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6.642 Continuum Electromechanics
Fall 2008

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Problem Set 1 - Solutions

Problem 1

a)

$$\nabla^2 \Phi = -\rho_f / \epsilon \Rightarrow \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = -\frac{\rho_0}{\epsilon} \sin(ax)$$

$$\Phi_p = \Phi_p(x) \Rightarrow \frac{d^2 \Phi_p}{dx^2} = -\frac{\rho_0}{\epsilon} \sin(ax) \Rightarrow \Phi_p = \frac{\rho_0}{a^2 \epsilon} \sin(ax)$$

b)

$$\Phi_T = \Phi_P + \Phi_h = +\frac{\rho_0 \sin(ax)}{a^2 \epsilon} + \sin(ax)[A \sinh(ay) + B \cosh(ay)]$$

$$\Phi_T(y=0) = 0 = \sin(ax) \left[\frac{\rho_0}{\epsilon a^2} + B \right] \Rightarrow B = -\frac{\rho_0}{\epsilon a^2}$$

$$\Phi_T(y=d) = 0 = \sin(ax) \left[\frac{\rho_0}{\epsilon a^2} + A \sinh(ad) + B \cosh(ad) \right]$$

$$A = \frac{-\frac{\rho_0}{\epsilon a^2} - B \cosh(ad)}{\sinh(ad)} = -\frac{\rho_0}{\epsilon a^2} \frac{1 - \cosh(ad)}{\sinh(ad)}$$

$$\Phi_T(x, y) = \frac{\rho_0}{\epsilon a^2} \sin(ax) \left[1 - \frac{1 - \cosh(ad)}{\sinh(ad)} \sinh(ay) - \cosh(ay) \right]$$

$$= \frac{\rho_0}{\epsilon a^2} \sin(ax) \left[1 + \frac{\sinh a(y-d) - \sinh ay}{\sinh ad} \right]$$

Check:

$$\Phi_T(x, y=0) = \frac{\rho_0}{\epsilon a^2} [1 - 1] \sin ax = 0$$

$$\Phi_T(x, y=d) = \frac{\rho_0}{\epsilon a^2} \sin ax [1 - 1] = 0$$

c)

$$\mathbf{E} = E_x \mathbf{i}_x + E_y \mathbf{i}_y$$

$$E_x = -\frac{\partial \Phi_T}{\partial x} = -\frac{\rho_0}{\epsilon a} \cos ax \left[1 + \frac{\sinh a(y-d) - \sinh ay}{\sinh ad} \right]$$

$$E_y = -\frac{\partial \Phi_T}{\partial y} = -\frac{\rho_0}{\epsilon a} \sin ax \left[\frac{\cosh a(y-d) - \cosh ay}{\sinh ad} \right]$$

$$\sigma_s(y=0) = \epsilon E_y(y=0) = -\frac{\rho_0 \sin ax (\cosh(ad) - 1)}{a \sinh(ad)}$$

$$\sigma_s(y=d) = -\epsilon E_y(y=d) = +\frac{\rho_0 \sin ax (1 - \cosh(ad))}{a \sinh(ad)}$$

d)

$$\begin{aligned} \frac{dy}{dx} &= \frac{E_y}{E_x} = \frac{\sin ax}{\cos ax} \cdot \frac{\cosh a(y-d) - \cosh ay}{\sinh ad + \sinh a(y-d) - \sinh ay} \\ \frac{\sinh ad + \sinh a(y-d) - \sinh ay}{\cosh a(y-d) - \cosh ay} dy &= \frac{\sin ax}{\cos ax} dx \\ \int \frac{\sin ax}{\cos ax} dx &= -\frac{\ln(\cos ax)}{a} \\ \int \frac{\sinh ad + \sinh a(y-d) - \sinh ay}{\cosh a(y-d) - \cosh ay} dy &= \frac{1}{a} \left\{ \left[2 \tanh^{-1} \frac{\cosh \left(\frac{a}{2}(d-y) \right)}{\cosh \frac{ay}{2}} \right] \cosh \left(\frac{ad}{2} \right) + \ln \left[-\sinh \frac{1}{2} a(d-2y) \right] \right\} \\ &+ \frac{\ln(\cos ax)}{a} + \frac{1}{a} \left\{ 2 \tanh^{-1} \left[\frac{\cosh \frac{a}{2}(d-y)}{\cosh \frac{ay}{2}} \right] \cosh \left(\frac{ad}{2} \right) + \ln \left[\sinh \frac{a}{2}(2y-d) \right] \right\} = C(x_0, y_0) \\ C(x_0, y_0) &= \frac{1}{a} \left[\ln(\cos ax_0) + 2 \tanh^{-1} \left[\frac{\cosh \frac{a}{2}(d-y_0)}{\cosh \frac{ay_0}{2}} \right] \cosh \left(\frac{ad}{2} \right) + \ln \left[\sinh \frac{a}{2}(2y_0-d) \right] \right] \end{aligned}$$

For plotting purposes, it is best to use non-dimensional variables:

$$\begin{aligned} \tilde{E}_x &= E_x \epsilon a / \rho_0, \quad \tilde{E}_y = E_y \epsilon a / \rho_0, \quad \tilde{x} = ax, \quad \tilde{y} = ay, \quad \tilde{d} = ad, \quad \tilde{C}(\tilde{x}_0, \tilde{y}_0) = aC(x_0, y_0) \\ \tilde{E}_x &= -\cos \tilde{x} [\sinh \tilde{d} + \sinh(\tilde{y} - \tilde{d}) - \sinh \tilde{y}] / \sinh \tilde{d} \\ \tilde{E}_y &= -\sin \tilde{x} [\cosh(\tilde{y} - \tilde{d}) - \cosh \tilde{y}] / \sinh \tilde{d} \end{aligned}$$

Field lines:

$$\begin{aligned} \ln(\cos \tilde{x}) + 2 \tanh^{-1} \left[\frac{\cosh \left(\frac{\tilde{d}-\tilde{y}}{2} \right)}{\cosh \left(\frac{\tilde{y}}{2} \right)} \right] \cosh \left(\frac{\tilde{d}}{2} \right) + \ln \left[\sinh \left(\tilde{y} - \frac{\tilde{d}}{2} \right) \right] \\ = \ln(\cos \tilde{x}_0) + 2 \tanh^{-1} \left[\frac{\cosh \left(\frac{\tilde{d}-\tilde{y}_0}{2} \right)}{\cosh \left(\frac{\tilde{y}_0}{2} \right)} \right] \cosh \left(\frac{\tilde{d}}{2} \right) + \ln \left[\sinh \left(\tilde{y}_0 - \frac{\tilde{d}}{2} \right) \right] \end{aligned}$$

e)

