MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

SINGAPORE-MIT ALLIANCE
Program on Advanced Materials for Micro and Nano Systems

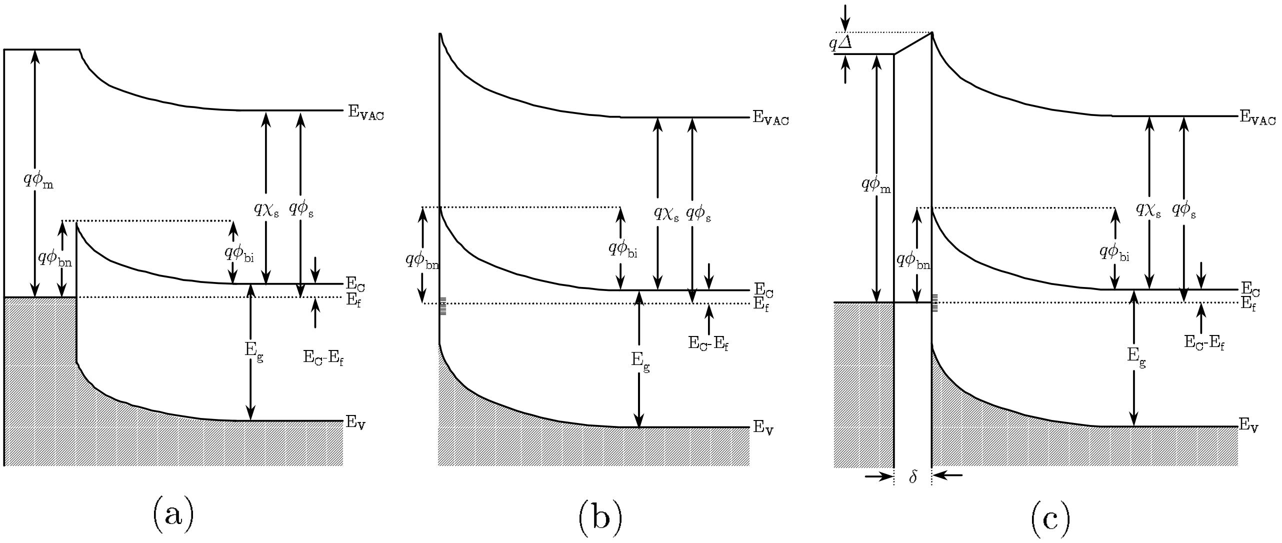
MIT 6.772/SMA 5111: COMPOUND SEMICONDUCTOR DEVICES

Problem Set No. 1 Solutions

Issued: February 10, 2003 **Due:** February 24, 2003

Problem 1.1

Note: We will be using $\varepsilon_r = 12$, so $\varepsilon = \varepsilon_r \varepsilon_o = 10^{-12} \frac{F}{cm}$.



(a) Diagram for parts a and b. (b) Diagram for part c. (c) Diagram for part d.

Part a: Evaluate the Semiconductor Work Function

$$\begin{aligned} q\phi_s &= q\chi_s + [E_C - E_V] && (1) \\ n &= N_C \cdot e^{\frac{-q(E_C - E_V)}{k_B T}} \cong N_D && (2) \\ E_C - E_V &= \frac{k_B T}{q} \cdot ln\left(\frac{N_D}{N_C}\right) && (3) \\ q\phi_s &= q\chi_s + \frac{k_B T}{q} \cdot ln\left(\frac{N_D}{N_C}\right) = 3.68\ eV && (4) \end{aligned}$$

Part b:

Solving for the Barrier Height:

$$q\phi_{bn} = q\phi_m - q\chi_s = 0.5\ eV \qquad (5)$$

Solving for the equilibrium depletion region width:

$$
\begin{aligned}
x_D &= \sqrt{\frac{2\varepsilon}{qN_D}\phi_{bi}} && (6)\\
\phi_{bi} &= \phi_{bn} - \frac{(E_c - E_f)}{q} = 0.32\ eV && (7)\\
x_D &= 0.206\ \mu m && (8)
\end{aligned}
$$

Part c:

Condition of the surface: The surface is depleted. The states are acceptor-like because the states are neutral when unoccupied: the sites can hold static charges only if the electrons occupy these states. Electrons from the surrounding area are drawn away from the surrounding areas to fill these states, depleting the area of free carriers.

