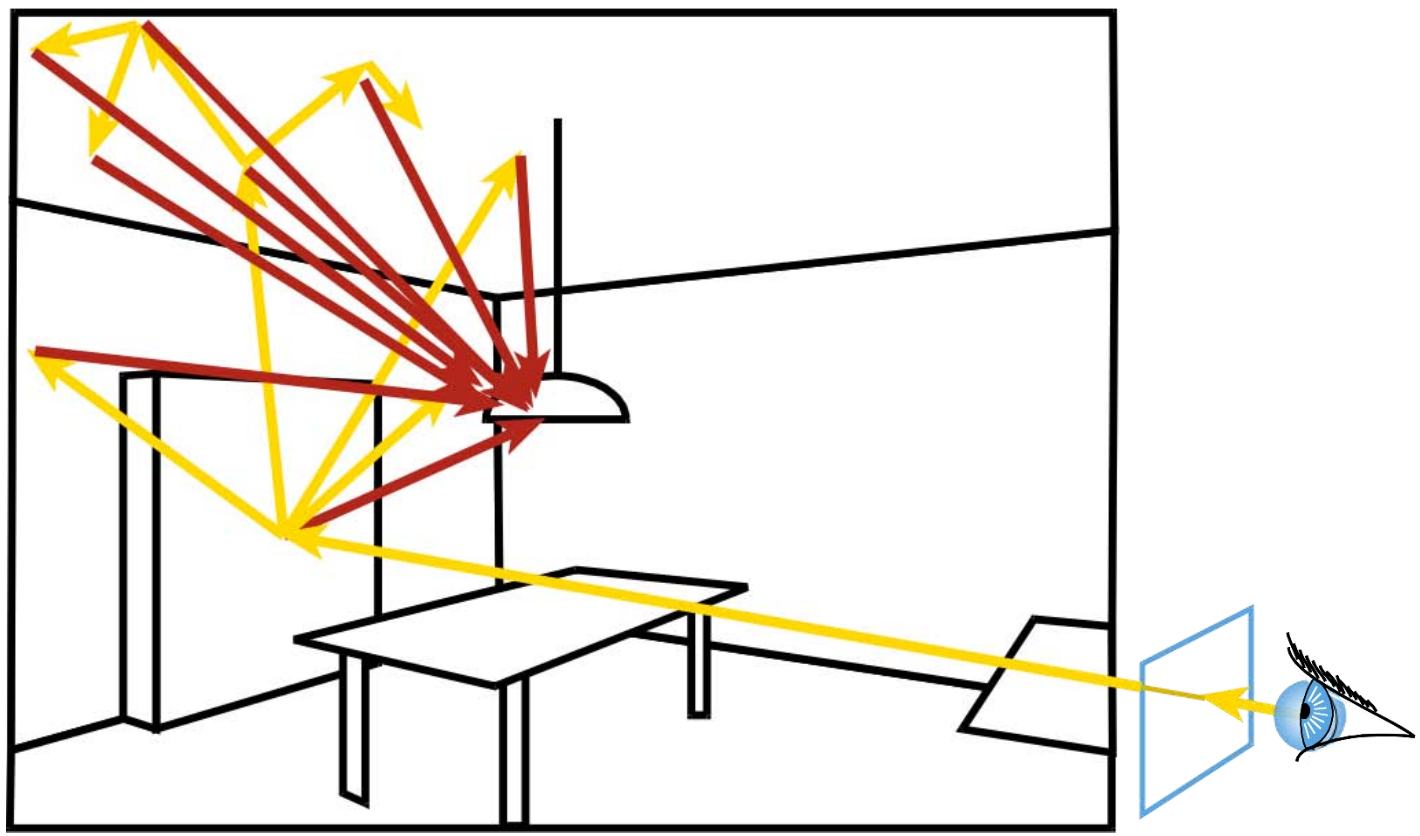
“Monte-Carlo Ray Tracing”

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
- Recurse



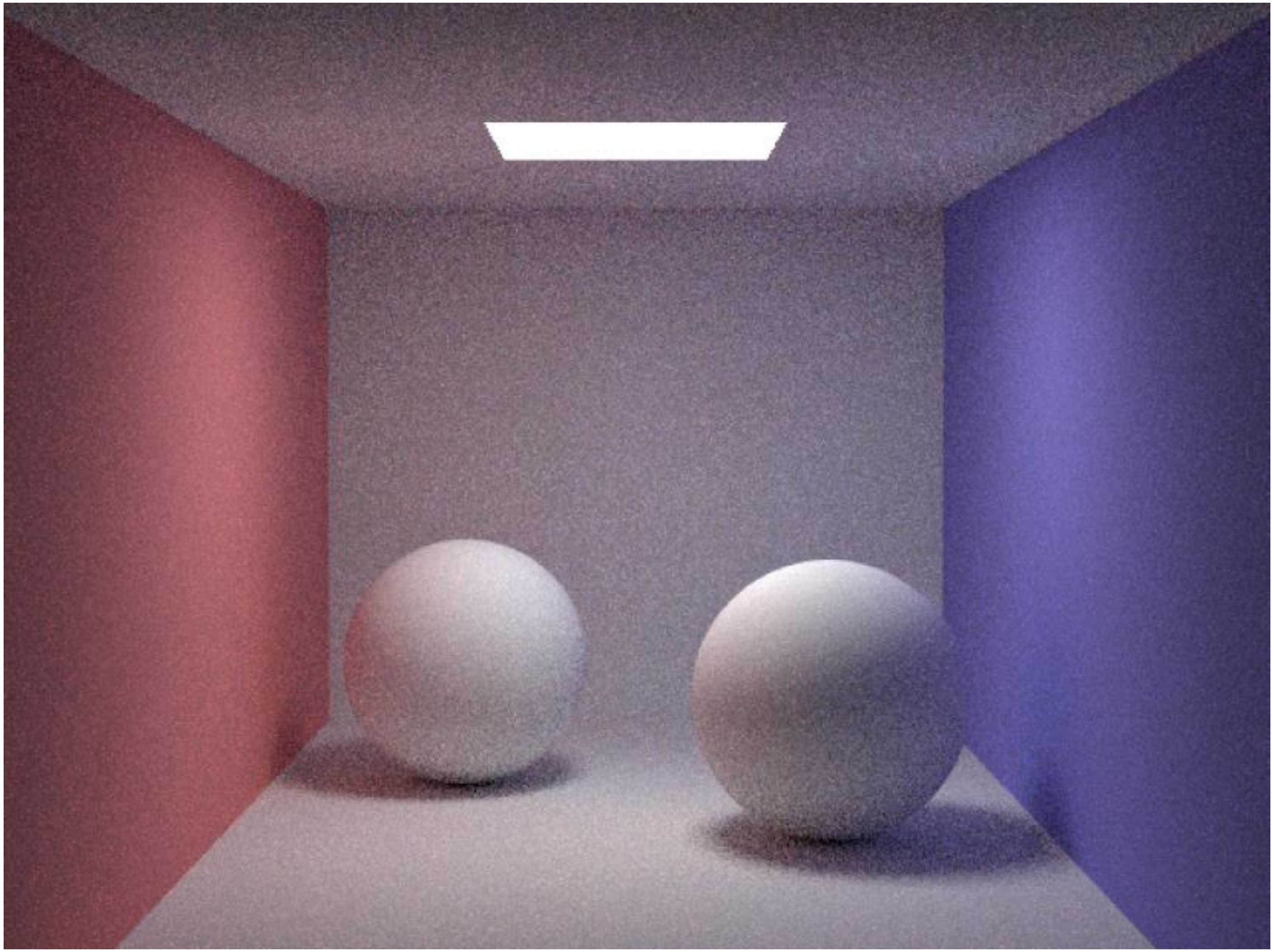
"Monte-Carlo Ray Tracing"

- Systematically sample light sources at each hit
  - Don't just wait the rays will hit it by chance



Results

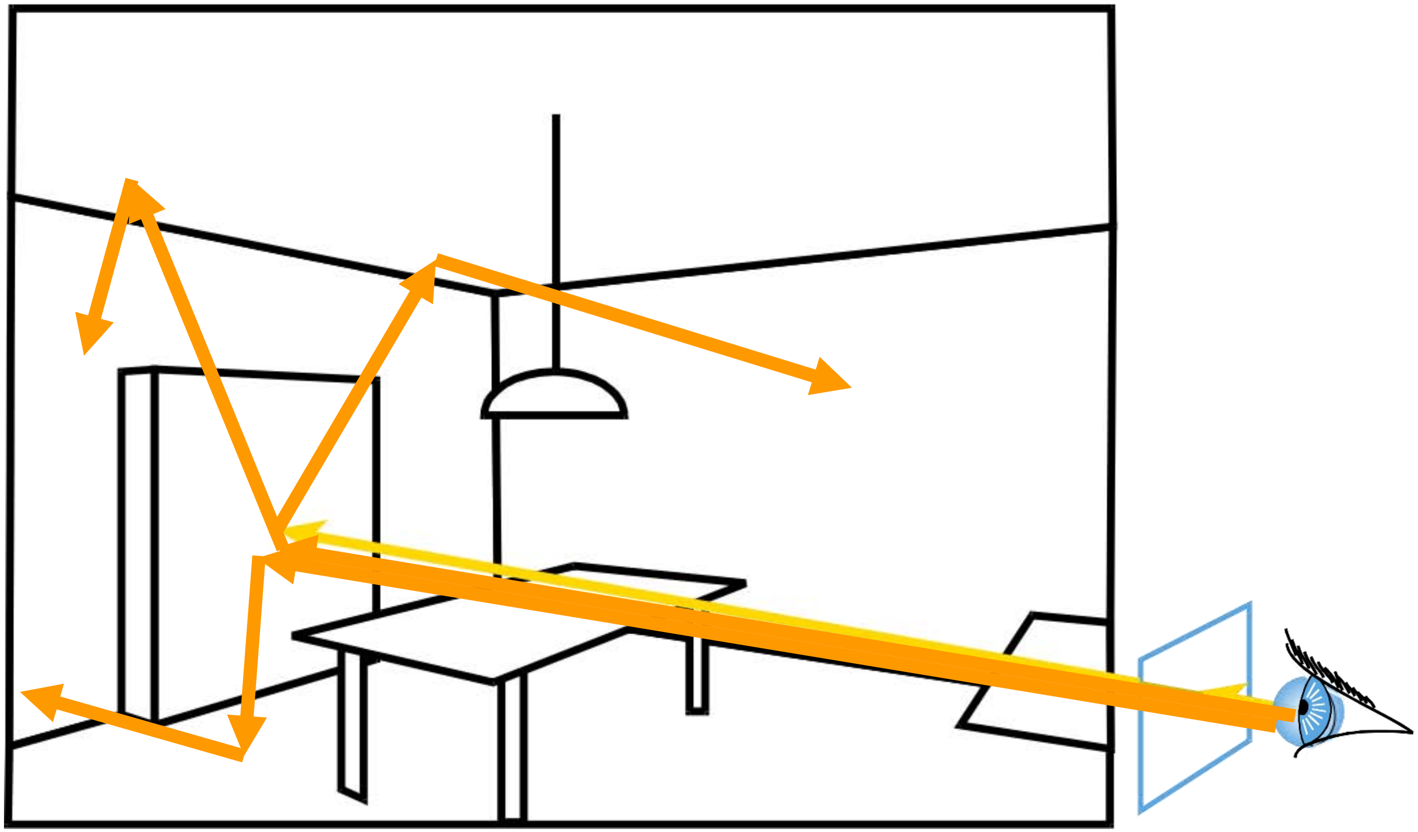
Henrik Wann Jensen



Courtesy of Henrik Wann Jensen. Used with permission.

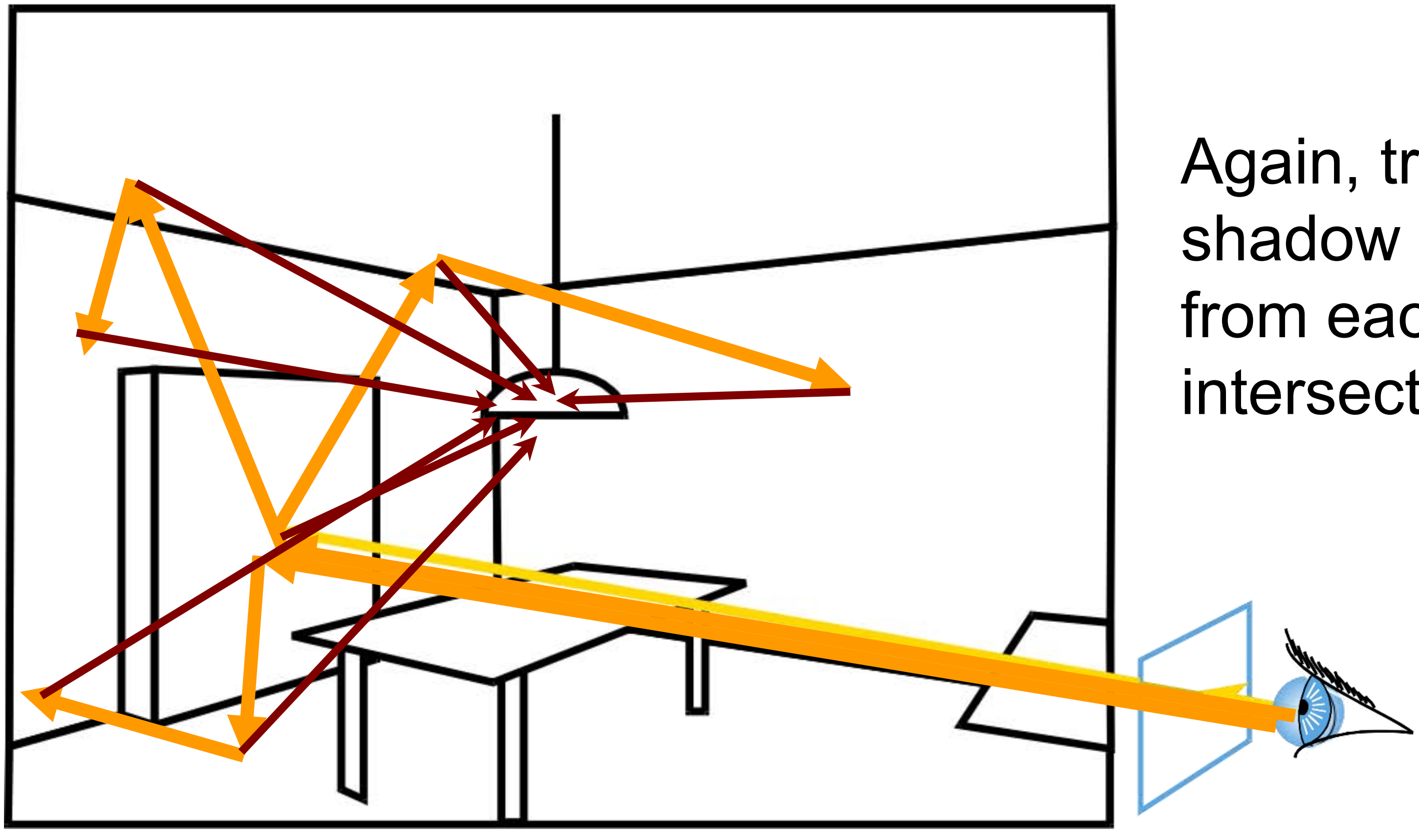
Monte Carlo Path Tracing

- Trace only one secondary ray per recursion
  - Otherwise number of rays explodes!
- But send many primary rays per pixel (antialiasing)