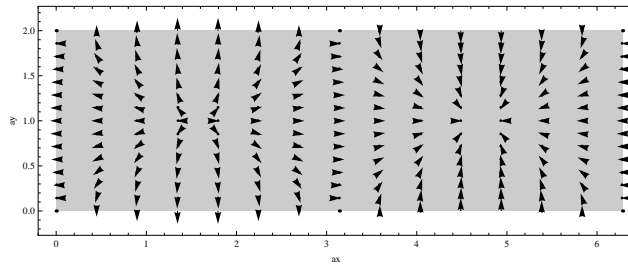


Figure 1: Electric field vector arrow plot.

$$\frac{d\tilde{y}}{d\tilde{x}} = \frac{\tilde{E}_y}{\tilde{E}_x} = \frac{\sin \tilde{x}}{\cos \tilde{x}} \left[\frac{\cosh(\tilde{y} - \tilde{d}) - \cosh \tilde{y}}{\sinh \tilde{d} + \sinh(\tilde{y} - \tilde{d}) - \sinh \tilde{y}} \right]$$

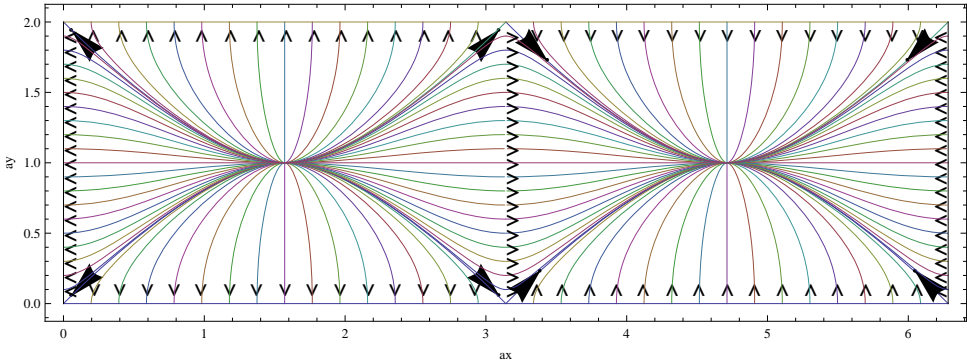


Figure 2: Electric field line plot.

Mathematica File for Computations Necessary to Generate Electric Field Vector Arrow and Electric Field Line Plots

$$f[y_] = \frac{\text{Sinh}[a * d] + \text{Sinh}[a * (y - d)] - \text{Sinh}[a * y]}{\text{Cosh}[a * (y - d)] - \text{Cosh}[a * y]}$$

Integrate[f[y], y]

$$2i \text{ArcTan} \left[\frac{(\text{Cosh}[a] + \text{Sinh}[a]) \left(\text{Cosh}[a] \text{Cosh} \left[\frac{ay}{2} \right] - \text{Sinh}[a] \text{Sinh} \left[\frac{ay}{2} \right] \right)}{i \text{Cosh}[a] \text{Cosh} \left[\frac{ay}{2} \right] + i \text{Cosh} \left[\frac{ay}{2} \right] \text{Sinh}[a]} \right] \text{Cosh}[a] + \frac{\text{Log}[-\text{Sinh}[a - ay]]}{a}$$

Simplify[%]

$$2 \text{ArcTanh} \left[\text{Cosh}[a] - \text{Sinh}[a] \text{Tanh} \left[\frac{ay}{2} \right] \right] \text{Cosh}[a] + \text{Log}[-\text{Sinh}[a - ay]]$$

$$g[x_] = \frac{\text{Sin}[a * x]}{\text{Cos}[a * x]}$$

Tan[ax]

Integrate[g[x], x]

$$-\text{Log}[\text{Cos}[ax]]$$

DSolve[y'[x] == Tan[a * x] * (Cosh[a * (y[x] - d)] - Cosh[a * y[x]]) / (Sinh[a * d] + Sinh[a * (y[x] - d)] - Sinh[a * y[x]]), y, x]

Solve::tdep : The equations appear to involve the variables to be solved for in an essentially non-algebraic way.))

$$\text{Solve} \left[-\frac{2i e^{2 \text{ArcTanh}[\text{Cosh}[a] - \text{Sinh}[a] \text{Tanh}[\frac{1}{2} a y[x]]] \text{Cosh}[a]} \text{Cos}[ax] (\text{Cosh}[a(-2 + y[x]) - \text{Cosh}[a y[x]])]}{a} == C[1], y[x] \right]$$

$$\text{ExT} = -\text{Cos}[xT] * (1 + (\text{Sinh}[yT - dT] - \text{Sinh}[yT]) / \text{Sinh}[dT])$$

$$-\text{Cos}[xT] (1 + \text{Csch}[2] (-\text{Sinh}[2 - yT] - \text{Sinh}[yT]))$$

$$\text{EyT} = -\text{Sin}[xT] * (\text{Cosh}[yT - dT] - \text{Cosh}[yT]) / \text{Sinh}[dT]$$

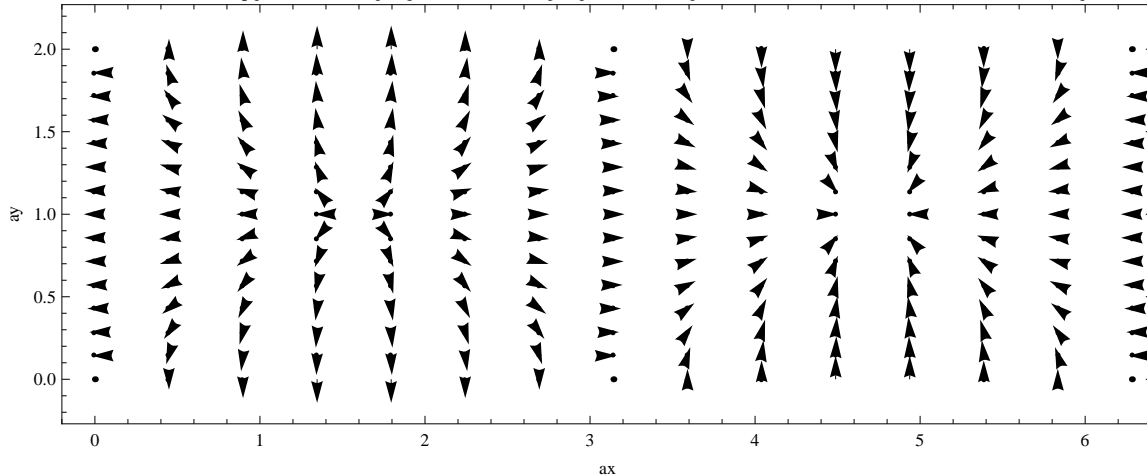
$$-(\text{Cosh}[2 - yT] - \text{Cosh}[yT]) \text{Csch}[2] \text{Sin}[xT]$$