Solving for the Depletion Region width:

$$
\begin{aligned}
q\phi_{bn} &= E_g - 0.3E_g = 0.98\ eV && (9)\\
q\phi_{bi} &= q\phi_{bn} - (E_c\ - E_f) = 0.80\ eV && (10)\\
x_D &= \sqrt{\frac{2\varepsilon}{qN_D}\phi_{bi}} = 0.326\ \mu m && (11)
\end{aligned}
$$

Solving for the fraction of states occupied in Thermal Equilibrium: The density of surface states is $N_{surface}$. The number of electrons depleted from the interior must be equal to the number of electrons occupying the surface states (N_S) to maintain quasi-neutrality.

$$
\begin{aligned}
N_s &= N_D x_D = 3.26 \times 10^{11} cm^{-2} && (12)\\
\frac{N_s}{N_{surface}} &= 0.033 && (13)
\end{aligned}
$$

Part d:

Solving for the Schottky Barrier Height: The Fermi level is pinned at the surface states, so the barrier height should remain unchanged from part c.

$$
q\phi_{bn} = E_g - 0.3E_g = 0.98\ eV \qquad (14)
$$

Solving for the Depletion Region width: Since $q\phi_{bn}$ remains unchanged, x_D does not change.

$$
\begin{aligned}
q\phi_{bi} &= q\phi_{bn} - (E_c\ - E_f) = 0.80\ eV && (15)\\
x_D &= \sqrt{\frac{2\varepsilon}{qN_D}\phi_{bi}} = 0.326\ \mu m && (16)
\end{aligned}
$$

Solving for the fraction of states occupied in Thermal Equilibrium: There are three regions of charge: the sheet charge induced on the metal, the charge of the carriers caught in the surface states, and the depletion region charge. These regions are separated by a distance δ and by the potential difference $q\Delta$.

$$q\Delta = q\chi + q\phi_{bn} - q\phi_m = 0.48\ eV \tag{17a}$$

$$C = \frac{\varepsilon}{\delta} = \frac{Q}{V} = \frac{Q}{\Delta} \tag{17b}$$

$$Q_{metal} = \frac{\Delta\varepsilon}{q\delta} = q \cdot 1.592 \times 10^{12} cm^{-2} \tag{17c}$$

$$0 = Q_{metal} + Q_s + Q_{depl} \tag{17d}$$

$$Q_s = -Q_{metal} - Q_{depl} = -q\left(1.592 \times 10^{12} + N_D x_D\right) = -q \cdot 1.918 \times 10^{12} cm^{-2} \tag{17e}$$

$$N_s = \frac{Q_s}{-q} = 2.918 \times 10^{12} cm^{-2} \tag{17f}$$

$$\frac{N_s}{N_{surface}} = 0.192 \tag{17g}$$

Note that the charge at the metal surface (Q_{metal}) is positive, not negative.

Problem 1.2

Please read Sze's book, *Physics of Semiconductor Devices*, for a thorough explanation[1].

Since the semiconductor is terminated by a metal, the force induced by an electron located a distance x away from the interface is given by:

$$F = -\frac{q^2}{4\pi\varepsilon\left(2x\right)^2} = -\frac{q^2}{16\pi\varepsilon x^2} \tag{18}$$

This electron is in the depletion region, where it is also experiences forces due to the space charge. To simplify the derivation, the electric field is the maximum field in the space charge region and is independent of position. The total potential is given by:

$$V(x) = \frac{q^2}{16\pi\varepsilon x^2} + q\mathcal{E}_{max}x \tag{19}$$

The magnitude of barrier lowering, $\Delta\phi$, is given by the condition $\frac{dV}{dx} = 0$. The expressions for the location, x_m, and $\Delta\phi$ of the image force lowering are:

$$x_m = \sqrt{\frac{q}{16\pi\varepsilon\mathcal{E}_{max}}} \tag{20}$$

$$\Delta\phi = \sqrt{\frac{q\mathcal{E}_{max}}{4\pi\varepsilon}} \tag{21}$$

$$\mathcal{E}_{max} = \frac{qN_D x_D}{\varepsilon} \tag{22}$$

$$x_D = \sqrt{\frac{2\varepsilon}{qN_D}\phi_{bi}} = \sqrt{\frac{2\varepsilon}{qN_D}\left(q\phi_{bo} - \frac{k_BT}{q} \cdot ln\left(\frac{N_D}{N_C}\right)\right)} \tag{23}$$