Monte Carlo Path Tracing

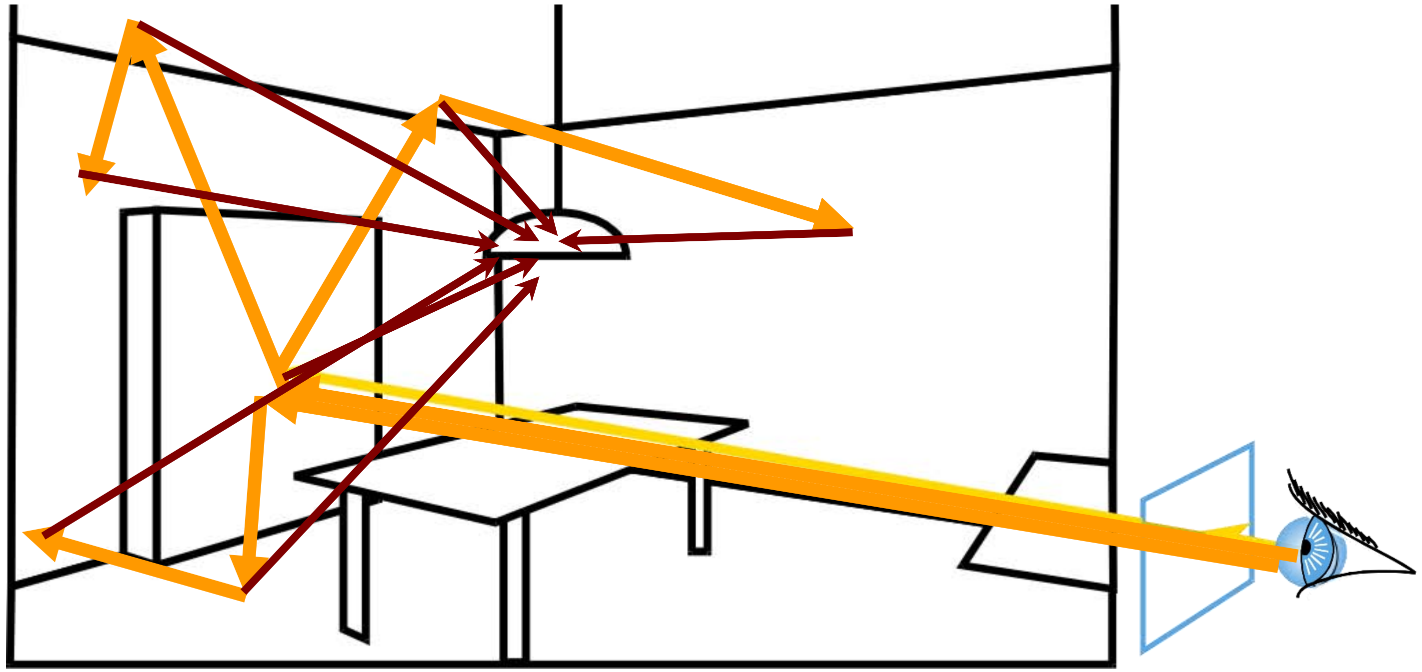
- Trace only one secondary ray per recursion
  - Otherwise number of rays explodes!
- But send many primary rays per pixel (antialiasing)



Again, trace shadow rays from each intersection

Monte Carlo Path Tracing

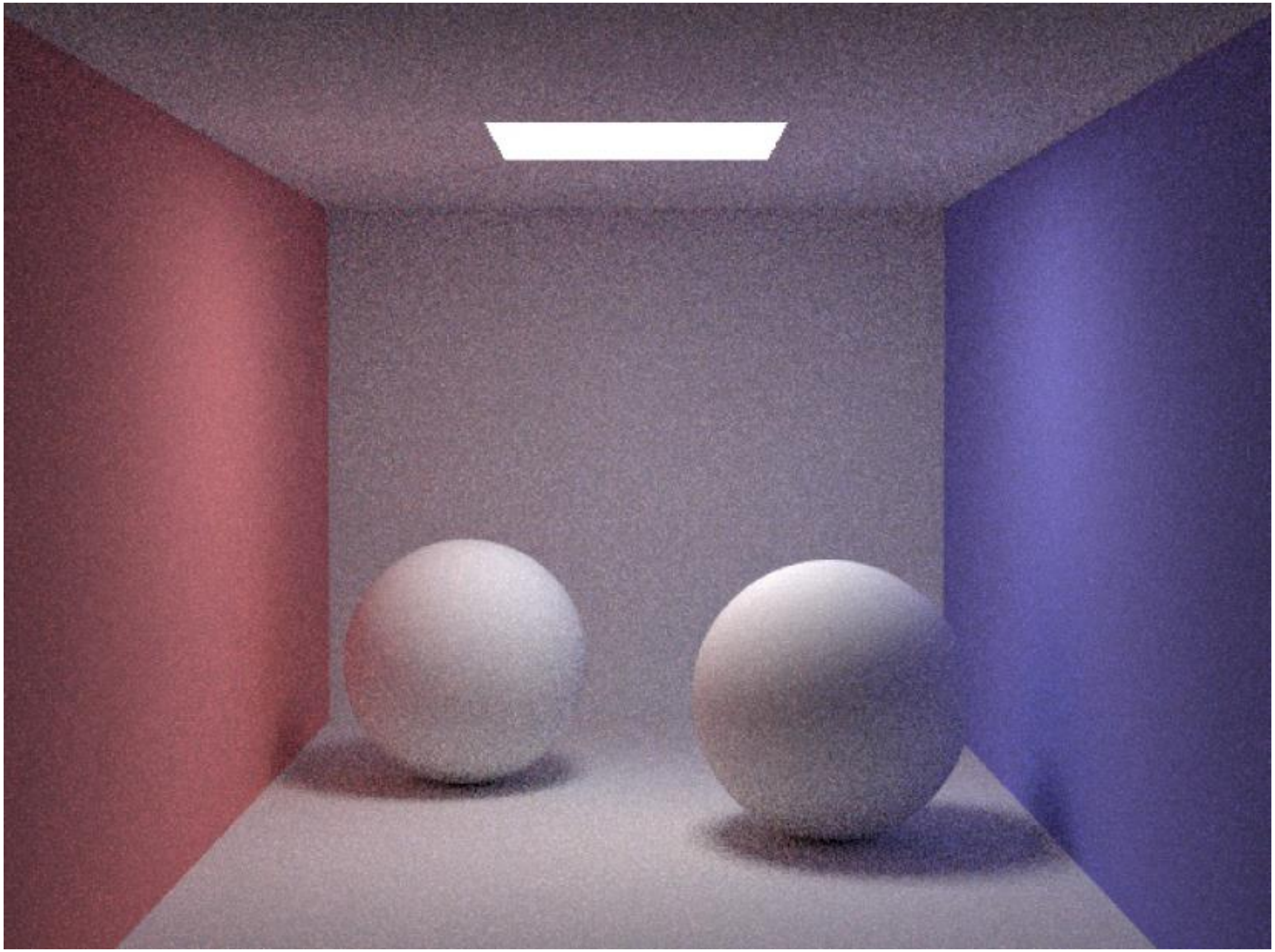
- We shoot one path from the eye at a time
  - Connect every surface point on the way to the light by a shadow ray
  - We are randomly sampling the space of all possible light paths between the source and the camera



Path Tracing Results

- 10 paths/pixel

Henrik Wann Jensen

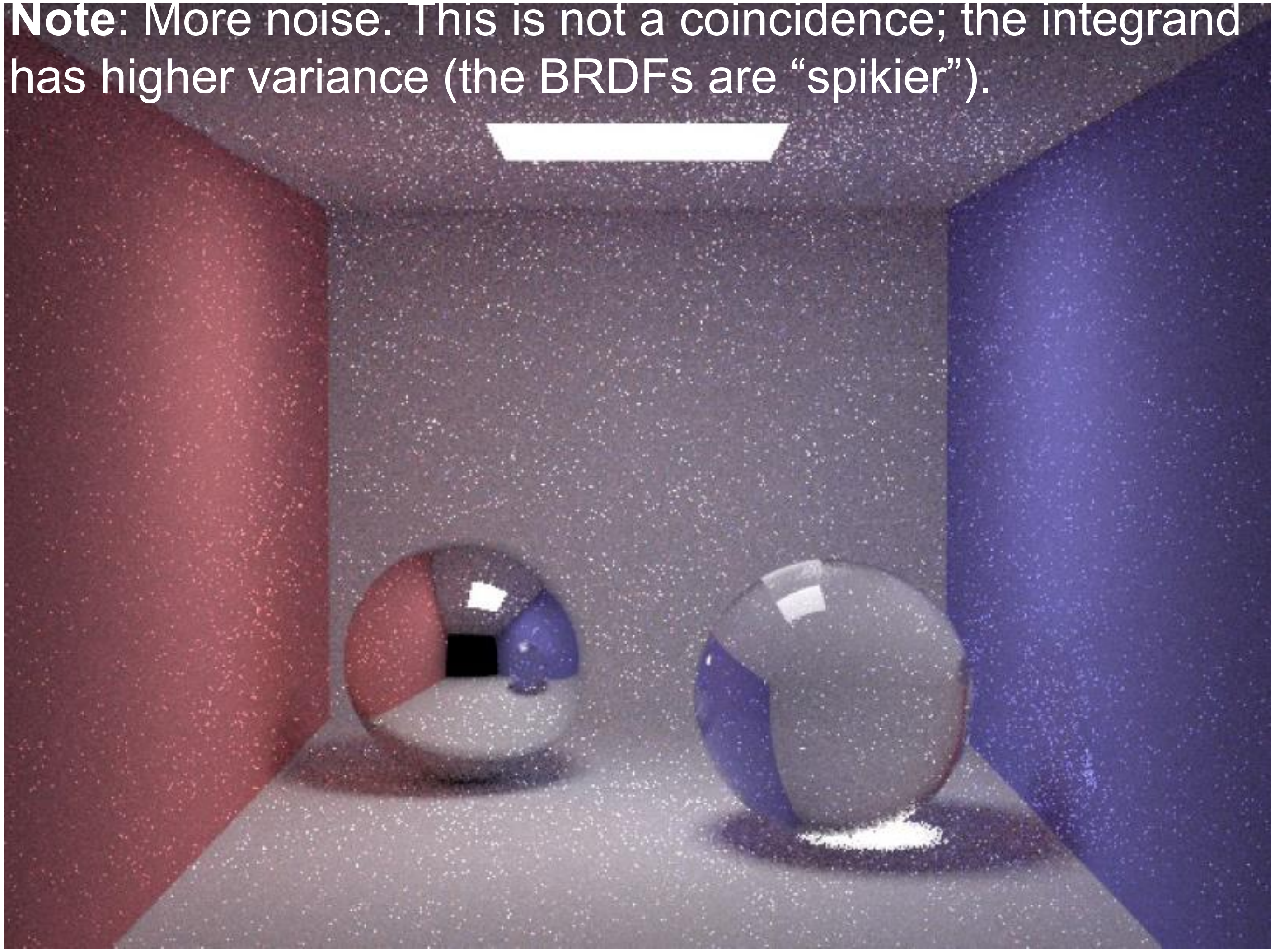


Courtesy of Henrik Wann Jensen. Used with permission.

Path Tracing Results: Glossy Scene

- 10 paths/pixel

Note: More noise. This is not a coincidence; the integrand has higher variance (the BRDFs are “spikier”).



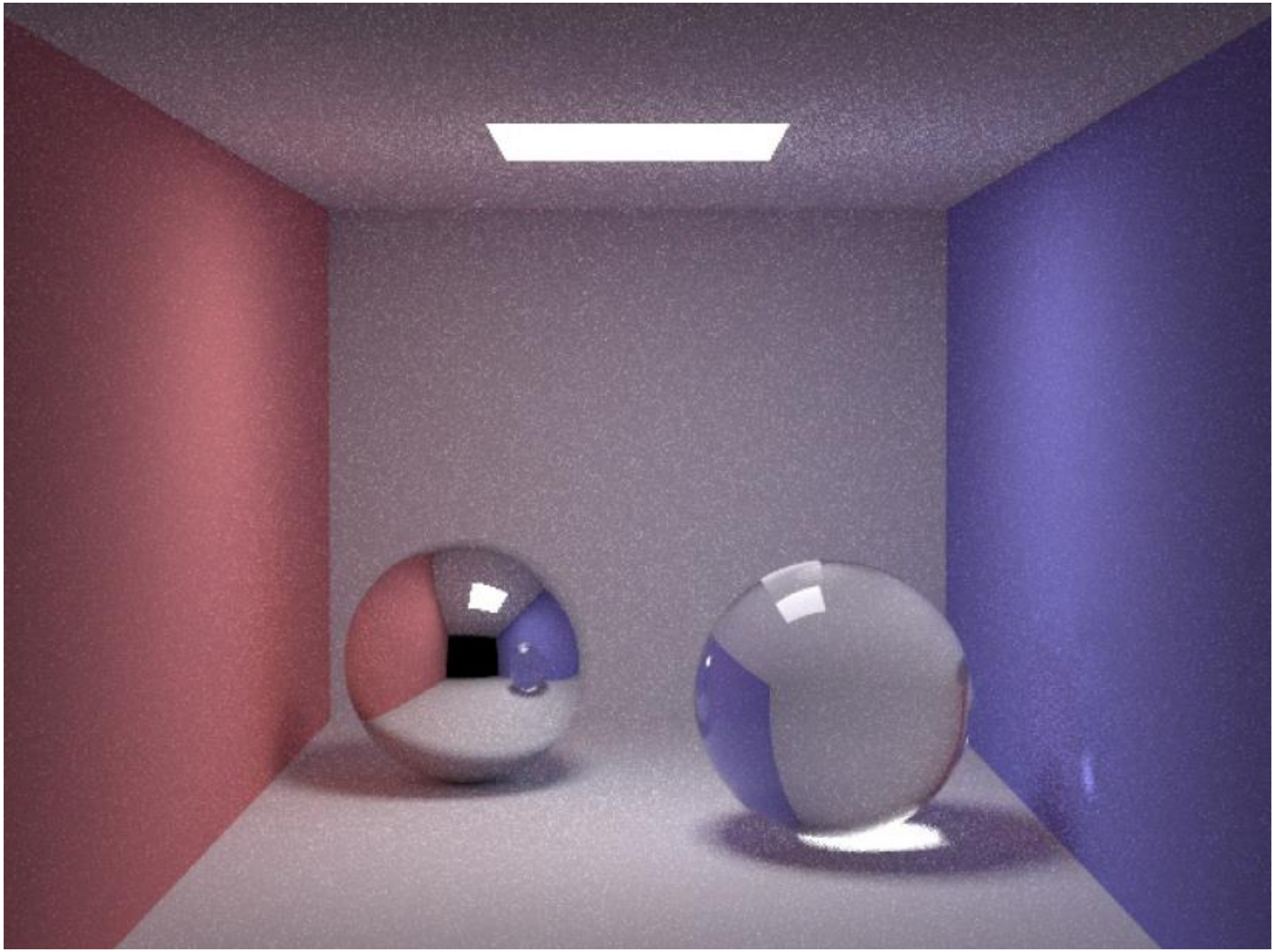
Henrik Wann Jensen

Courtesy of Henrik Wann Jensen. Used with permission.

Path Tracing Results: Glossy Scene

- 100 paths/pixel

Henrik Wann Jensen



Courtesy of Henrik Wann Jensen. Used with permission.

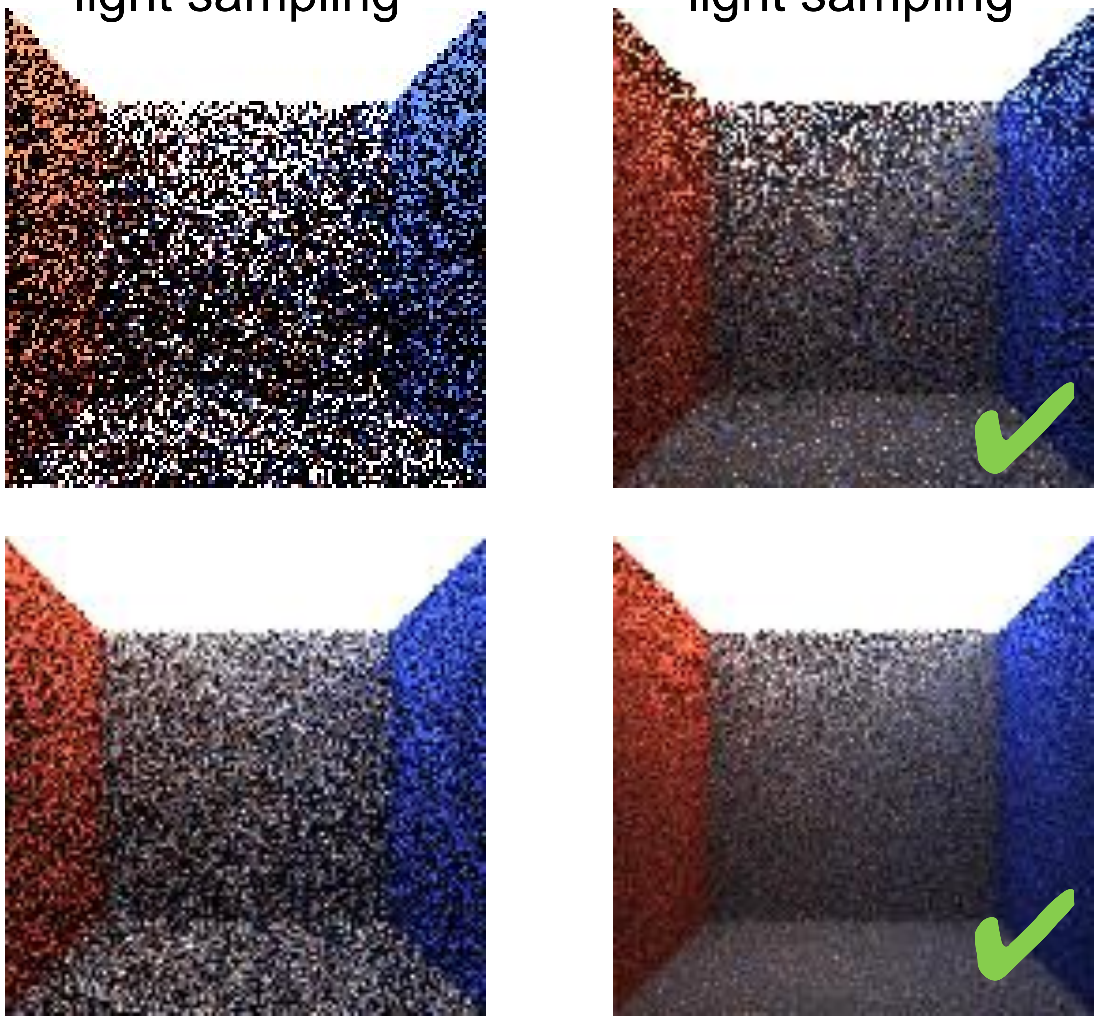
Importance of Sampling the Light

Without explicit light sampling

With explicit light sampling

1 path per pixel

4 paths per pixel



Why Use Random Numbers?