$$dT = 2$$

2

LoadVectorFieldPlots

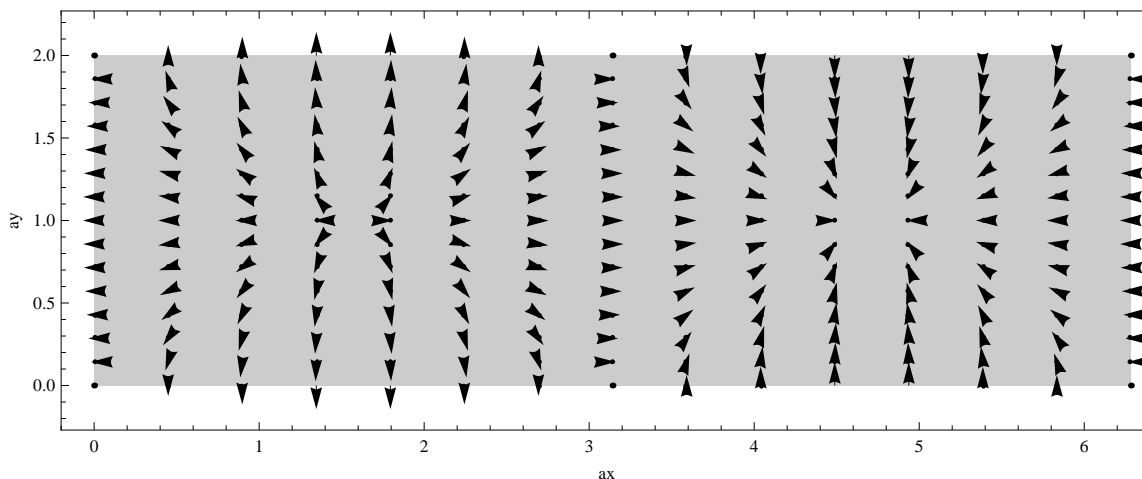
plot1 = PlotVectorField[{ExT, EyT}, {xT, 0, 2 * Pi}, {yT, 0, dT}, Frame -> True, FrameLabel -> {ax, ay}]



plot2 = {GrayLevel[.8], Rectangle[{0, 0}, {2 * Pi, dT}]}

{GrayLevel[0.8], Rectangle[{0, 0}, {2π, 2}]}

Show[Graphics[plot2], plot1, Frame -> True, FrameLabel -> {ax, ay}]

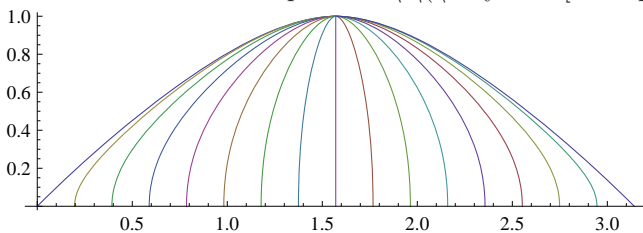


```
g[x0_, y0_, y_, d_] =
ArcCos[Exp[Log[Cos[x0]] + 2 * Cosh[d/2] * (ArcTanh[Cosh[(d - y0)/2]/Cosh[y0/2]] -
ArcTanh[Cosh[(d - y)/2]/Cosh[y/2]]) + Log[Sinh[d/2 - y0]/(Sinh[d/2 - y])]]
ArcCos [e2(-ArcTanh[Cosh[ $\frac{2-y}{2}$ ]/Sech[ $\frac{y}{2}$ ]]) + ArcTanh[Cosh[ $\frac{2-y0}{2}$ ]/Sech[ $\frac{y0}{2}$ ]]) Cosh[1] Cos[x0] Csch[1 - y] Sinh[1 - y0]]
g1 = Graphics[{Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {1.5875, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {1.385, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {1.78, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {1.195, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {1.98, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {.61, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {2.18, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {.99, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {2.57, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {.8, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {2.37, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {2.763, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {2.935, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {.415, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {.235, .1}, {0, 0}, {0, -1}]]
```

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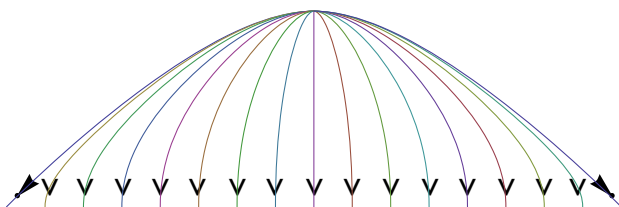
```
q1 = ParametricPlot[{{g[0, 0, y, 2], y}, {Graphics[{Text["X", {.5, .2}]}]}, {g[Pi/16, 0, y, 2], y}, {g[Pi/8, 0, y, 2], y},
{g[3 * Pi/16, 0, y, 2], y}, {g[Pi/4, 0, y, 2], y}, {g[5 * Pi/16, 0, y, 2], y}, {g[3 * Pi/8, 0, y, 2], y},
{g[7 * Pi/16, 0, y, 2], y}, {g[Pi/2, 0, y, 2], y}, {g[9 * Pi/16, 0, y, 2], y}, {g[5 * Pi/8, 0, y, 2], y},
{g[11 * Pi/16, 0, y, 2], y}, {g[3 * Pi/4, 0, y, 2], y}, {g[13 * Pi/16, 0, y, 2], y}, {g[7 * Pi/8, 0, y, 2], y},
{g[15 * Pi/16, 0, y, 2], y}, {g[Pi, 0, y, 2], y}}, {y, 0, 1}]
```