Let $\frac{k_BT}{q} \cdot ln\left(\frac{N_D}{N_C}\right) = K$. K is a weak function of N_D: a order of magnitude change of doping will only yield a 60 mV change in potential. Therefore K is essentially a constant.

$$\Delta\phi = \sqrt{\frac{q\mathcal{E}_{max}}{4\pi\varepsilon}} = \sqrt{\frac{q\frac{qN_Dx_D}{\varepsilon}}{4\pi\varepsilon}} = \frac{q}{2\varepsilon}\sqrt{\frac{N_D}{\pi}\sqrt{\frac{2\varepsilon}{qN_D}}\left(q\phi_{bo} - K\right)} = \frac{q}{2\varepsilon}\sqrt[4]{\frac{2\varepsilon N_D}{q\pi^2}\left(q\phi_{bo} - K\right)} \tag{24}$$

N_D and $(q\phi_{bo} - K)$ varies with an $x^{\frac{1}{4}}$ dependence.

Problem 1.3

p-n junction: The saturation current density of a p-n junction diode is given by:

$$
\begin{aligned}
J_{s,pn} &= q \cdot n_i^2 \left[\frac{D_e}{w_p^* \cdot N_A} + \frac{D_h}{w_n^* \cdot N_D} \right] && (25) \\
J_{s,pn}(N_A = 10^{18} cm^{-3}) &= 3.148 \times 10^{-11} \frac{Amps}{cm^2} && (26) \\
J_{s,pn}(N_A = 10^{16} cm^{-3}) &= 6.876 \times 10^{-11} \frac{Amps}{cm^2} && (27)
\end{aligned}
$$

where w_p^* and w_n^* are the effective lengths of the p and n regions. Since the minority carrier lifetime is infinite (the carriers do not recombine), $w_p^* = w_n^* = 1\mu m$.

Metal Semiconductor Diode: Again, Sze's book, *Physics of Semiconductor Devices*, contains a complete examination of metal semiconductor junctions[2]. The saturation current density of a metal-semiconductor diode is:

$$J_{s,MS} = A^* T^2 e^{-\frac{q\phi_{bn}}{k_B T}}. \qquad (28)$$

where A^* is the effective Richardson constant for thermionic emission, defined as:

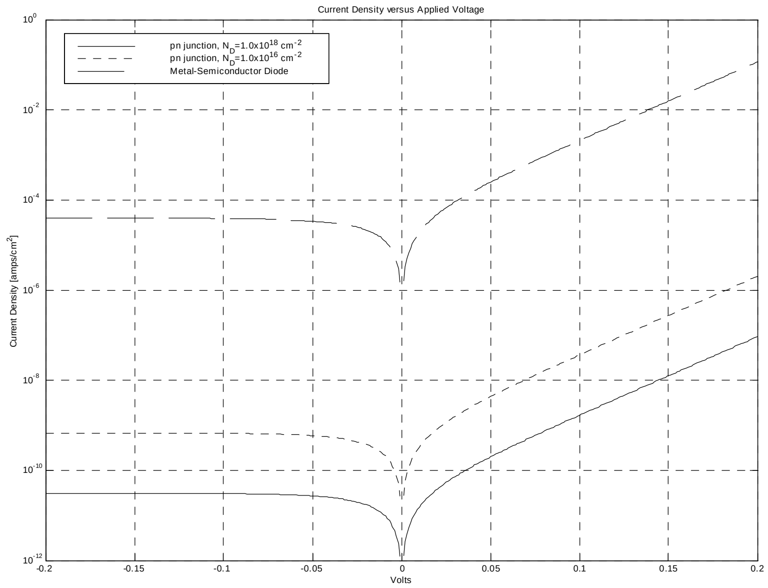
$$A^* = \frac{4\pi q m^* k_B^2}{h^3} \qquad (29)$$

Note the m^* is the effective mass, and is dependent on crystal orientation. For this problem, the <100> direction is used, so:

$$m^* = 2m_t + 4\sqrt{m_t m_l} \qquad (30)$$

where m_t and m_l are the transverse and longitudinal masses. Thus:

$$
\begin{aligned}
A^* &= 253.092 \frac{Amps}{K^2 cm^2} && (31) \\
J_{s,ms} &= 3.964 \times 10^{-5} \frac{Amps}{cm^2} && (32)
\end{aligned}
$$



Current Characteristics of the three devices.