- Fixed random sequence
- We see the structure in the error

Henrik Wann Jensen

Demo

- http://madebyevan.com/webgl-path-tracing/

Image removed due to copyright restrictions. Please see the above link for further details.

For more demo/experimentation

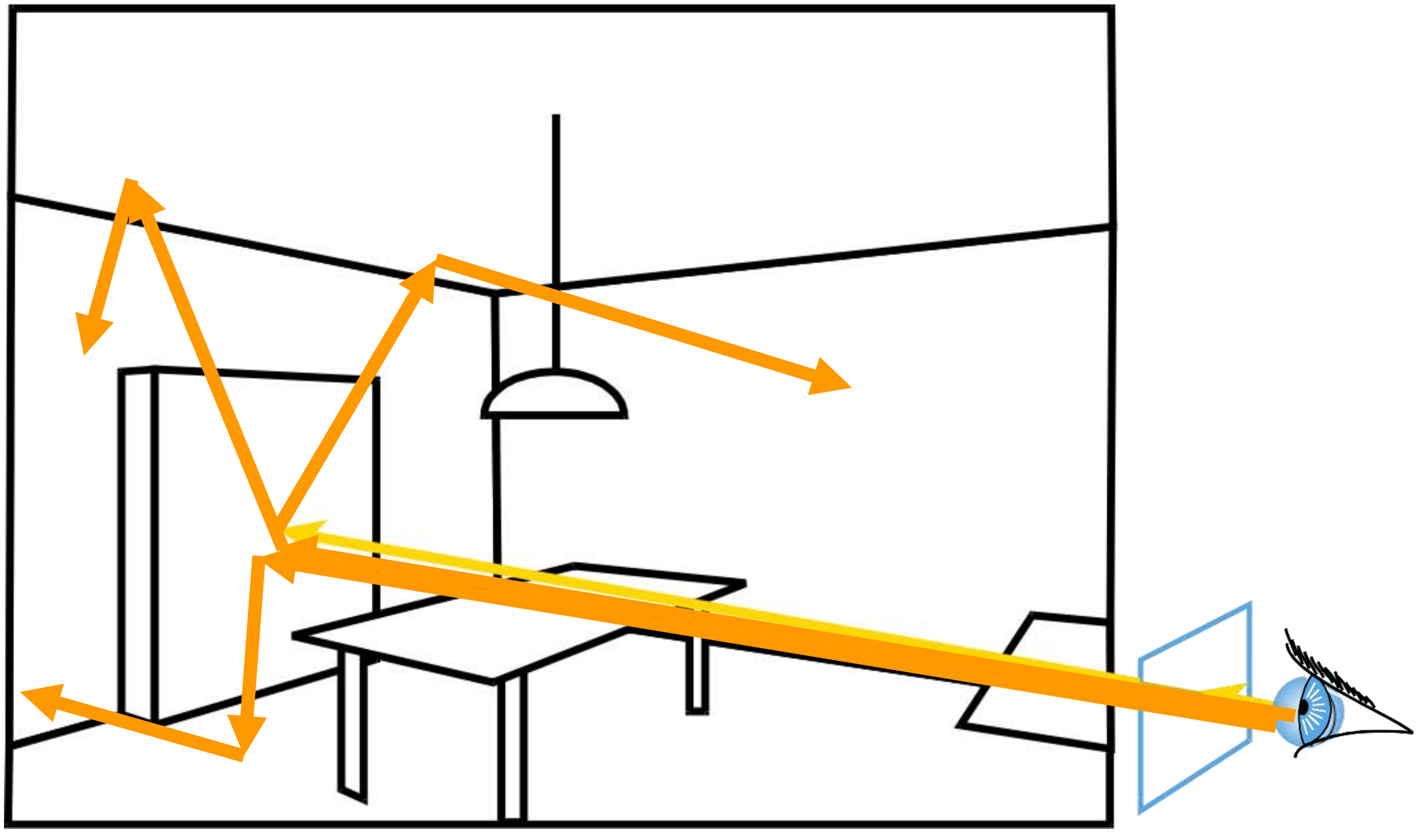
- http://www.mitsuba-renderer.org/
- http://www.pbrt.org/
- http://www.luxrender.net/en_GB/index

Questions?

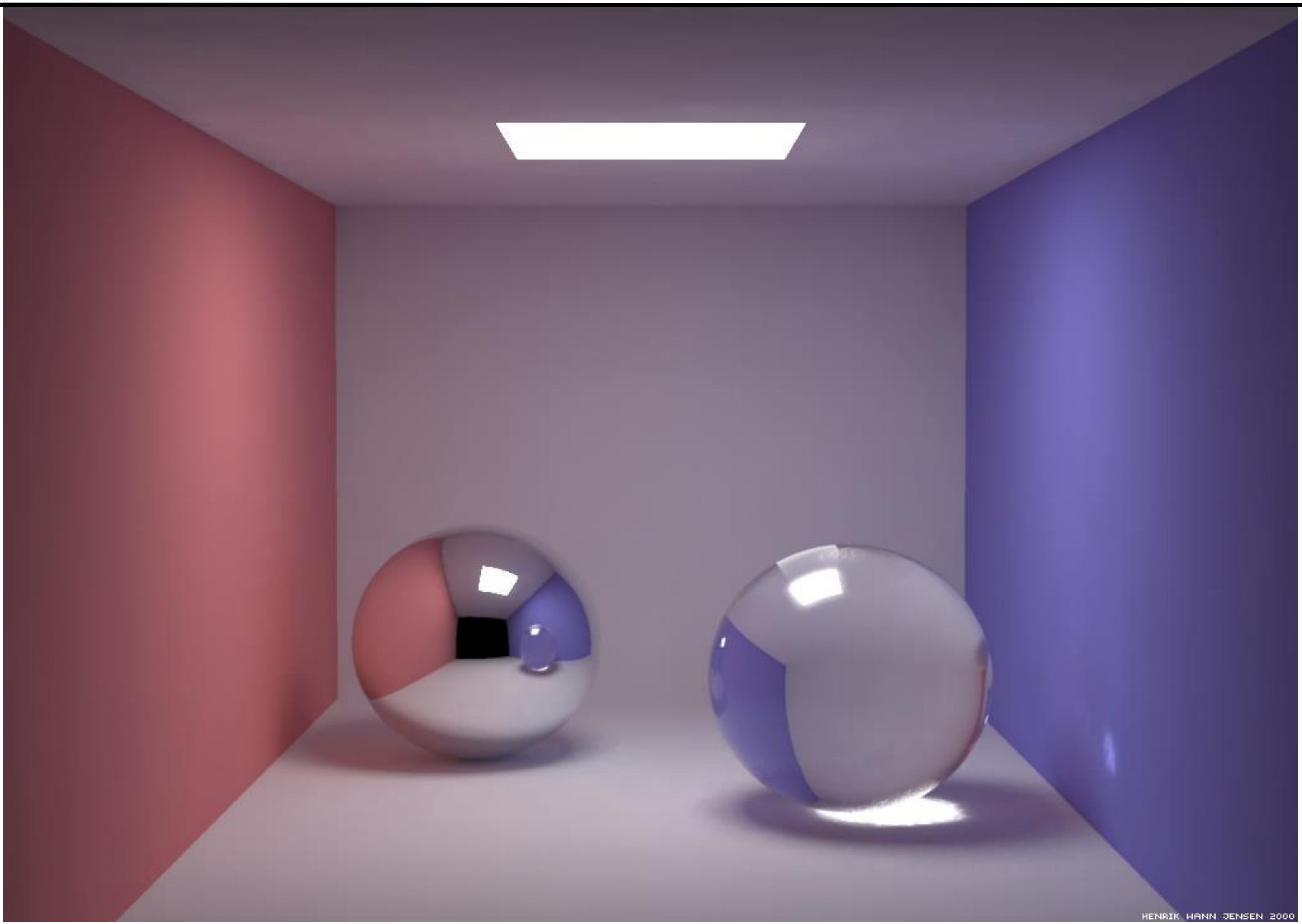
- Vintage path tracing by Kajiya

Path Tracing is costly

- Needs tons of rays per pixel!

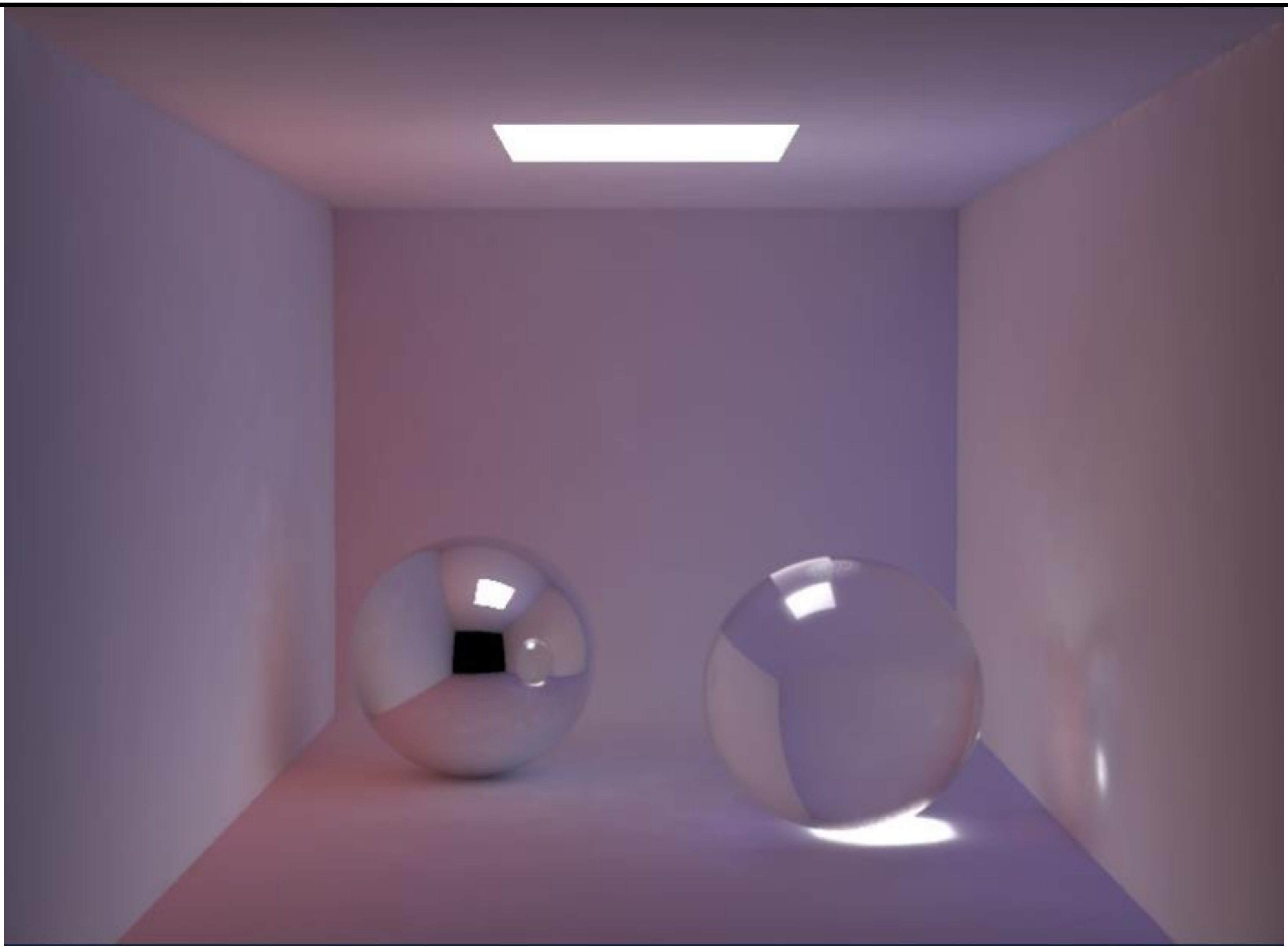


Global Illumination (with Indirect)



Courtesy of Henrik Wann Jensen. Used with permission.

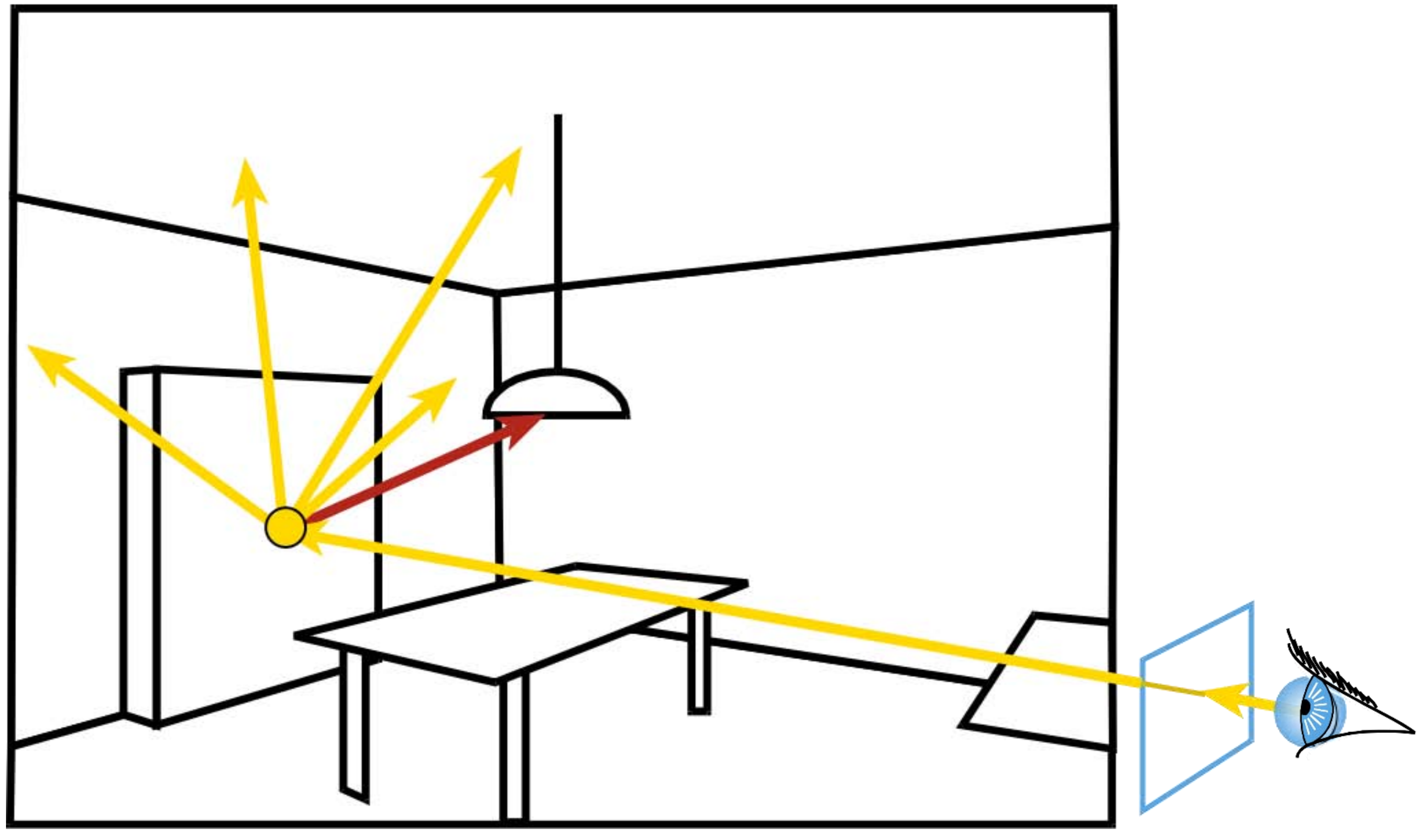
Indirect Lighting is Mostly Smooth



Courtesy of Henrik Wann Jensen. Used with permission.