∞::indet : Indeterminate expression $\sqrt{\sqrt{\text{StyleBox}[0\text{ComplexInfinitySinh}[1], \text{"MT"}]}}$ encountered.)



```
a1 = {Graphics[Arrow[{{.06, .06}, {0.05, .05}}]], Graphics[Arrow[{{3.09, .06}, {3.10, .05}}]]}
```

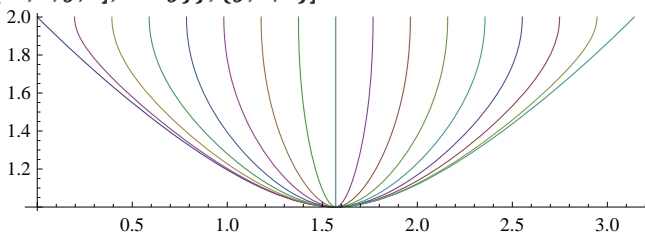
```
{,}
s1 = Show[g1, a1, q1]
```



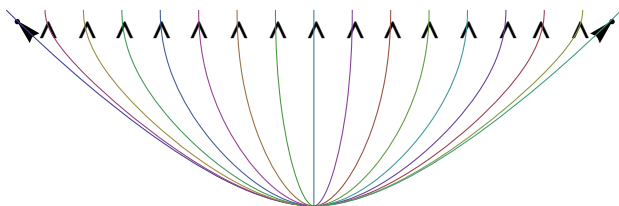
```
g2 = Graphics[{Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {1.57, 1.9}], {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {1.375, 1.9}], {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {1.77, 1.9}], {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {1.175, 1.9}], {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {1.96, 1.9}], {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {.59, 1.9}], {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {2.15, 1.9}], {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {.97, 1.9}], {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {2.55, 1.9}], {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {.78, 1.9}], {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {2.35, 1.9}], {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {2.72, 1.9}], {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {2.92, 1.9}], {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {.4, 1.9}], {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {.20, 1.9}], {0, 0}, {0, 1}]]
```

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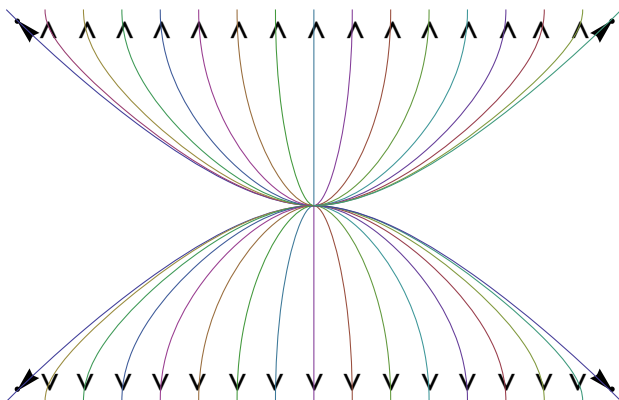
```
q2 = ParametricPlot[{{g[0, 0, y, 2], 2 - y}, {g[Pi/16, 0, y, 2], 2 - y}, {g[Pi/8, 0, y, 2], 2 - y}, {g[3 * Pi/16, 0, y, 2], 2 - y},
{g[Pi/4, 0, y, 2], 2 - y}, {g[5 * Pi/16, 0, y, 2], 2 - y}, {g[3 * Pi/8, 0, y, 2], 2 - y}, {g[7 * Pi/16, 0, y, 2], 2 - y},
{g[Pi/2, 0, y, 2], 2 - y}, {g[9 * Pi/16, 0, y, 2], 2 - y}, {g[5 * Pi/8, 0, y, 2], 2 - y}, {g[11 * Pi/16, 0, y, 2], 2 - y},
{g[3 * Pi/4, 0, y, 2], 2 - y}, {g[13 * Pi/16, 0, y, 2], 2 - y}, {g[7 * Pi/8, 0, y, 2], 2 - y}, {g[15 * Pi/16, 0, y, 2], 2 - y},
{g[Pi, 0, y, 2], 2 - y}}, {y, 0, 1}]
```



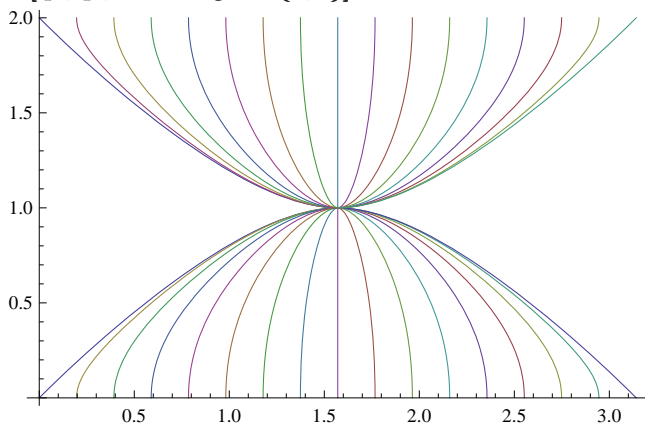
```
a2 = {Graphics[Arrow[{{.06, 1.94}, {.05, 1.95}}]], Graphics[Arrow[{{3.09, 1.94}, {3.10, 1.95}}]]]
{,}
s2 = Show[g2, a2, q2]
```



Show[s1, s2]



Show[q1, q2, PlotRange -> {0, 2}]

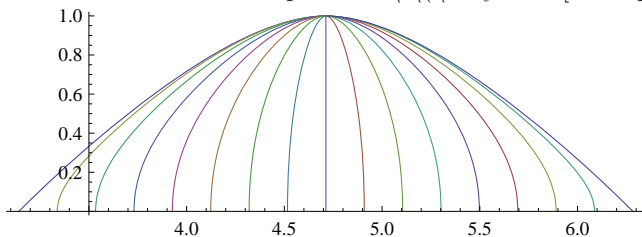


```
g3 = Graphics[{Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {1.5875 + Pi, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {1.385 + Pi, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {1.78 + Pi, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {1.195 + Pi, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {1.98 + Pi, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {.61 + Pi, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2.18 + Pi, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {.99 + Pi, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2.57 + Pi, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {.8 + Pi, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2.37 + Pi, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2.763 + Pi, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2.935 + Pi, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {.415 + Pi, .1}, {0, 0}, {0, -1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {.235 + Pi, .1}, {0, 0}, {0, -1}]]
```


