6.728 Applied Quantum and Statistical Physics:

Department of Electrical Engineering and Computer Science
Massachusetts Institute of Technology

PROBLEM SET 9

Problem Set Out: 11/15/06   Problem Set Due: 11/22/06 at the beginning of class

Problem 9.1 In this problem we will model how the electrons near the surface of a MOSFET become a two dimensional electron gas. We will model electrons in the conduction band as free electrons confined to a box length L in the y and z-directions, and experiencing a linear potential in the x-direction:

$$V(x, y, z) = \begin{cases} eEx & x \geq 0 \\ 0 & 0 < y < L \\ 0 & 0 < z < L \\ \infty & \text{otherwise} \end{cases}$$

Where $E \approx 10^8 \,\text{V/m}$ near the oxide surface at $x = 0$.

a) Argue that the eigenenergies are

$$E = E_N^x + \frac{\hbar^2 k_y^2}{2m^*} + \frac{\hbar^2 k_z^2}{2m^*}$$

Where E_N^x are the eigenenergies of the one dimensional potential $V(x) = eEx$ for $x \geq 0$. Assume you have already found the energies $E_{N,1}^x, E_{N,2}^x, E_{N,3}^x$. What does the wavefunction $\psi(x, y, z)$ look like roughly, and what value do k_y and k_z take on?

b) If an electron has an energy $E_{N,1}^x \leq E \leq E_{N,2}^x$ it is said to be in the first subband. Explain why the electrons in the first subband act like a two dimensional gas of free particles.

c) What is the maximum areal density of particles, n (particles per unit area) that can fit in the first subband?

d) If you knew the wave functions, explain how you would estimate how far from the surface $(x = 0)$ the two dimensional electron gas is and also estimate how "thick" the two dimensional layer is.

e) Sketch the density of states (number of particles per unit area per unit energy) for $0 < E < E_{N,4}^x$.

The problem set is continued on the back.

Problem 9.2 *Density of Electronic States: 1d, 2d, and 3d*

Problem 23.4 in the text. This problem is a good review of the density of states. You may use periodic boundary conditions.

Problem 9.3 *Applying Fermi's Golden Rule*

Do problem 24.2 in the text. This problem is almost the same (with the same algebra) that we worked out in class for the two atomic states interacting with the electromagnetic field and in Section 24.9. Just use Fermi's golden rule to calculate the the decay rate.

Note also, that you can use periodic boundary condition for the 3D Harmonic oscillator if you like.