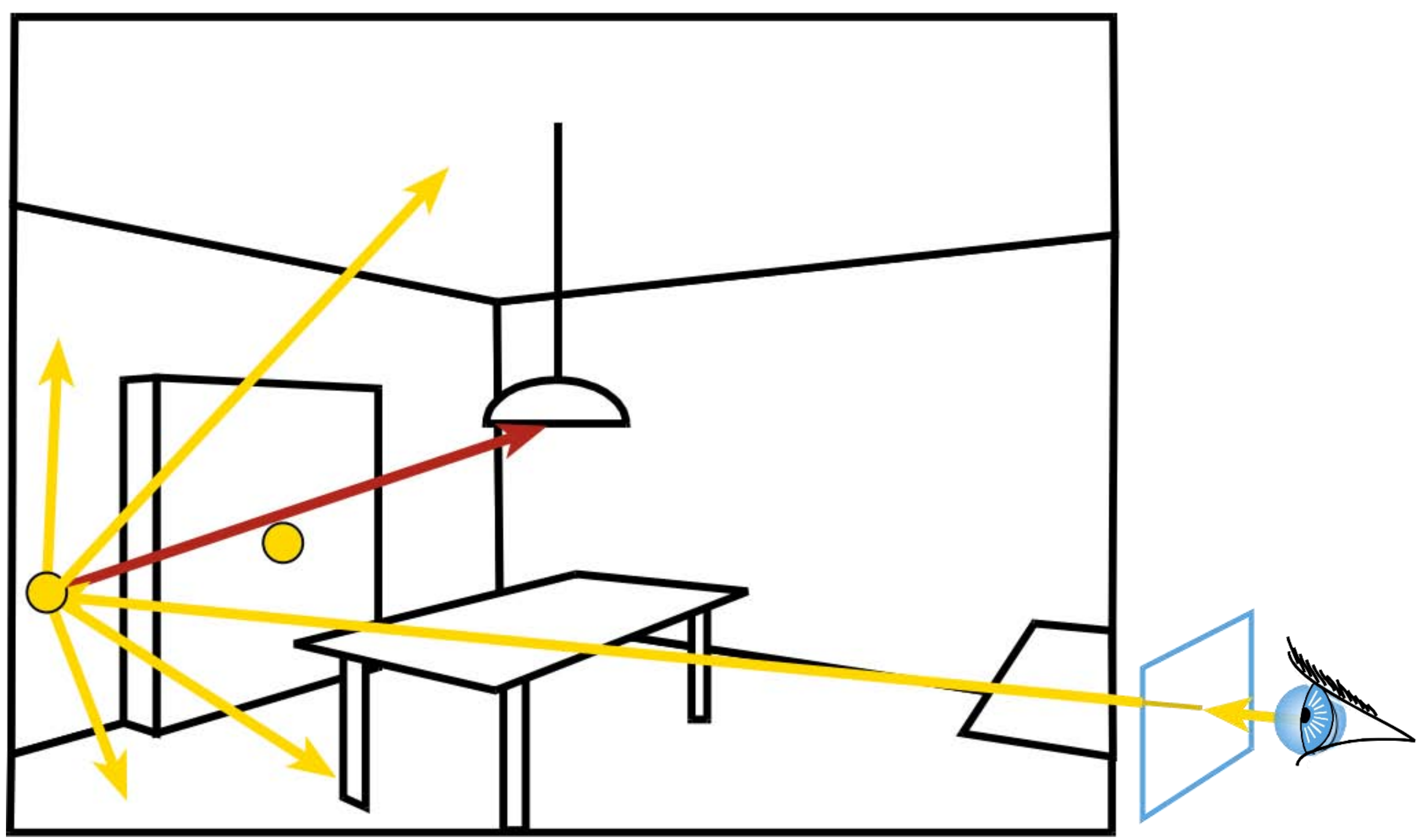
Irradiance Caching

- Indirect illumination is smooth



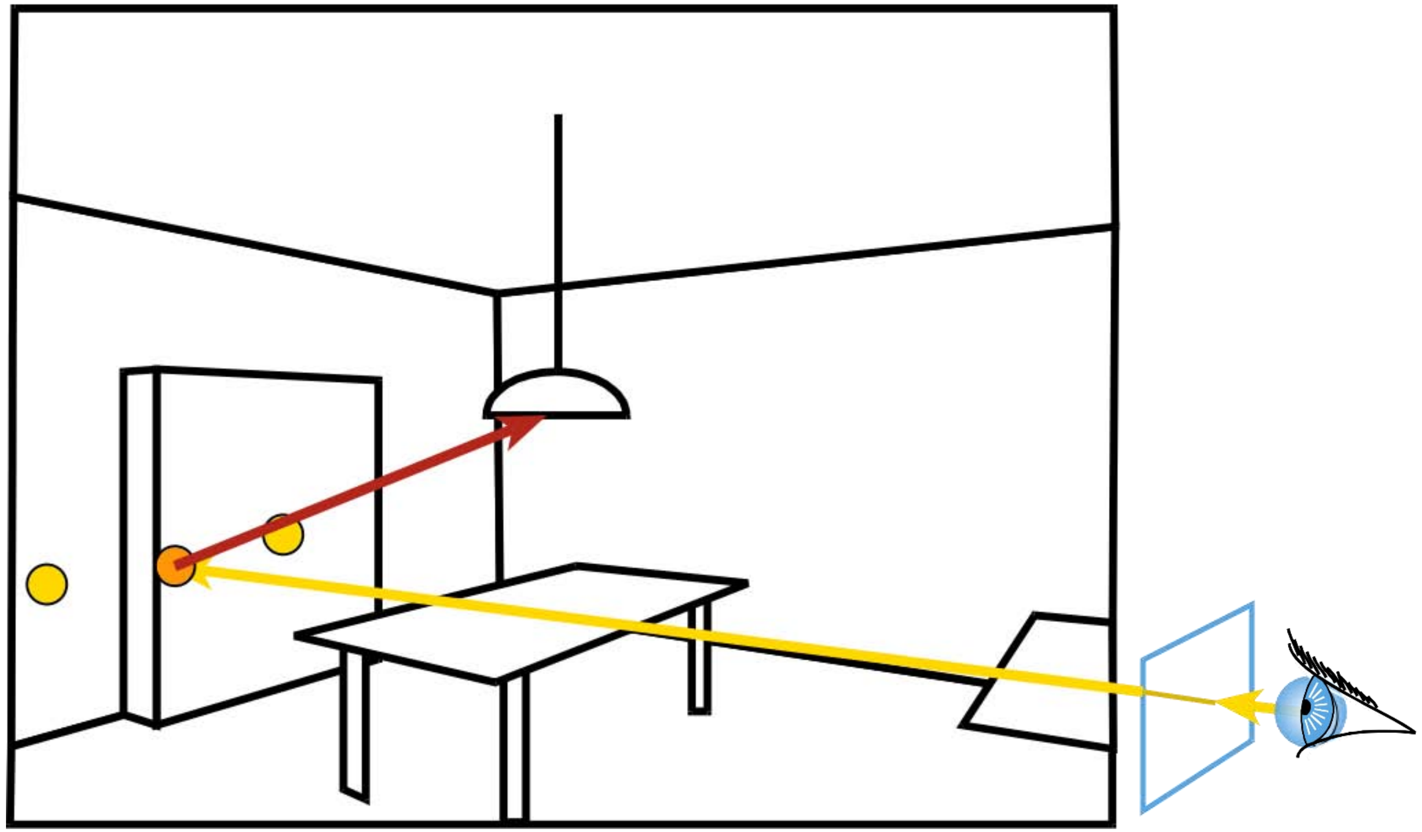
Irradiance Caching

- Indirect illumination is smooth



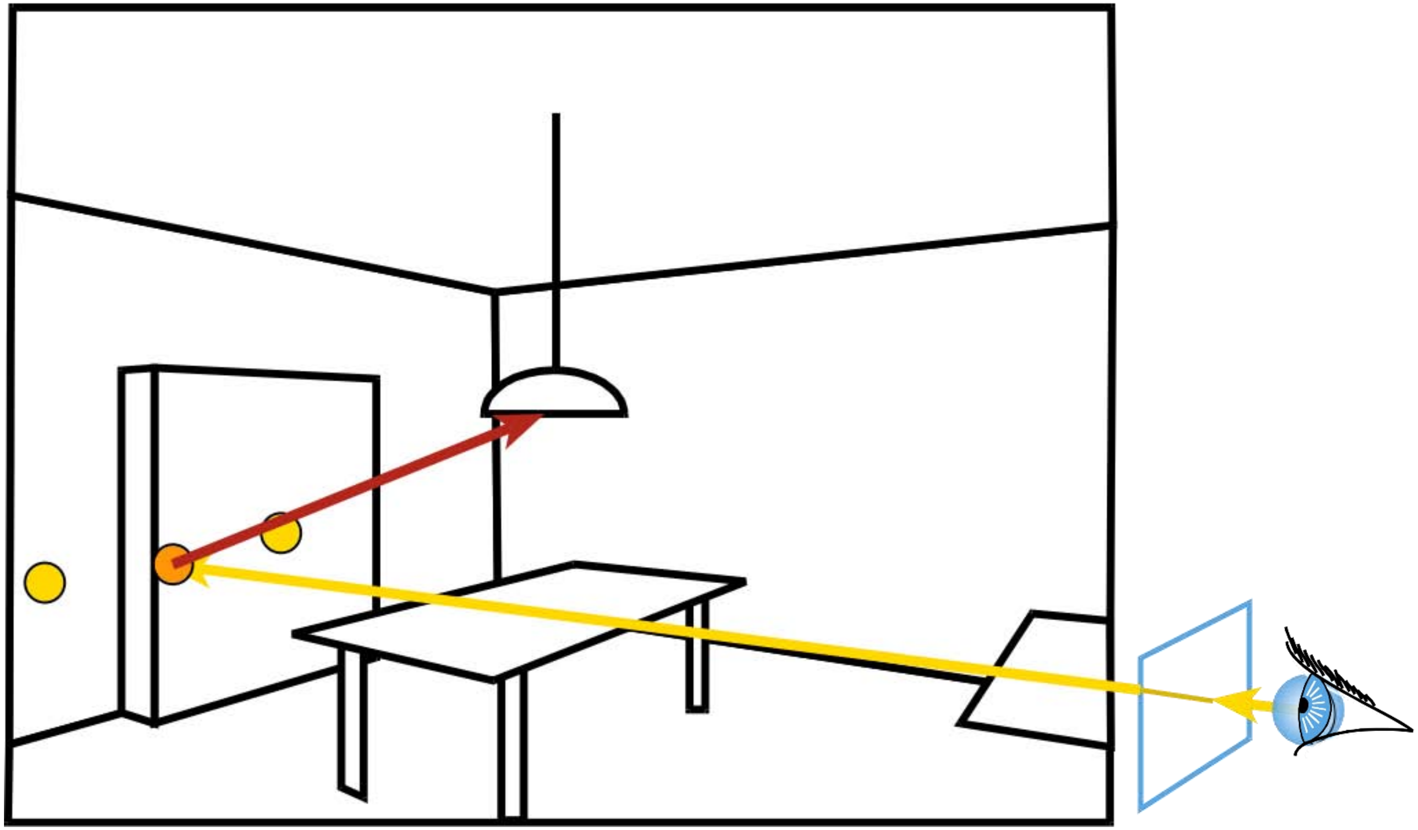
Irradiance Caching

- Indirect illumination is smooth
  ==> Sample sparsely, interpolate nearby values



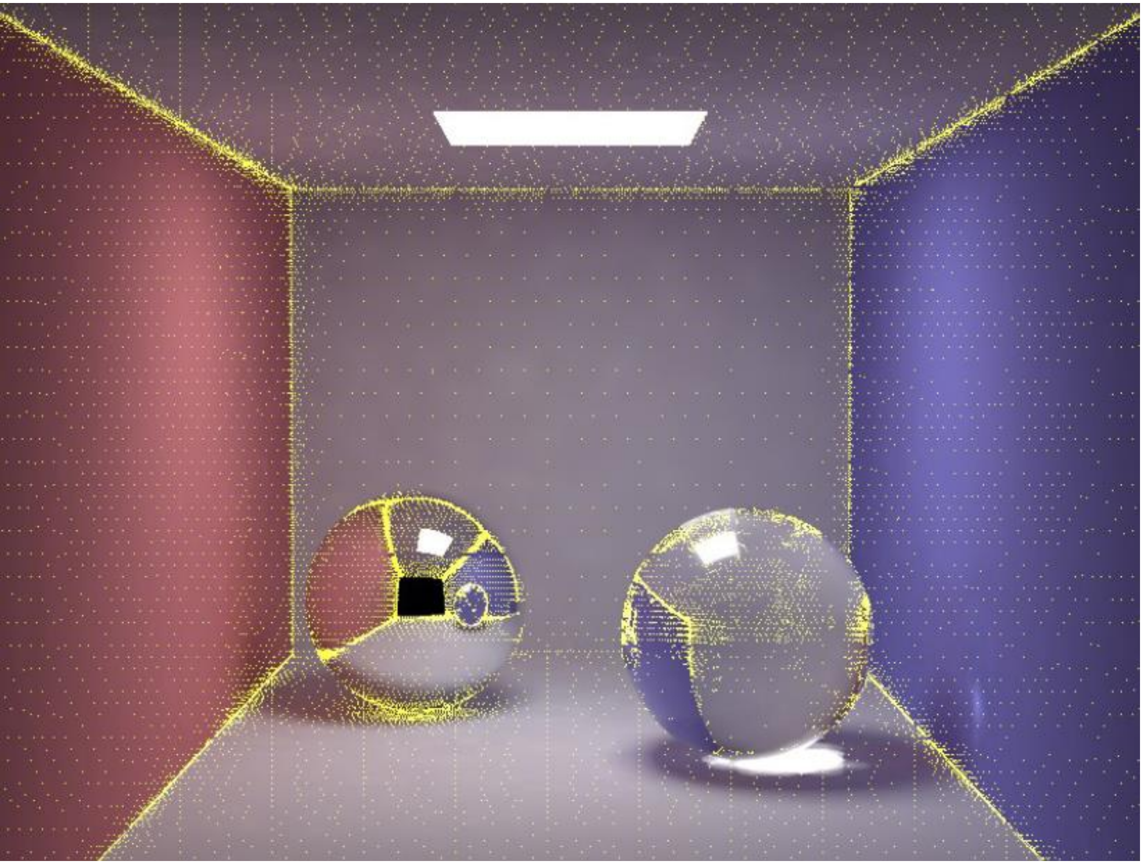
Irradiance Caching

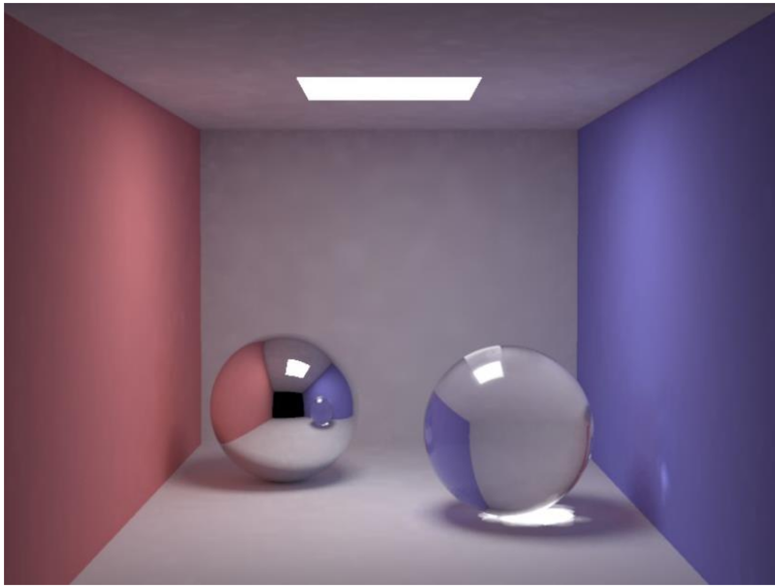
- Store the indirect illumination
- Interpolate existing cached values
- But do full calculation for direct lighting



Irradiance Caching

- Yellow dots:
  indirect diffuse sample points





The irradiance cache tries to adapt sampling density to expected frequency content of the indirect illumination (denser sampling near geometry)

Radiance by Greg Ward

- The inventor of irradiance caching
- http://radsite.lbl.gov/radiance/

Image removed due to copyright restrictions. Please see above link for further details.