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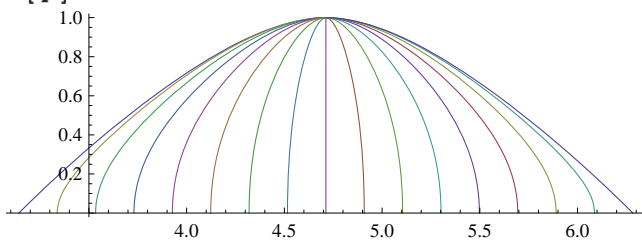
```
q3 = ParametricPlot[{{Pi + g[0, 0, y, 2], y}, {Graphics[{Text["X", {.5, .2}]}]}, {Pi + g[Pi/16, 0, y, 2], y},
{Pi + g[Pi/8, 0, y, 2], y}, {Pi + g[3 * Pi/16, 0, y, 2], y}, {Pi + g[Pi/4, 0, y, 2], y}, {Pi + g[5 * Pi/16, 0, y, 2], y},
{Pi + g[3 * Pi/8, 0, y, 2], y}, {Pi + g[7 * Pi/16, 0, y, 2], y}, {Pi + g[Pi/2, 0, y, 2], y}, {Pi + g[9 * Pi/16, 0, y, 2], y},
{Pi + g[5 * Pi/8, 0, y, 2], y}, {Pi + g[11 * Pi/16, 0, y, 2], y}, {Pi + g[3 * Pi/4, 0, y, 2], y}, {Pi + g[13 * Pi/16, 0, y, 2], y},
{Pi + g[7 * Pi/8, 0, y, 2], y}, {Pi + g[15 * Pi/16, 0, y, 2], y}, {Pi + g[Pi, 0, y, 2], y}}, {y, 0, 1}]
```

∞:indet : Indeterminate expression $\sqrt{\sqrt{\text{StyleBox}[0\text{ComplexInfinitySinh}[1], \text{"MT"}]}}$ encountered.)

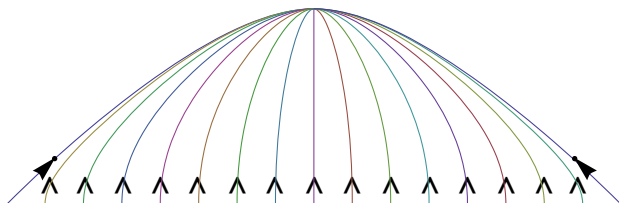


```
a3 = {Graphics[Arrow[{{.24 + Pi, .23}, {.25 + Pi, .24}]], Graphics[Arrow[{{2.91 + Pi, .23}, {2.90 + Pi, .24}]]]}
{, }
```

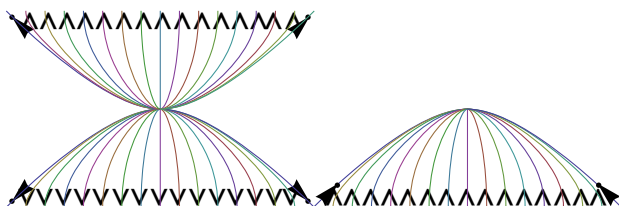
Show[q3]



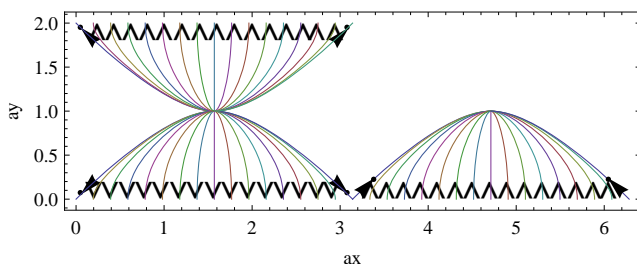
s3 = Show[g3, q3, a3]



Show[s1, s2, s3]



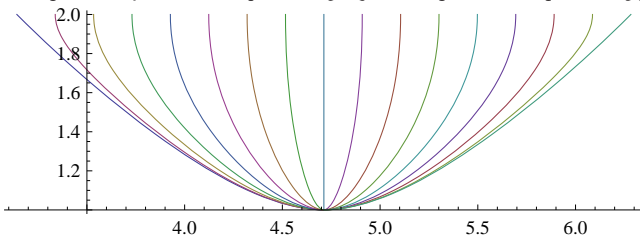
Show[%, Frame->True, FrameTicks -> True, FrameLabel -> {ax, ay}]



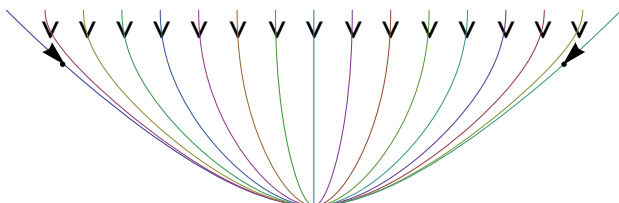
```
g4 = Graphics[{Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {1.56 + Pi, 2 - .1}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {1.37 + Pi, 2 - .1}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {1.76 + Pi, 2 - .1}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {1.17 + Pi, 2 - .1}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {1.95 + Pi, 2 - .1}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {.59 + Pi, 2 - .1}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2.15 + Pi, 2 - .1}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {.97 + Pi, 2 - .1}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2.54 + Pi, 2 - .1}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {.78 + Pi, 2 - .1}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2.34 + Pi, 2 - .1}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2.73 + Pi, 2 - .1}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2.915 + Pi, 2 - .1}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {.40 + Pi, 2 - .1}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {.21 + Pi, 2 - .1}, {0, 0}, {0, 1}]]
```

V V V V V V V V V V V V V V V

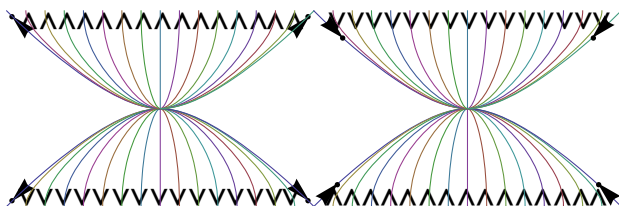
```
q4 = ParametricPlot[{{Pi + g[0, 0, y, 2], 2 - y}, {Pi + g[Pi/16, 0, y, 2], 2 - y}, {Pi + g[Pi/8, 0, y, 2], 2 - y},
{Pi + g[3 * Pi/16, 0, y, 2], 2 - y}, {Pi + g[Pi/4, 0, y, 2], 2 - y}, {Pi + g[5 * Pi/16, 0, y, 2], 2 - y},
{Pi + g[3 * Pi/8, 0, y, 2], 2 - y}, {Pi + g[7 * Pi/16, 0, y, 2], 2 - y}, {Pi + g[Pi/2, 0, y, 2], 2 - y},
{Pi + g[9 * Pi/16, 0, y, 2], 2 - y}, {Pi + g[5 * Pi/8, 0, y, 2], 2 - y}, {Pi + g[11 * Pi/16, 0, y, 2], 2 - y},
{Pi + g[3 * Pi/4, 0, y, 2], 2 - y}, {Pi + g[13 * Pi/16, 0, y, 2], 2 - y}, {Pi + g[7 * Pi/8, 0, y, 2], 2 - y},
{Pi + g[15 * Pi/16, 0, y, 2], 2 - y}, {Pi + g[Pi, 0, y, 2], 2 - y}}, {y, 0, 1}]
```