Problem 1.4

Part a:

i: What are they reporting? They are reporting that the band gap for InN is between 0.7 and 0.8 eV.

ii. What measurements do they use? They used three different measurement techniques: optical absorption, photoluminescence, and photomodulated reflection.

iii. Why are these results important? Most books and articles use 1.9 eV as the band gap for InN, for both the wurtzite and zinc-blende lattices. If their claim is valid, this discovery changes our understanding, processing, and modeling of the material, . A fairly recent article reported the band gaps as 1.1 eV[3].

Part b:

i: How should the energy band gap of GaN_xAs_{1-x} vary with x, for small concentrations of N? The band gap should vary quadratically with x.

$$E_g\left(A_{1-x}B_x\right) = (1-x)\,E_g(A) + xE_g(B) - Cx\,(x-1) \tag{33}$$

> where the so-called bowing parameter C accounts for the deviation from a linear interpolation (virtual-crystal approximation) between the two binaries A and B. The bowing parameter for III-V alloys is typically positive (i.e., the alloy band gap is smaller than the linear interpolation result) and can in principle be a function of temperature. The physical origin of the band gap bowing can be traced to the disorder effects caused by the presence of different cation (anions)[4].

ii: What is in the literature? There is no single accepted value of C, ranging from 14-20 eV. Most articles agree that C is a function of x. The band gap variation as a function of x can be divided into two regions: (i) a bandlike region where the bowing coefficient is constant, and (ii) an impuritylike region where the bowing coefficient is considerably larger and composition dependent[5][6]. For very small concentrations of Nitrogen ($x \leq 0.04$), the band gap varies linearly with x[4][7], and the cubic equation $E_g(x) = 20.4x^2 + 100x^3 eV$ for $x \leq 0.15$[4].

Part c:

Write a 25-50 word synopsis of velocity saturation: Scattering of low kinetic energy carriers transfers the energy to the lattice and generates acoustical phonons. Carriers subjected to high electric fields obtain a large kinetic energy and scattering produces optical phonons. Generating optical phonons is an effective method of transferring the carrier's kinetic energy to the lattice, and is the main cause of velocity saturation.

Resources Used:

Note: I did NOT use all of the listed resources to solve this problem set. These books contain information relevant to the problems introduced in this problem set: many of them have the same information.

- Muller, Richard S., Kamins, Theodore I., *Device Electronics for Integrated Circuits: 2^{nd} edition.* New York: John Wiley & Sons, 1986
- Neamen, Donald A., *Semiconductor Physics and Devices: Basic Principles.* Homewood, IL: Richard D. Irwin, INC., 1992
- Sze, S.M., *Physics of Semiconductor Devices.* New York: John Wiley & Sons, 1981

Bibliography

[1] Sze, S.M., *Physics of Semiconductor Devices.* New York: John Wiley & Sons, 1981, pgs 250-254

[2] Sze, S.M., *Physics of Semiconductor Devices.* New York: John Wiley & Sons, 1981, pgs 255-258

[3] Inushima, T., Mamutin, V.V., Vekshin, V.A., Ivanov, S.V., Sakon, T., Motokawa, M., Ohoya, S., Physical Properties of InN with the Band Gap Energy of 1.1 eV, Journal of Crystal Growth, **227-228** 481-485, 2001

[4] Vurgaftman, I., Meyer, J.R., Ram-Mohan, L.R., *Band Parameters for III-V Compound Semiconductors and their Alloys*, Journal of Applied Physics, **89** (11) 5815, 1 June 2001

[5] Wei, Su-Huai, Zunger, Alex, *Giant and Composition-Dependent Optical Bowing Coefficient in GaAsN Alloys,* Physical Review Letters, **74** (4) 664, 22 January 1996

[6] Bi, W.G., Tu, C.W., *Bowing Parameter of the Band-gap Energy of* $GaN_xAs_{1\text{-}x}$, Applied Physics Letters, **70** (12) 1608, 24 March 1997

[7] Pozina, G., Ivanov, I., Monemar, B., *Properties of Molecular-Beam Epitaxy-Grown GaNAs from Optical Spectroscopy*, Journal of Applied Physics, **84** (7) 3830, 1 October 1998