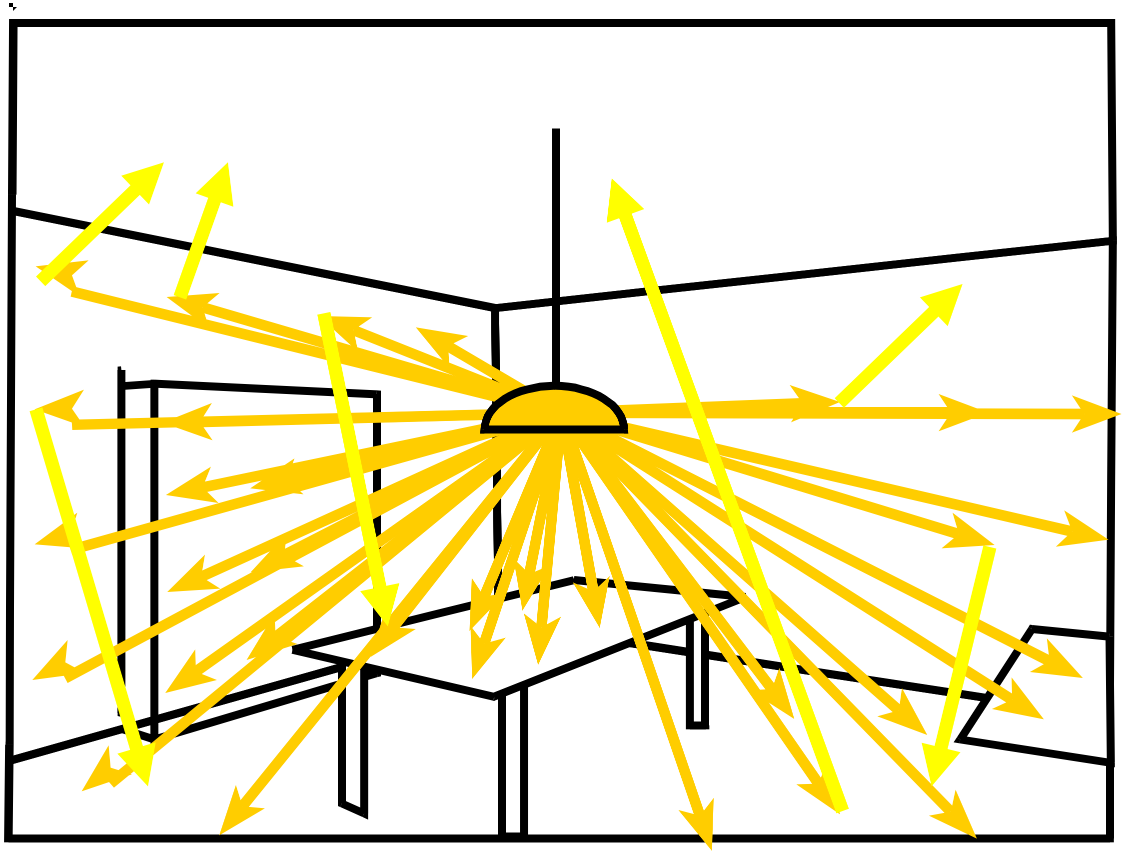
Questions?

Image of Y chair designed by H.J. Wegner has been removed due to copyright restrictions.
Please see http://tora_2097.cgsociety.org/portfolio/project-detail/786738/ for further details.

Image: Pure

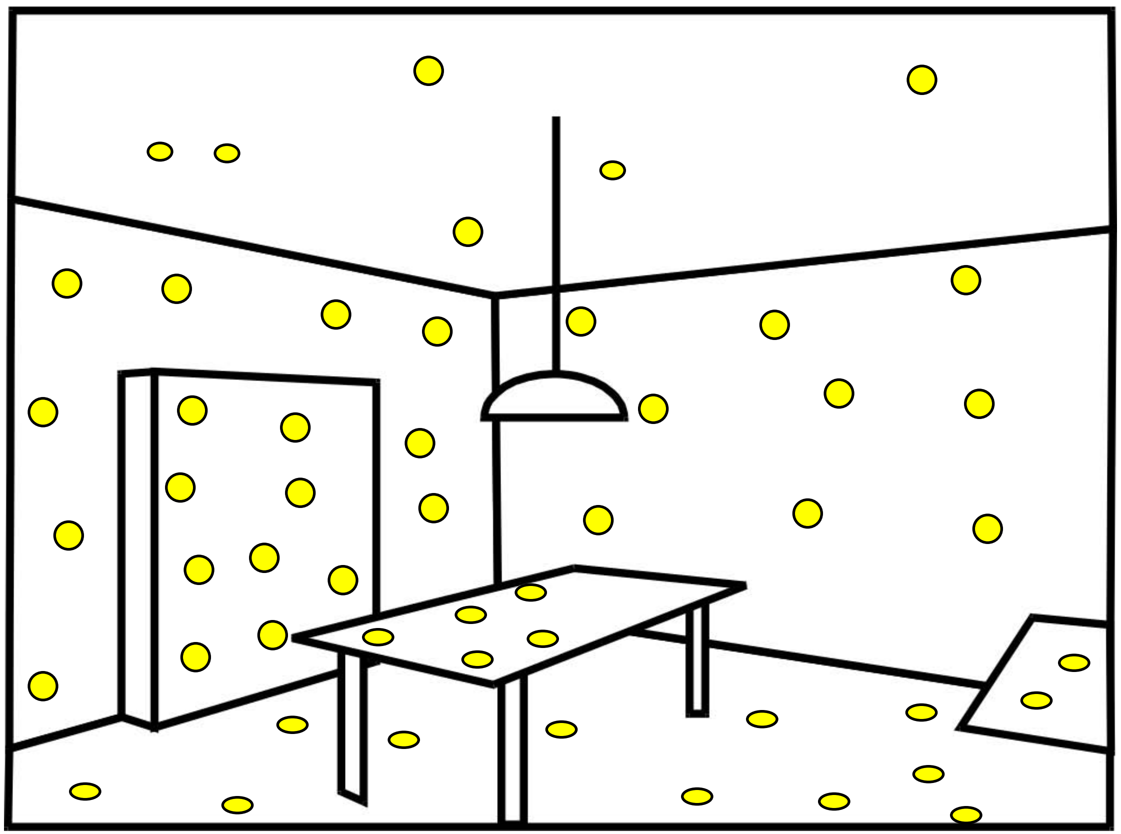
Photon Mapping

- Preprocess: cast rays from light sources, let them bounce around randomly in the scene
- Store “photons”



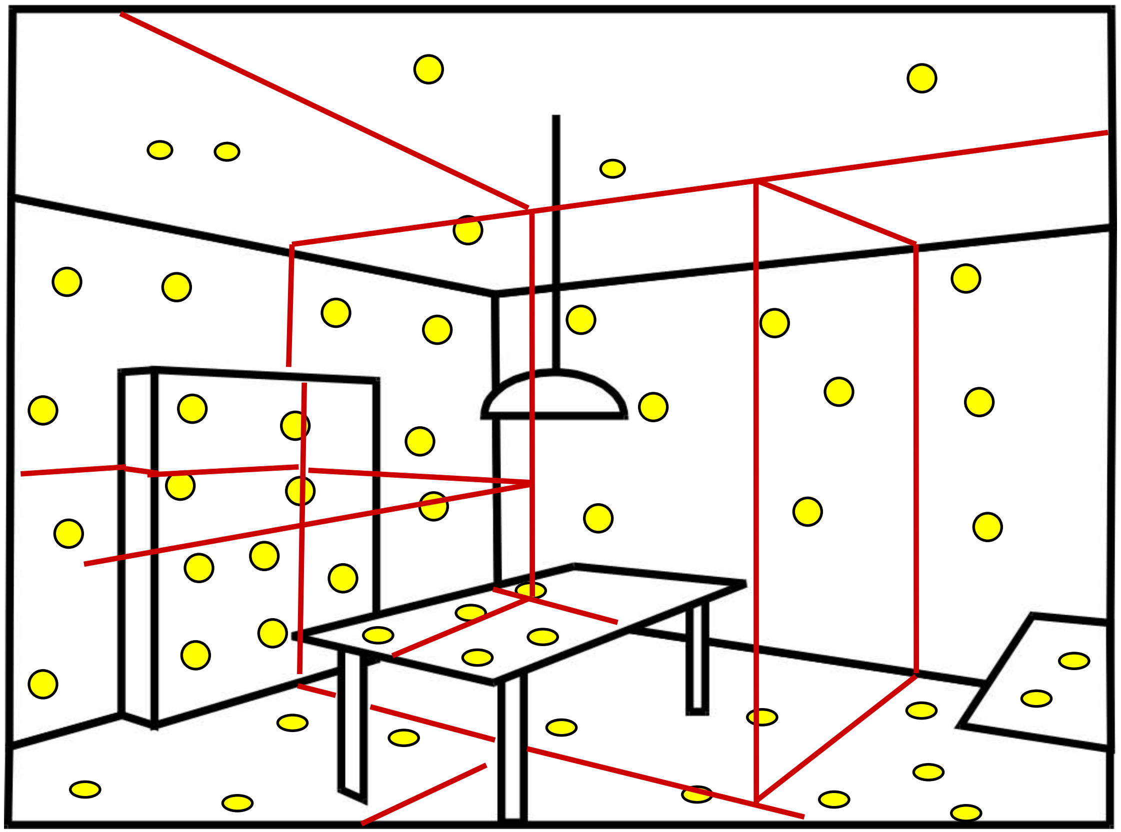
Photon Mapping

- Preprocess: cast rays from light sources
- Store photons (position + light power + incoming direction)



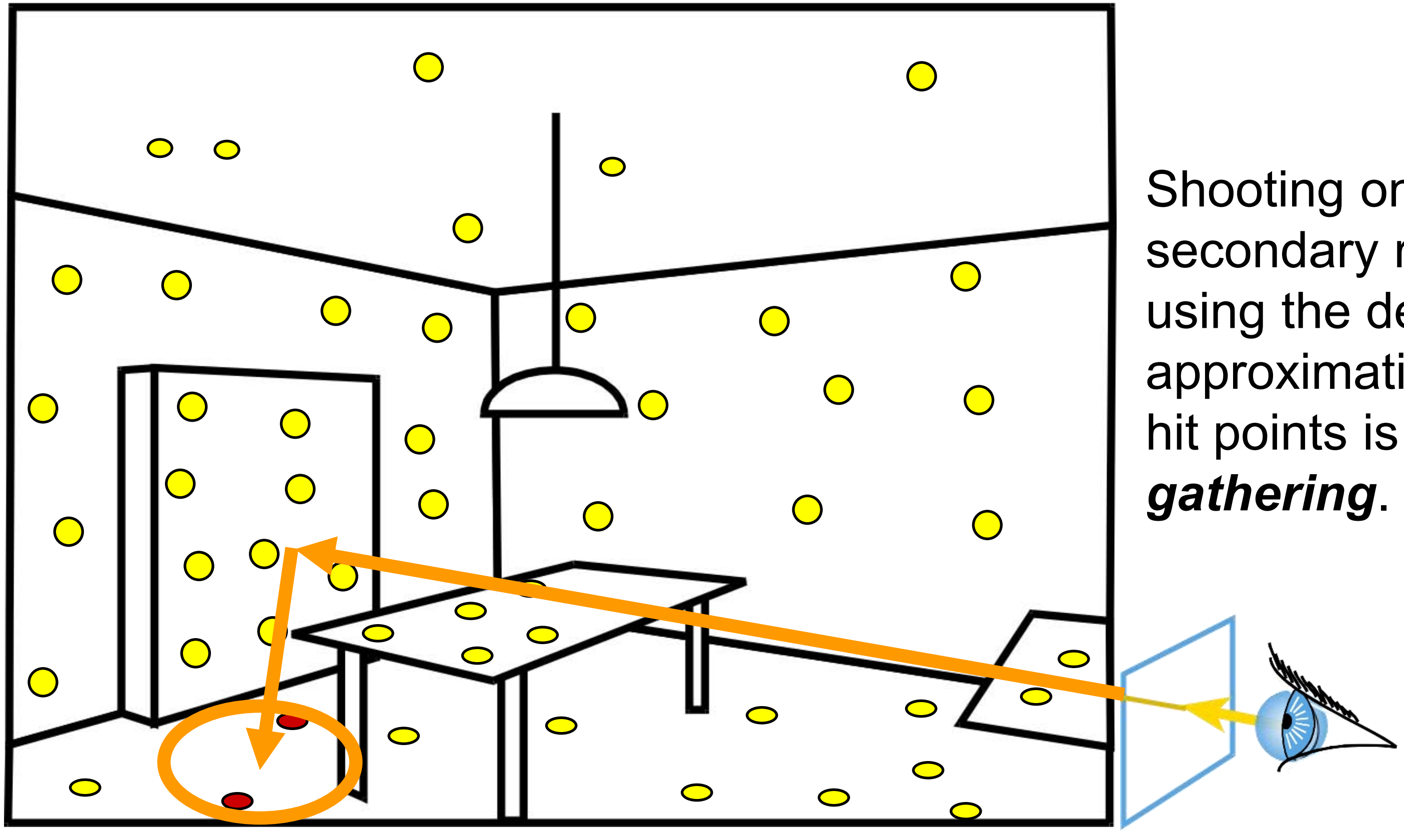
The Photon Map

- Efficiently store photons for fast access
- Use hierarchical spatial structure (kd-tree)



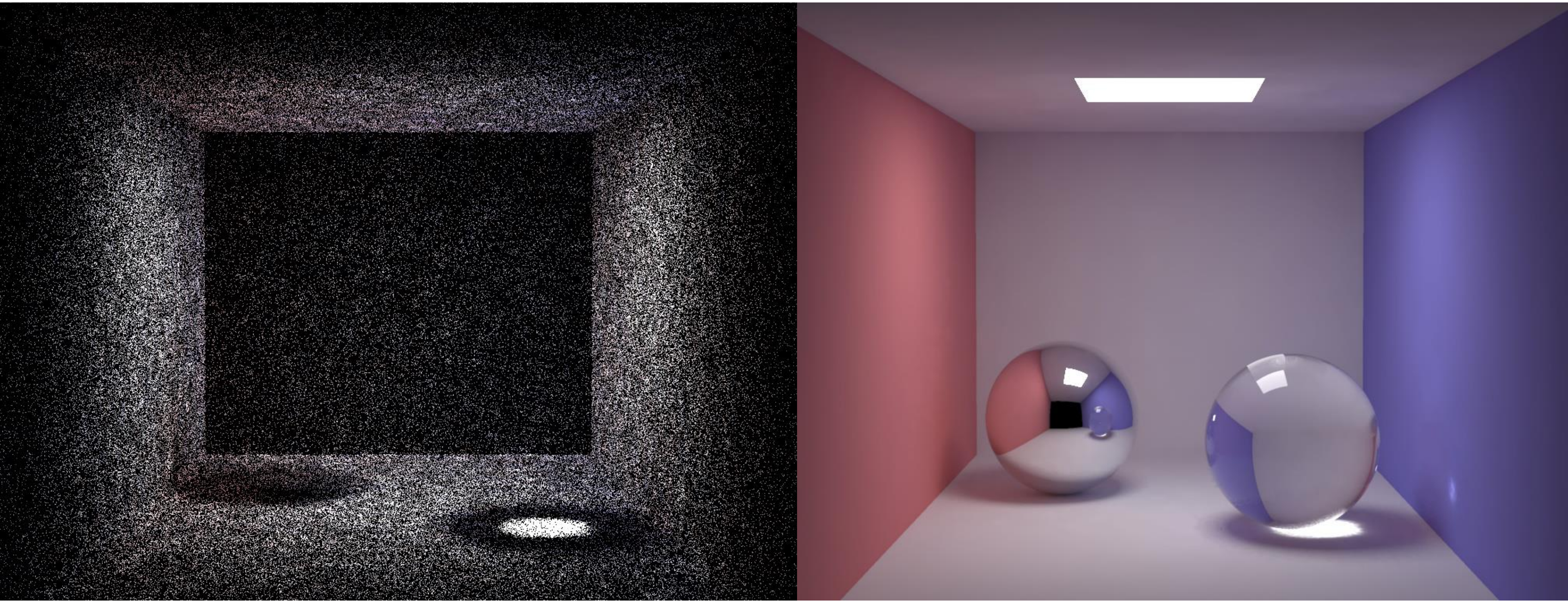
Photon Mapping - Rendering

- Cast primary rays
- For secondary rays
  - reconstruct irradiance using adjacent stored photon
  - Take the k closest photons
- Combine with irradiance caching and a number of other techniques



Shooting one bounce of secondary rays and using the density approximation at those hit points is called ***final gathering***.