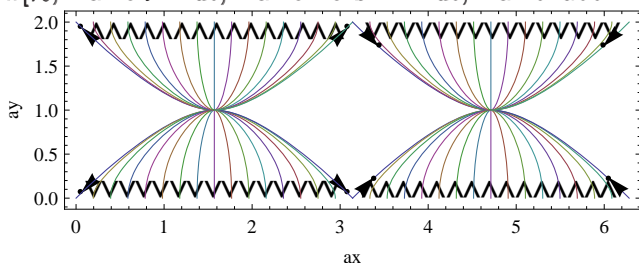
```
a4 = {Graphics[Arrow[{{.29 + Pi, 2 - .27}, {.30 + Pi, 2 - .28}}]],
Graphics[Arrow[{{2 * Pi - .29, 2 - .27}, {2 * Pi - .30, 2 - .28}}]]]
{, }
s4 = Show[g4, q4, a4]
```



Show[s1, s2, s3, s4]



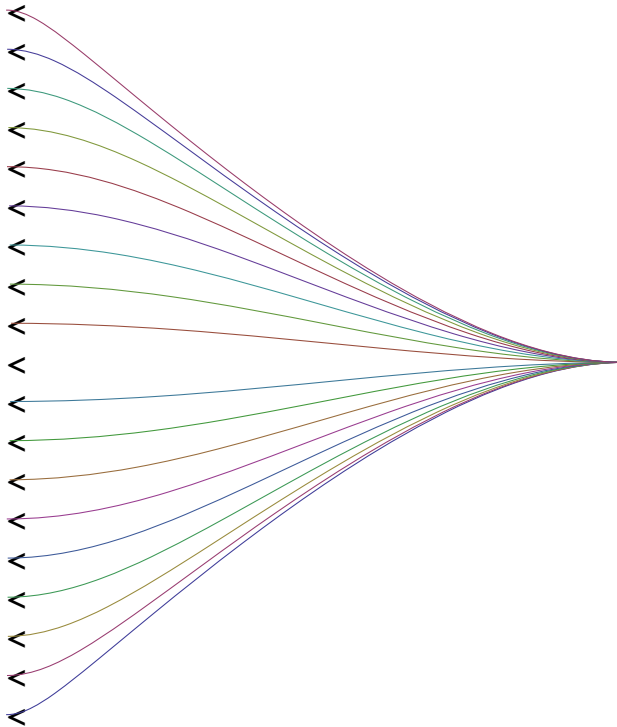
Show[%, Frame->True, FrameTicks -> True, FrameLabel -> {ax, ay}]



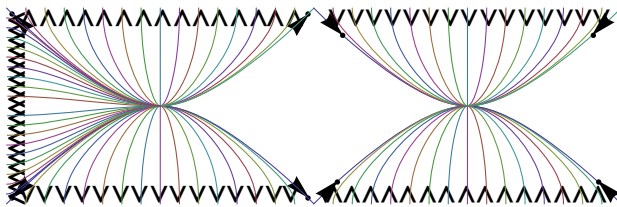
```

g5 = Graphics[{Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, .1}, {-1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, .2}, {-1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, .3}, {-1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, .4}, {-1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, .5}, {-1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, .6}, {-1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, .7}, {-1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, .8}, {-1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, .9}, {-1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, 1.}, {-1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, 1.1}, {-1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, 1.2}, {-1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, 1.3}, {-1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, 1.4}, {-1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, 1.5}, {-1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, 1.6}, {-1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, 1.7}, {-1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, 1.8}, {-1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {0, 1.9}, {-1, 0}]}]
    
```


s5 = Show[g5, q5]



Show[s1, s2, s3, s4, s5]

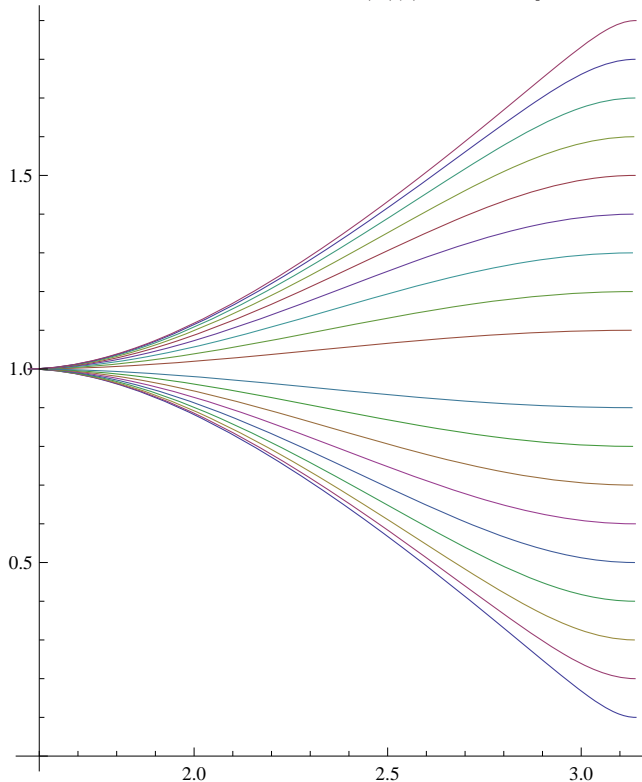


```
g6 = Graphics[{Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, .1}, {-1, 0}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, .2}, {-1, 0}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, .3}, {-1, 0}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, .4}, {-1, 0}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, .5}, {-1, 0}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, .6}, {-1, 0}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, .7}, {-1, 0}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, .8}, {-1, 0}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, .9}, {-1, 0}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, 1.}, {-1, 0}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, 1.1}, {-1, 0}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, 1.2}, {-1, 0}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, 1.3}, {-1, 0}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, 1.4}, {-1, 0}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, 1.5}, {-1, 0}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, 1.6}, {-1, 0}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, 1.7}, {-1, 0}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, 1.8}, {-1, 0}],
Text[StyleForm[" > ", FontSize -> 14, FontWeight -> "Bold"], {Pi, 1.9}, {-1, 0}]]
```