Photon Map Results



More Global Illumination Coolness

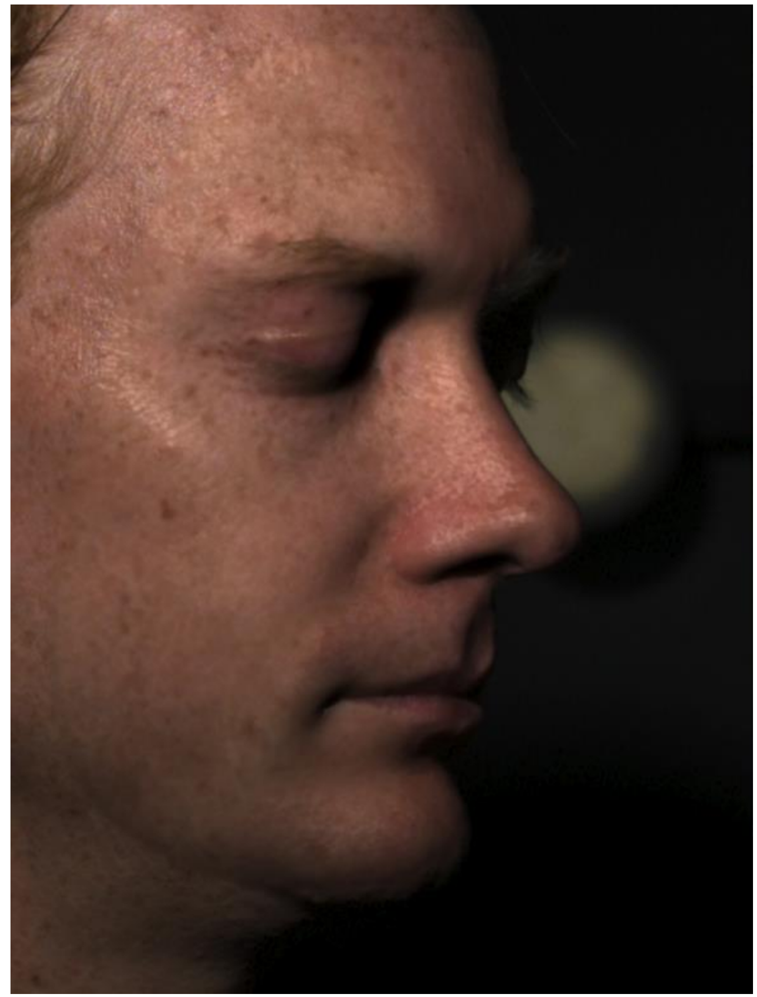
- Many materials exhibit *subsurface scattering*
  - Light doesn't just reflect off the surface
  - Light enters, scatters around, and exits at another point
  - Examples: Skin, marble, milk

Images: Jensen et al.

More Subsurface Scattering

Weyrich et al. 2006

Photograph



Rendering

That Was Just the Beginning

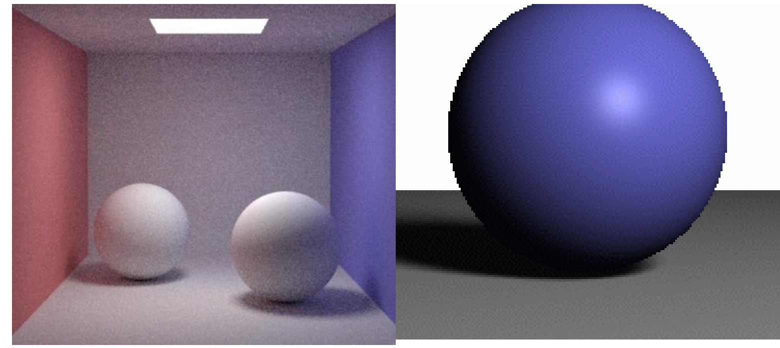
- Tons and tons of other Monte Carlo techniques
  - Bidirectional Path Tracing
    - Shoot random paths not just from camera but also light, connect the path vertices by shadow rays
  - Metropolis Light Transport
- And Finite Element Methods
  - Use basis functions instead of random sampling
  - Radiosity (with hierarchies & wavelets)
  - Precomputed Radiance Transfer

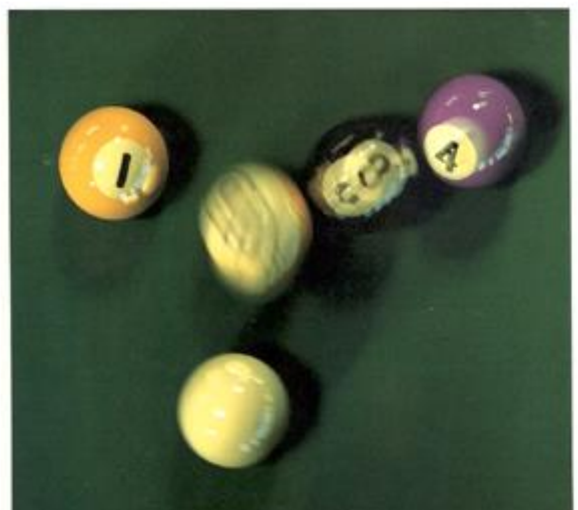
- This would warrant a class of its own!

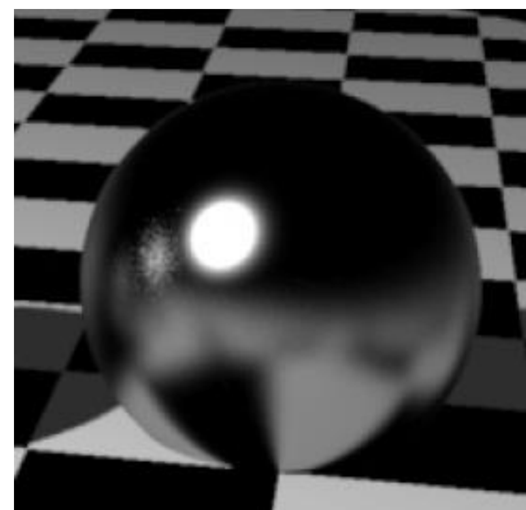
What Else Can We Integrate?

- Pixel: antialiasing
- Light sources: Soft shadows
- Lens: Depth of field
- Time: Motion blur
- BRDF: glossy reflection
- (Hemisphere: indirect lighting)

$$\int\int\int\int\int L(x, y, t, u, v)\,dx\,dy\,dt\,du\,dv$$



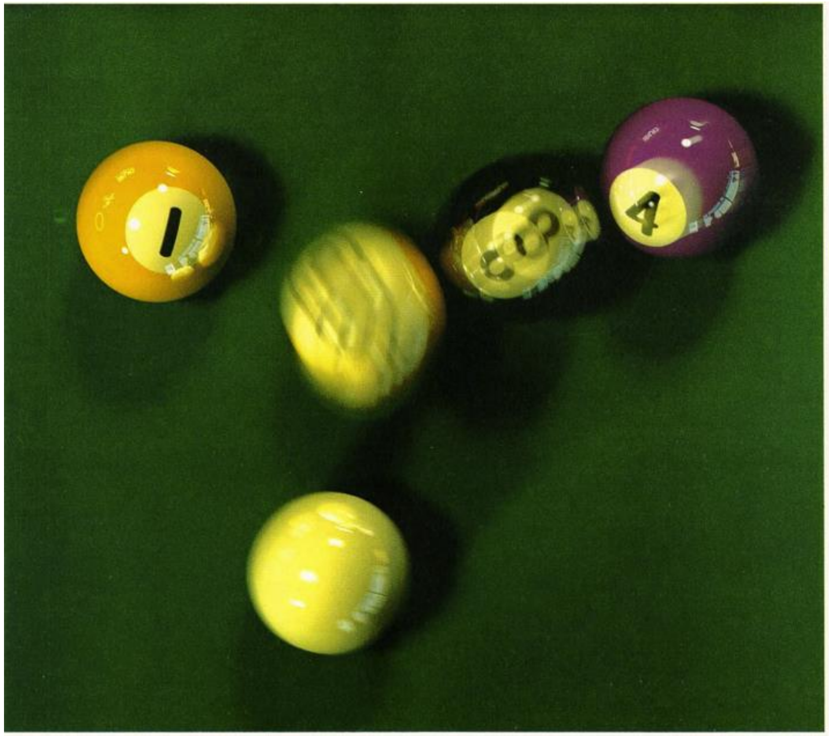




Domains of Integration

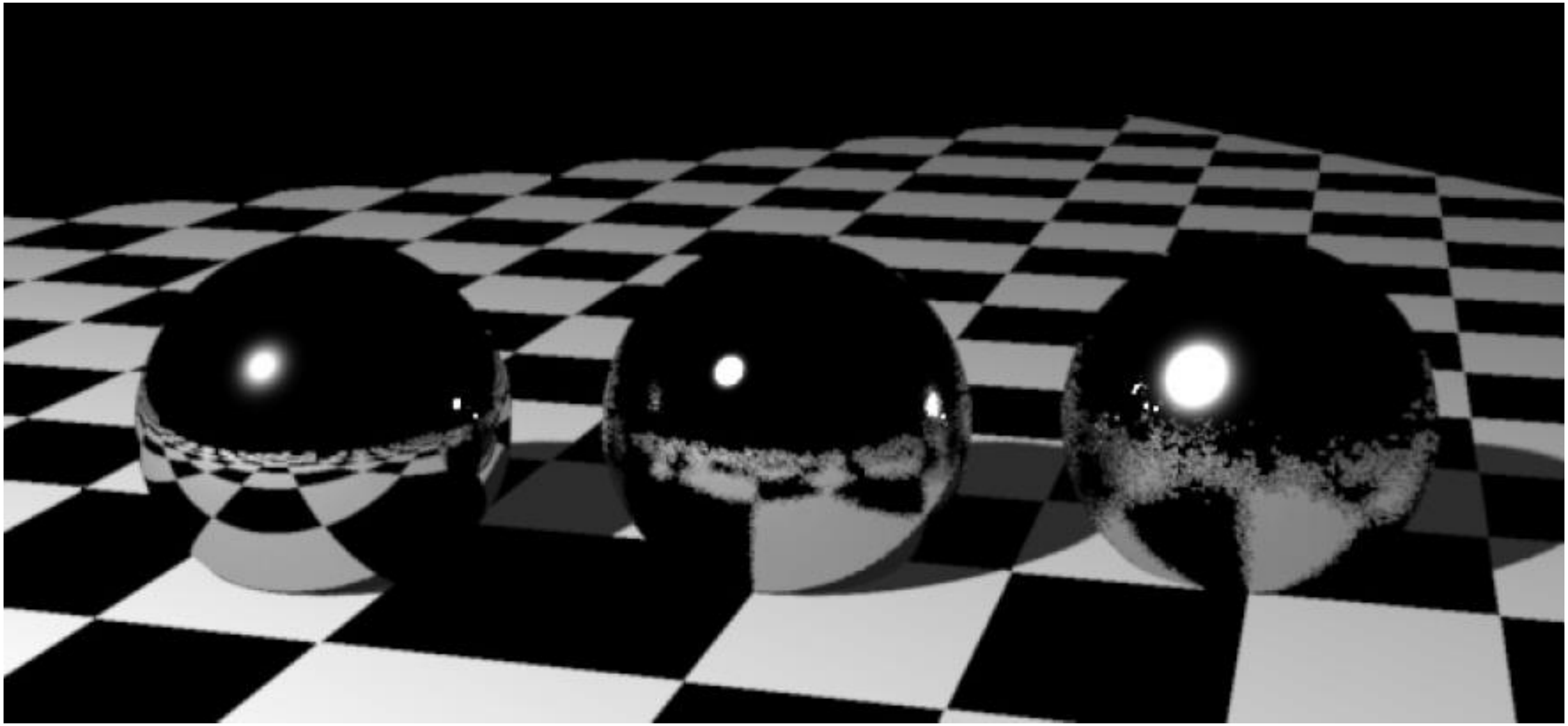
- Pixel, lens (Euclidean 2D domain)
  - Antialiasing filters, depth of field
- Time (1D)
  - Motion blur
- Hemisphere
  - Indirect lighting
- Light source
  - Soft shadows

Famous motion blur image from Cook et al. 1984



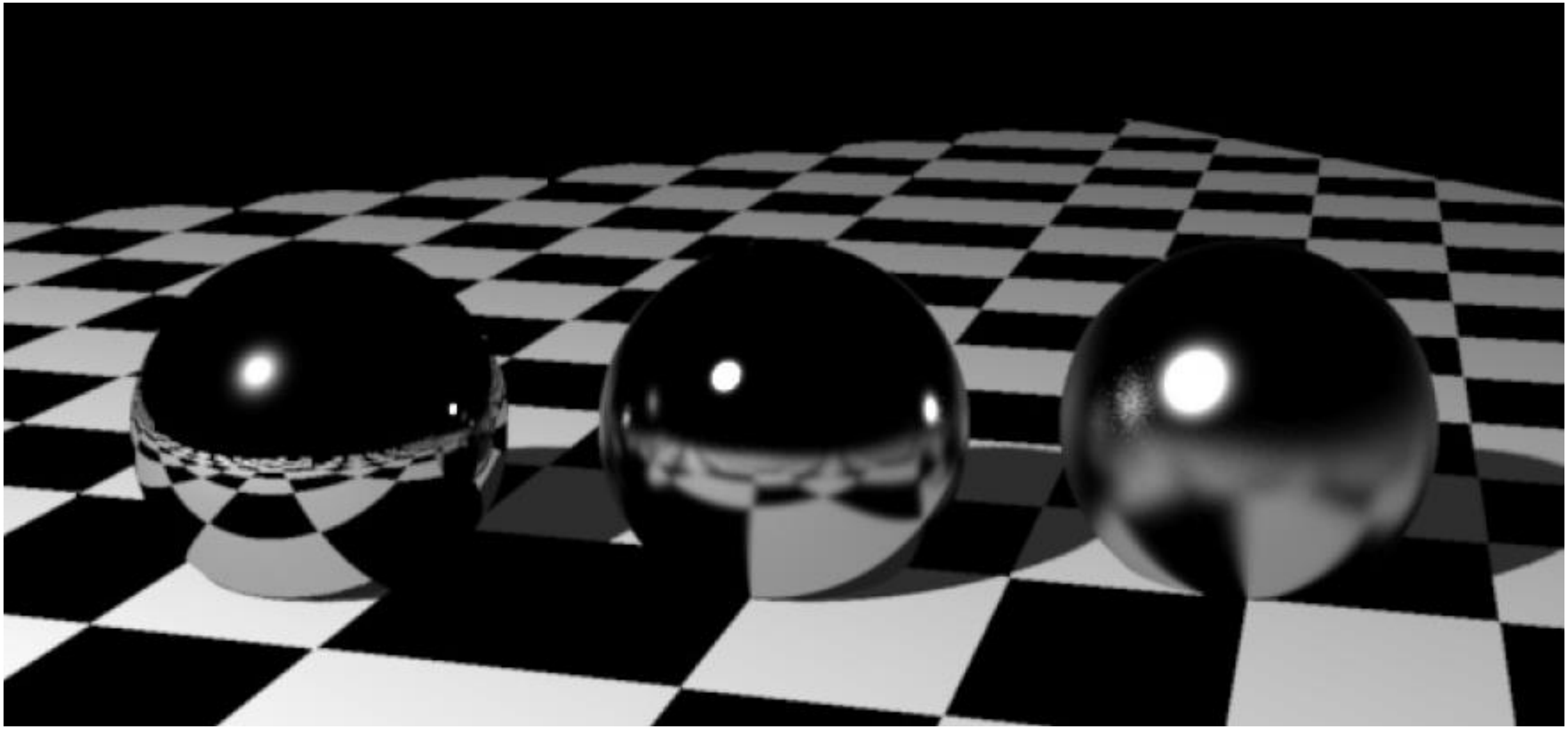
Motivational Eye Candy

- Rendering glossy reflections
- Random reflection rays around mirror direction
  - 1 sample per pixel



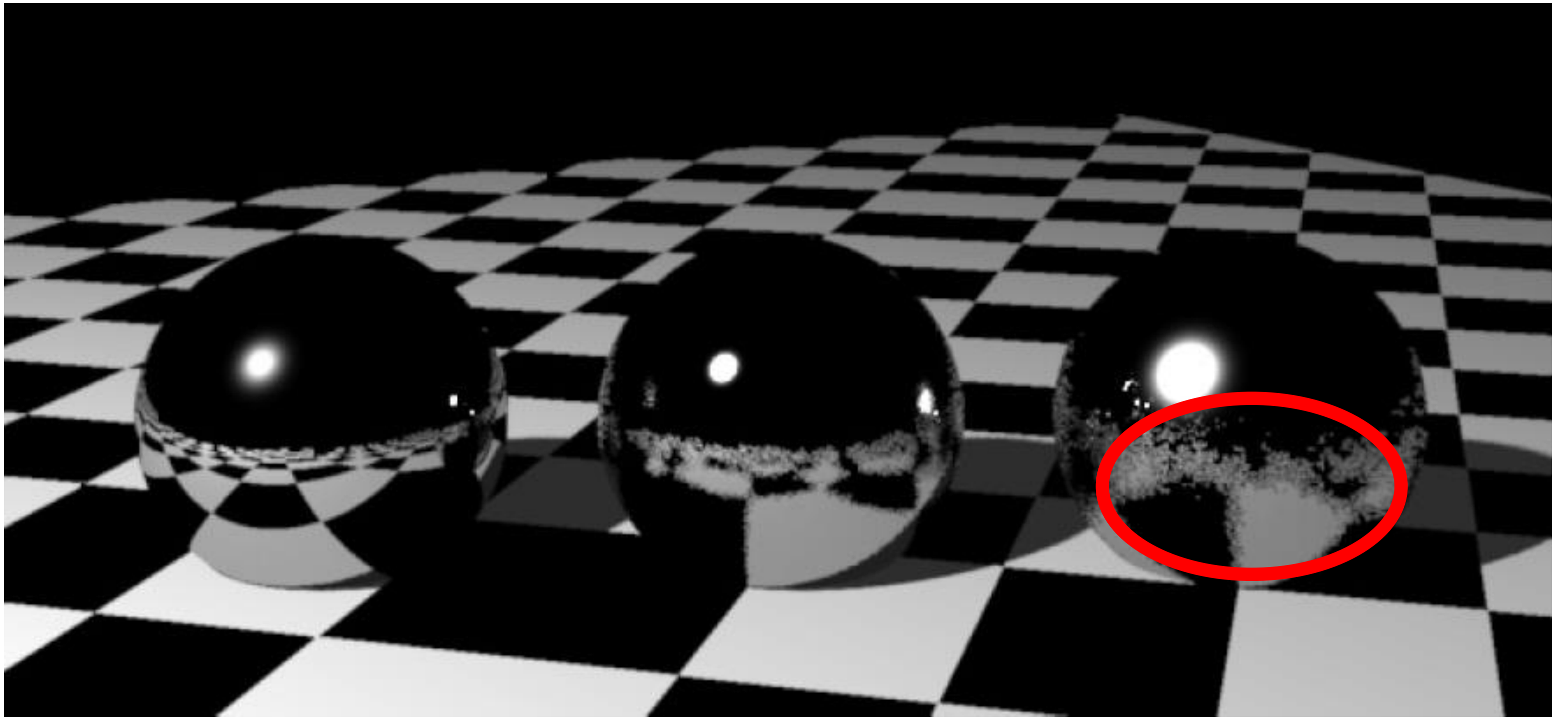
Motivational Eye Candy

- Rendering glossy reflections
- Random reflection rays around mirror direction
  - 256 samples per pixel



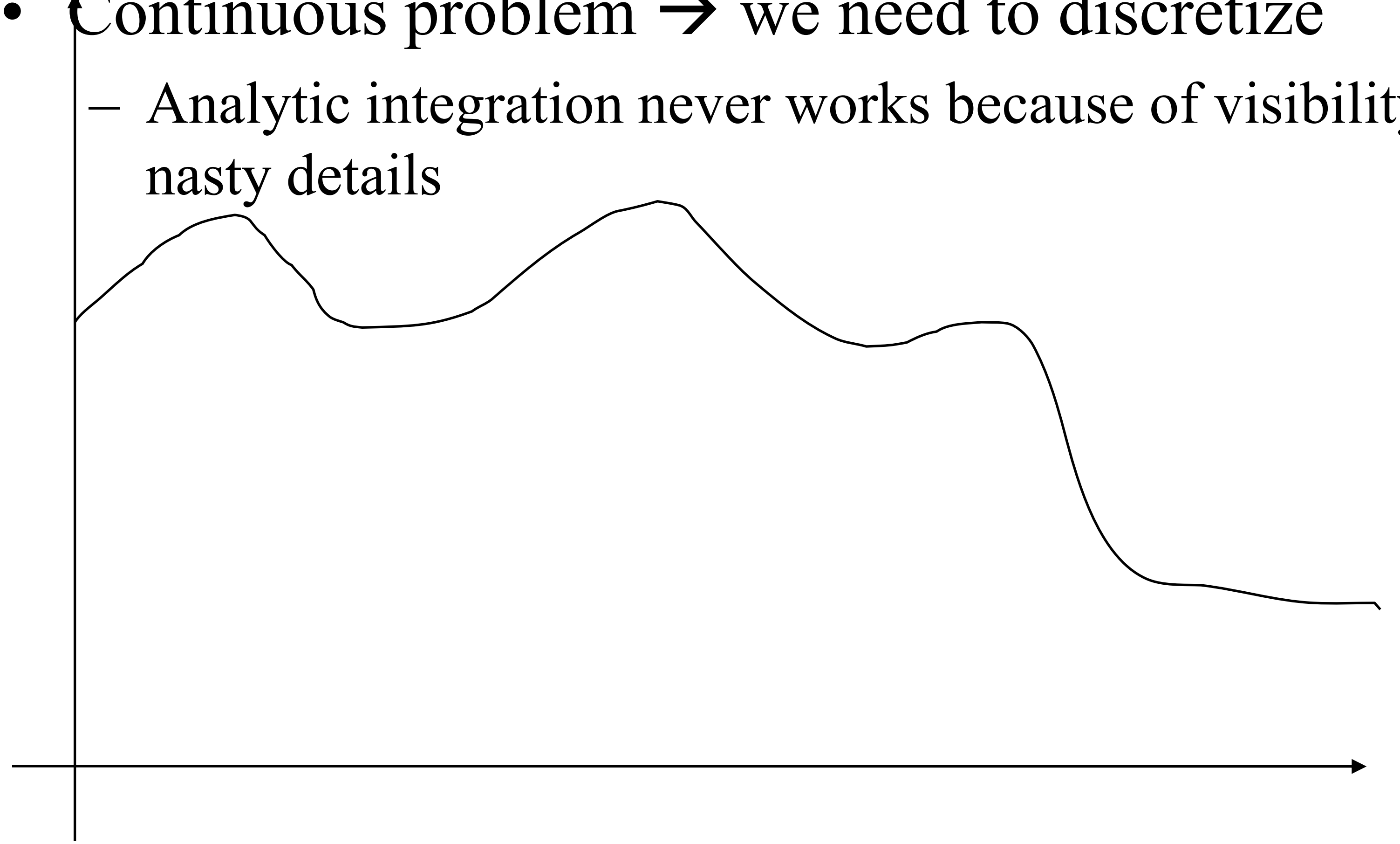
Error/noise Results in Variance

- We use random rays
  - Run the algorithm again → get different image
- What is the noise/variance/standard deviation?
  - And what's really going on anyway?



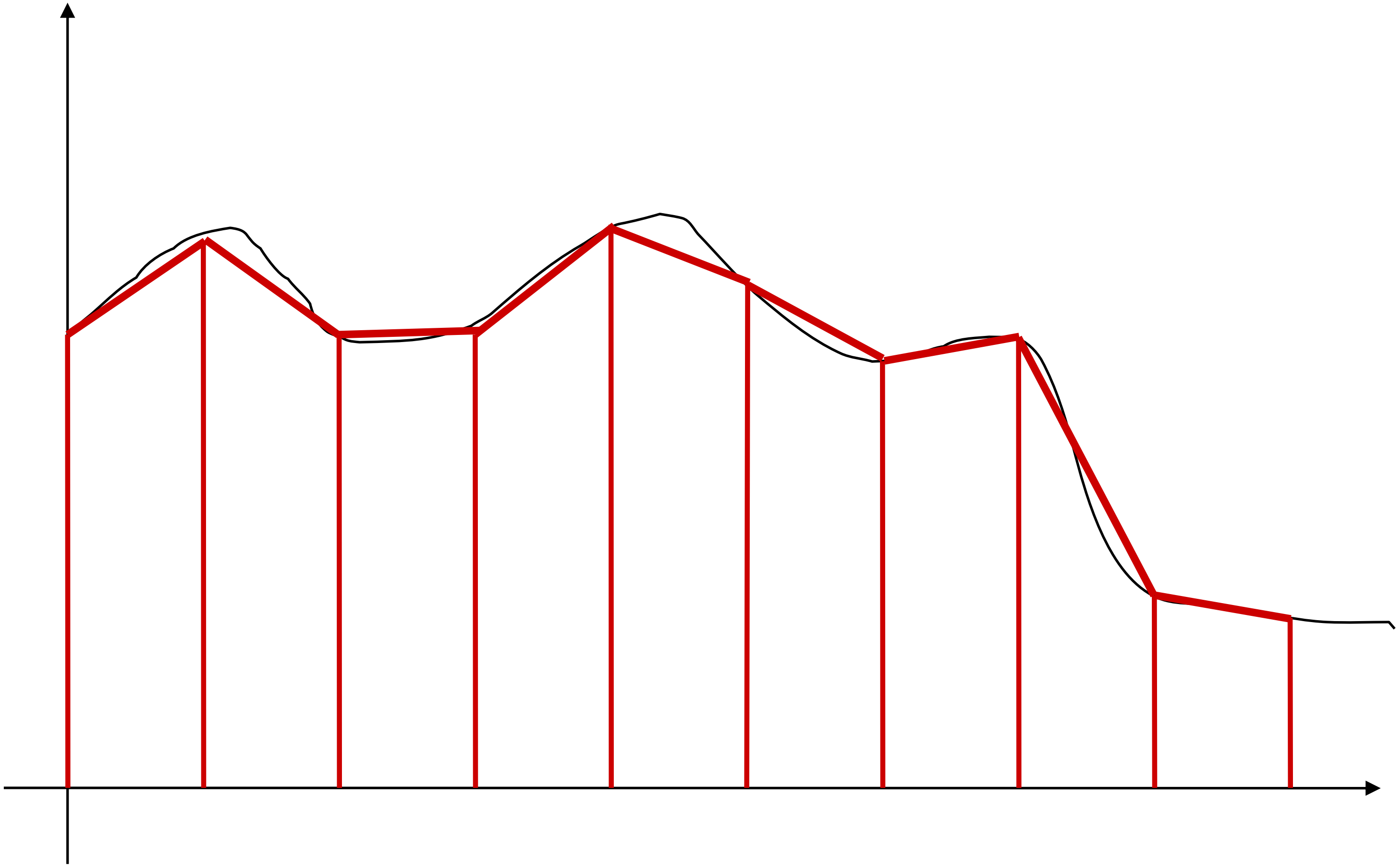
Integration

- Compute integral of arbitrary function
  - e.g. integral over area light source, over hemisphere, etc.
- Continuous problem → we need to discretize
  - Analytic integration never works because of visibility and other nasty details



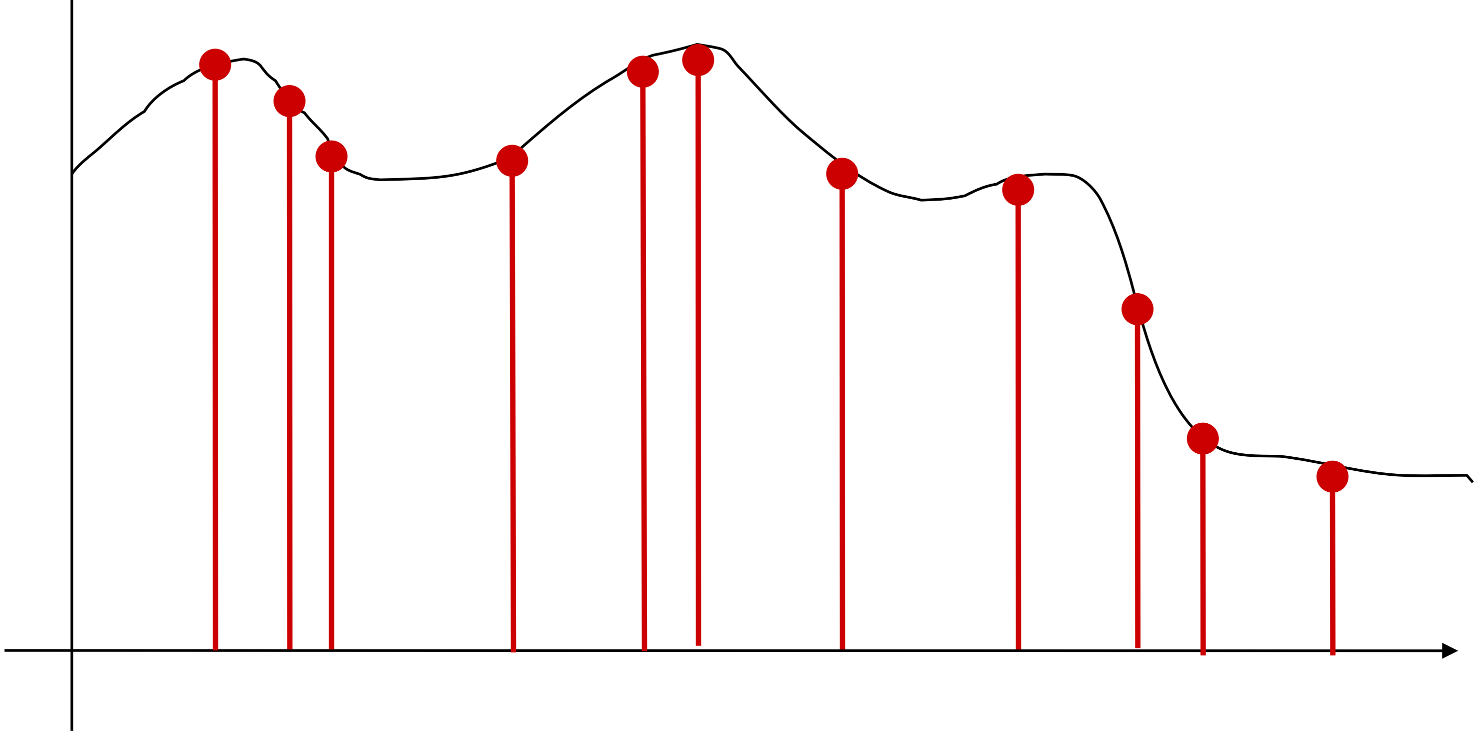
Integration

- You know trapezoid, Simpson’s rule, etc.