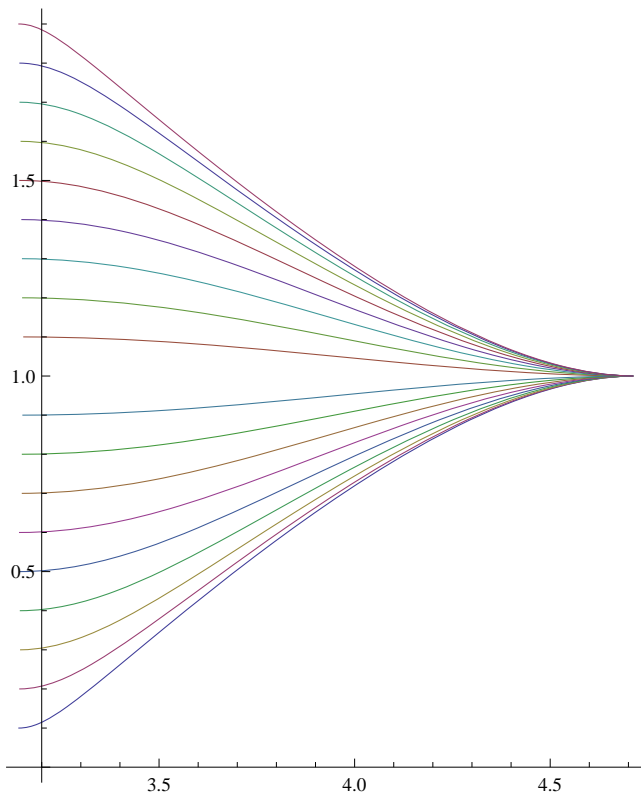
```
q6 = ParametricPlot[{{Pi - g[0, .1, y, 2], y}, {Pi - g[0, .2, y, 2], y}, {Pi - g[0, .3, y, 2], y}, {Pi - g[0, .4, y, 2], y},
{Pi - g[0, .5, y, 2], y}, {Pi - g[0, .6, y, 2], y}, {Pi - g[0, .7, y, 2], y}, {Pi - g[0, .8, y, 2], y}, {Pi - g[0, .9, y, 2], y},
{Pi - g[0, 1, y, 2], y}, {Pi - g[0, 1.1, y, 2], y}, {Pi - g[0, 1.2, y, 2], y}, {Pi - g[0, 1.3, y, 2], y}, {Pi - g[0, 1.4, y, 2], y},
{Pi - g[0, 1.5, y, 2], y}, {Pi - g[0, 1.6, y, 2], y}, {Pi - g[0, 1.7, y, 2], y}, {Pi - g[0, 1.8, y, 2], y}, {Pi - g[0, 1.9, y, 2], y}},
{y, .1, 1.9}]
```

∞::indet : Indeterminate expression ∞ encountered.)
 ∞::indet : Indeterminate expression ∞ encountered.)
 ∞::indet : Indeterminate expression ∞ encountered.)
 General::stop : Further output of ∞ will be suppressed during this calculation.)

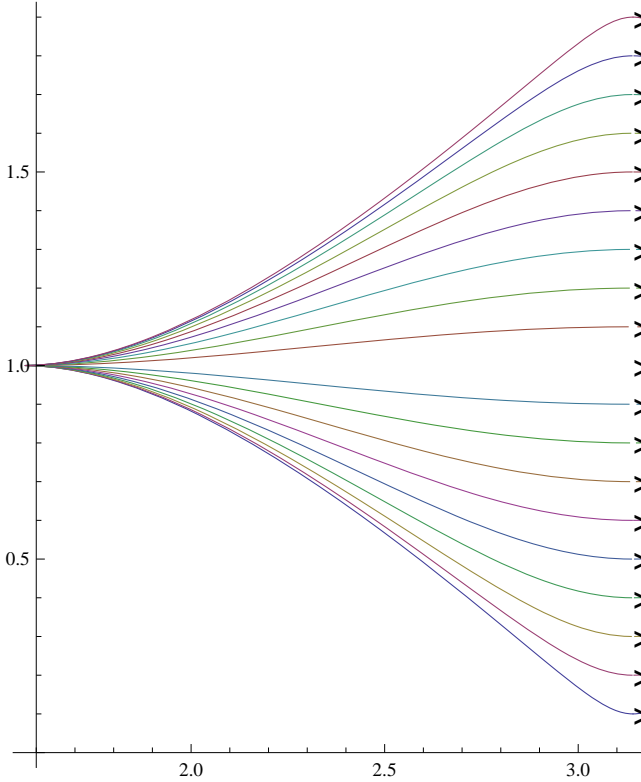


```
q7 = ParametricPlot[{{Pi + g[0, .1, y, 2], y}, {Pi + g[0, .2, y, 2], y}, {Pi + g[0, .3, y, 2], y}, {Pi + g[0, .4, y, 2], y},
{Pi + g[0, .5, y, 2], y}, {Pi + g[0, .6, y, 2], y}, {Pi + g[0, .7, y, 2], y}, {Pi + g[0, .8, y, 2], y}, {Pi + g[0, .9, y, 2], y},
{Pi + g[0, 1, y, 2], y}, {Pi + g[0, 1.1, y, 2], y}, {Pi + g[0, 1.2, y, 2], y}, {Pi + g[0, 1.3, y, 2], y}, {Pi + g[0, 1.4, y, 2], y},
{Pi + g[0, 1.5, y, 2], y}, {Pi + g[0, 1.6, y, 2], y}, {Pi + g[0, 1.7, y, 2], y}, {Pi + g[0, 1.8, y, 2], y}, {Pi + g[0, 1.9, y, 2], y}},
{y, .1, 1.9}]
```

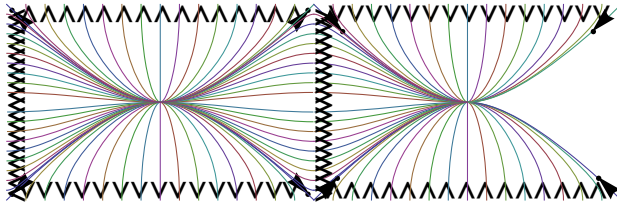
∞::indet : Indeterminate expression ∞ encountered.)
 ∞::indet : Indeterminate expression ∞ encountered.)
 ∞::indet : Indeterminate expression ∞ encountered.)
 General::stop : Further output of ∞ will be suppressed during this calculation.)