Monte Carlo Integration

- Monte Carlo integration: use random samples and compute average
  - We don't keep track of spacing between samples
  - But we kind of hope it will be 1/N on average



Monte Carlo Integration

$$\int_S f(x)\,\mathrm{d}x \;\approx\; \frac{\mathrm{Vol}(S)}{N}\sum_{i=1}^{N} f(x_i)$$

- S is the integration domain
  - Vol(S) is the volume (measure) of S
- $\{x_i\}$ are independent uniform random points in S

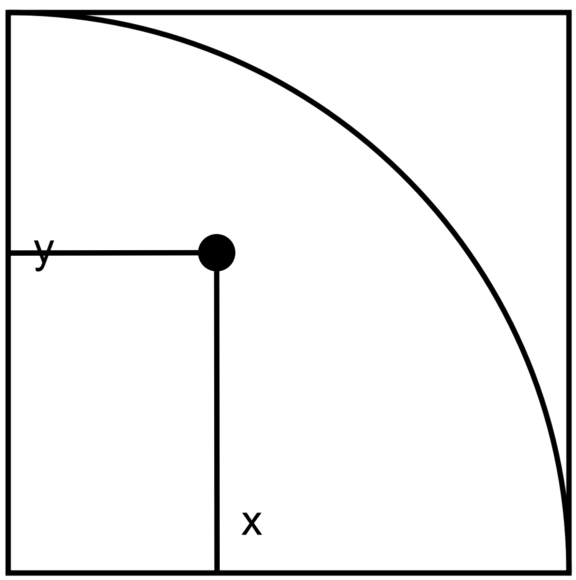
Monte Carlo Integration

$$\int_S f(x)\,\mathrm{d}x \approx \frac{\mathrm{Vol}(S)}{N} \sum_{i=1}^{N} f(x_i)$$

- S is the integration domain
  - Vol(S) is the volume (measure) of S
- $\{x_i\}$ are independent uniform random points in S
- The integral is the average of *f* times the volume of S
- Variance is proportional to 1/N
  - Avg. error is proportional 1/sqrt(N)
  - To halve error, need 4x samples

Monte Carlo Computation of π

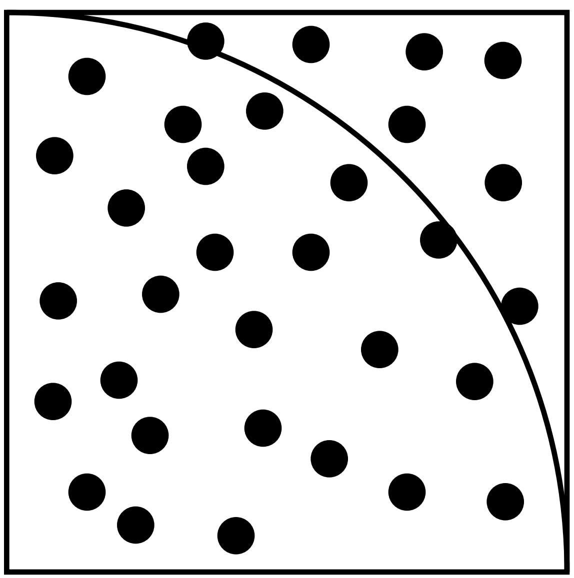
- Take a square
- Take a random point (x,y) in the square
- Test if it is inside the ¼ disc ($x^2+y^2 < 1$)
- The probability is π /4



Integral of the function that is one inside the circle, zero outside

Monte Carlo Computation of π

- The probability is π /4
- Count the inside ratio n = # inside / total # trials
- $\pi \approx n * 4$
- The error depends on the number or trials



Demo

```
def piMC(n):
  success = 0
  for i in range(n):                    x=random.random()
            y=random.random()
            if x*x+y*y<1: success = success+1       return
4.0*float(success)/float(n)
```

Why Not Use Simpson Integration?

- You're right, Monte Carlo is not very efficient for computing π
- When is it useful?
  - High dimensions: Convergence is independent of dimension!
  - For d dimensions, Simpson requires N^d domains (!!!)
  - Similar explosion for other quadratures (Gaussian, etc.)

Advantages of MC Integration

- Few restrictions on the integrand
  - Doesn’t need to be continuous, smooth, ...
  - Only need to be able to evaluate at a point
- Extends to high-dimensional problems
  - Same convergence
- Conceptually straightforward
- Efficient for solving at just a few points

Disadvantages of MC

- Noisy
- Slow convergence
- Good implementation is hard
  - Debugging code
  - Debugging math
  - Choosing appropriate techniques

Questions?

- Images by Veach and Guibas, SIGGRAPH 95

Naïve sampling strategy

Optimal sampling strategy

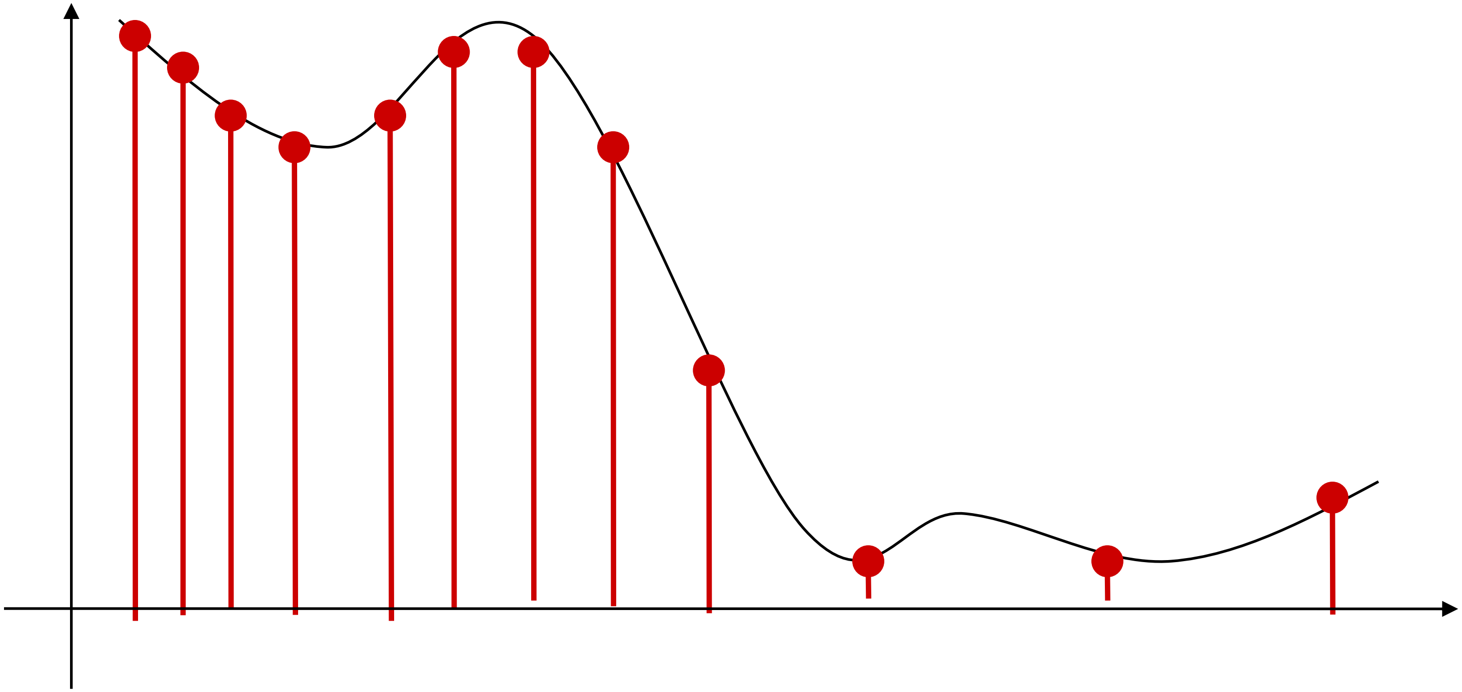
Hmmh...

- Are uniform samples the best we can do?

Smarter Sampling

Sample a non-uniform probability

- Called “importance sampling”
- Intuitive justification: Sample more in places where there are likely to be larger contributions to the integral



Example: Glossy Reflection

Slide courtesy of Jason Lawrence

- Integral over hemisphere
- BRDF times cosine times incoming light

Sampling a BRDF

Slide courtesy of Jason Lawrence

Image removed due to copyright restrictions – please see Jason Lawrence's slide 9-12 in the talk slides on "Efficient BRDF Importance Sampling Using a Factored Representation," available at http://www.cs.virginia.edu/~jdl/.

Sampling a BRDF

Slide courtesy of Jason Lawrence

Image removed due to copyright restrictions – please see Jason Lawrence's slide 9-12 in the talk slides on "Efficient BRDF Importance Sampling Using a Factored Representation," available at http://www.cs.virginia.edu/~jdl/.

Sampling a BRDF

Slide courtesy of Jason Lawrence

Image removed due to copyright restrictions – please see Jason Lawrence's slide 9-12 in the talk slides on "Efficient BRDF Importance Sampling Using a Factored Representation," available at http://www.cs.virginia.edu/~jdl/.

Importance Sampling Math

$$\int_S f(x)\,\mathrm{d}x \approx \frac{\mathrm{Vol}(S)}{N} \sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)}$$

- Like before, but now $\{x_i\}$ are not uniform but drawn according to a probability distribution *p*
  - Uniform case reduces to this with $p(x)$ = const.
- The problem is designing *p*s that are easy to sample from and mimic the behavior of *f*

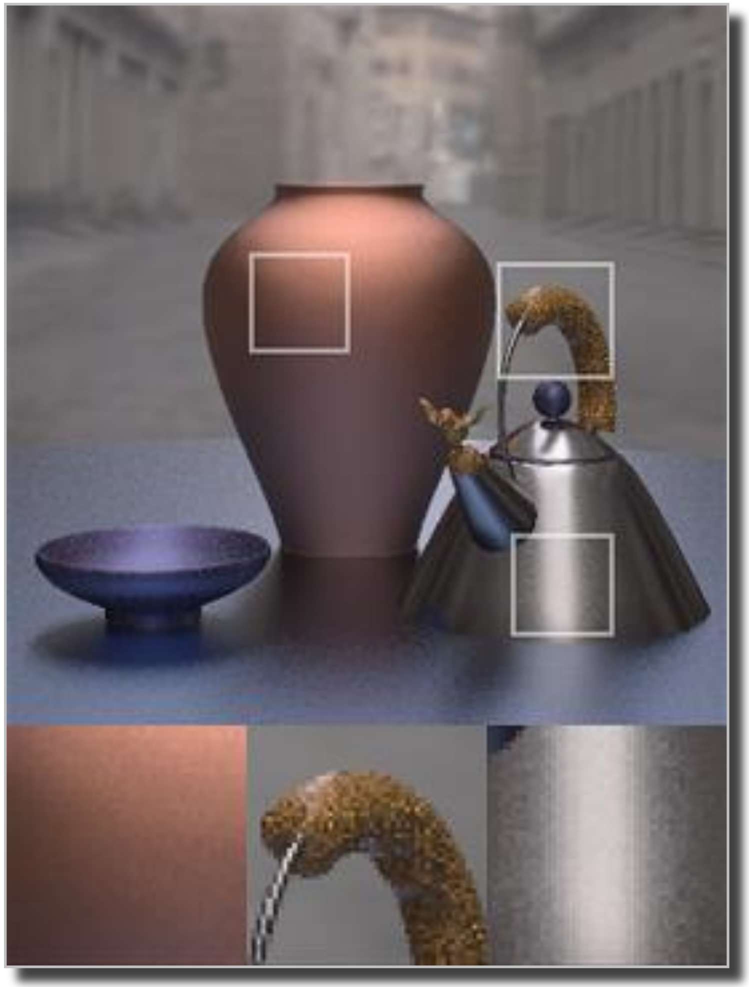
Monte Carlo Path Tracing

Video removed due to copyright restrictions – please see the link below for further details.

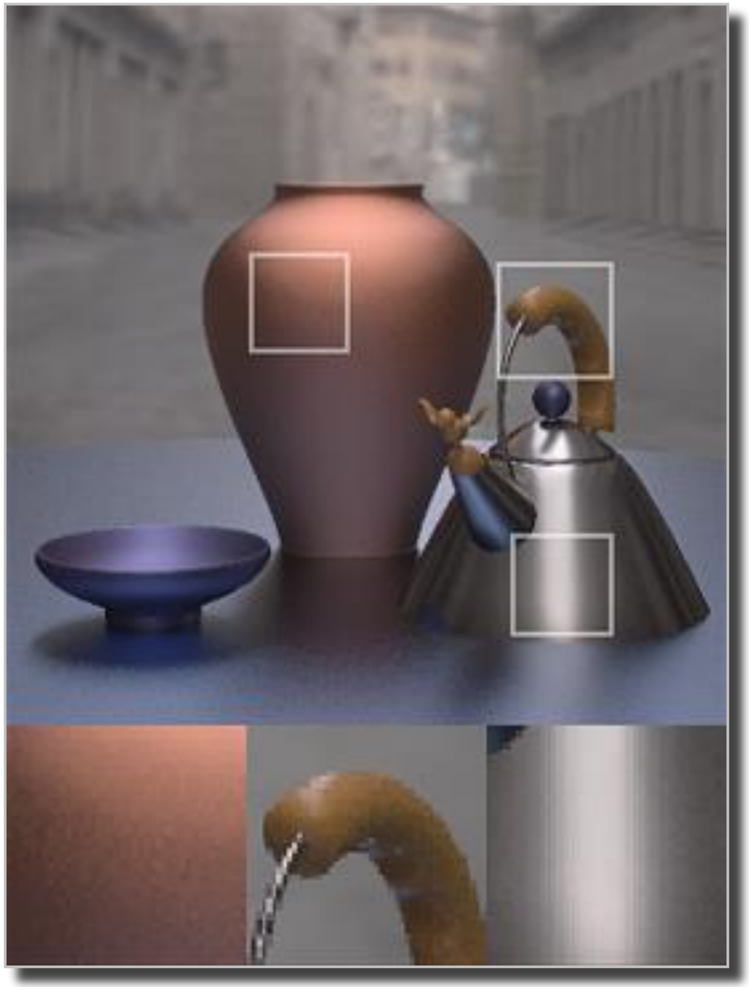
http://www.youtube.com/watch?v=mYMkAnm-PWw

Questions?

1200 Samples/Pixel

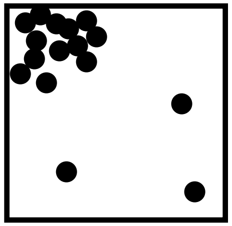


Traditional importance function

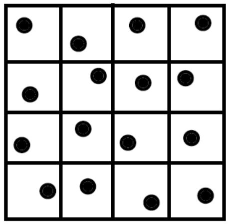


Better importance by Lawrence et al.

Stratified Sampling



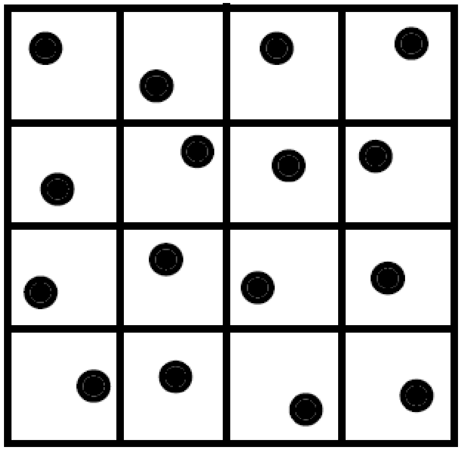
- With uniform sampling, we can get unlucky
  - E.g. all samples clump in a corner
  - If we don't know anything of the integrand, we want a relatively uniform sampling
    - Not regular, though, because of aliasing!



- To prevent clumping, subdivide domain Ω into non-overlapping regions Ω_i
  - Each region is called a *stratum*
- Take one random sample per Ω_i

Stratified Sampling Example

- When supersampling, instead of taking KxK regular sub-pixel samples, do random jittering within each KxK sub-pixel



Stratified Sampling Analysis

- Cheap and effective
- But mostly for low-dimensional domains
  - Again, subdivision of N-D needs N^d domains like trapezoid, Simpson's, etc.!
- With very high dimensions, Monte Carlo is pretty much the only choice

Questions?

- Image from the ARNOLD Renderer by Marcos Fajardo

For Further Information...

- 6.839!
- Eric Veach's PhD dissertation
  http://graphics.stanford.edu/papers/veach_thesis/

- Physically Based Rendering
  by Matt Pharr, Greg Humphreys

References

Images of the following book covers have been removed due to copyright restrictions:
-*Advanced Global Illumination* by Philip Dutre, Philippe Bekaert, and Kavita Bala
-*Realistic Ray Tracing* by Peter Shirley and R. K. Morley
-*Realistic Image Synthesis Using Photon Mapping* by Henrik Wann Jensen
Please check the books for further details.

That’s All for today

Image removed due to copyright restrictions -- please Fig. 13 in Fournier A. and W.T. Reeves. "A Simple Model of Ocean Waves." SIGGRAPH '86 Proceedings of the 13th Annual Conference on Computer Graphics and Interactive Techniques; Pages 75-84.

Image: Fournier and Reeves, SIGGRAPH 86

6.837 Computer Graphics
Fall 2012