`s6 = Show[q6, q7, g6]`



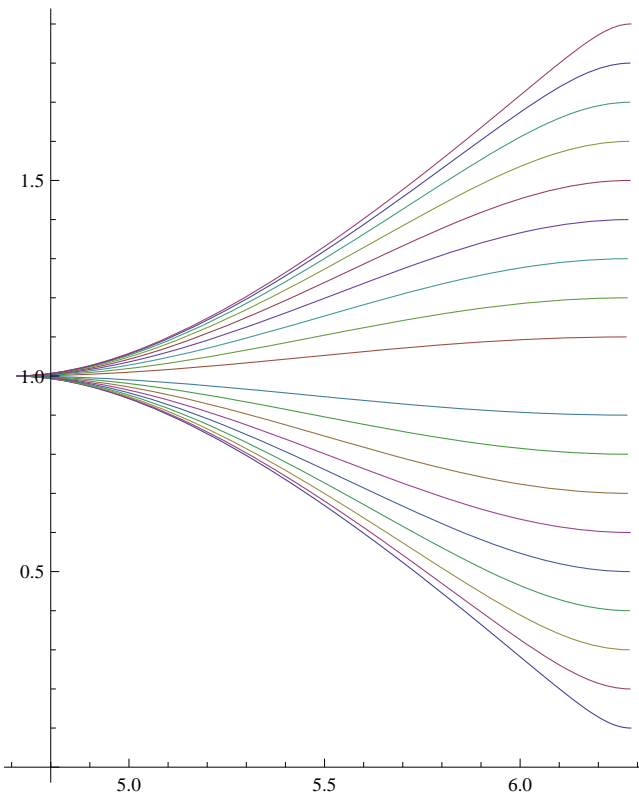
`Show[s1, s2, s3, s4, s5, s6]`



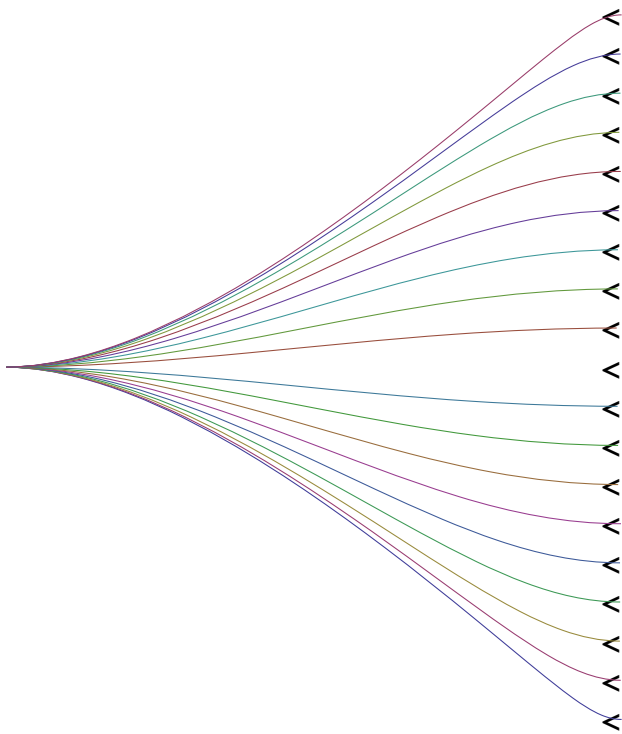
```
g8 = Graphics[{Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, .1}, {1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, .2}, {1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, .3}, {1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, .4}, {1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, .5}, {1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, .6}, {1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, .7}, {1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, .8}, {1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, .9}, {1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, 1.}, {1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, 1.1}, {1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, 1.2}, {1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, 1.3}, {1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, 1.4}, {1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, 1.5}, {1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, 1.6}, {1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, 1.7}, {1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, 1.8}, {1, 0}],
Text[StyleForm[" < ", FontSize -> 14, FontWeight -> "Bold"], {2 * Pi, 1.9}, {1, 0}]]
```



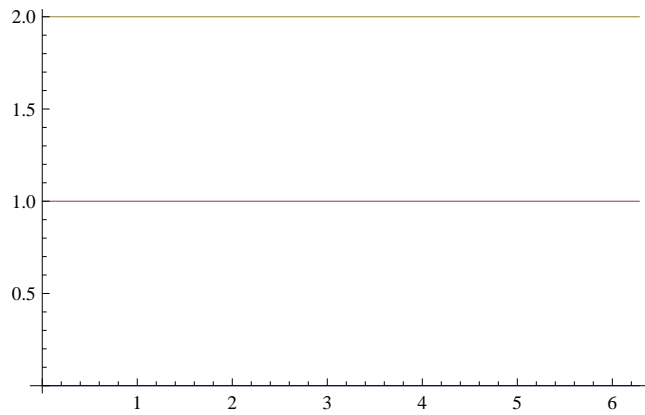
```
q8 = ParametricPlot[{{2 * Pi - g[0, .1, y, 2], y}, {2 * Pi - g[0, .2, y, 2], y}, {2 * Pi - g[0, .3, y, 2], y}, {2 * Pi - g[0, .4, y, 2], y},
{2 * Pi - g[0, .5, y, 2], y}, {2 * Pi - g[0, .6, y, 2], y}, {2 * Pi - g[0, .7, y, 2], y}, {2 * Pi - g[0, .8, y, 2], y},
{2 * Pi - g[0, .9, y, 2], y}, {2 * Pi - g[0, 1, y, 2], y}, {2 * Pi - g[0, 1.1, y, 2], y}, {2 * Pi - g[0, 1.2, y, 2], y},
{2 * Pi - g[0, 1.3, y, 2], y}, {2 * Pi - g[0, 1.4, y, 2], y}, {2 * Pi - g[0, 1.5, y, 2], y}, {2 * Pi - g[0, 1.6, y, 2], y},
{2 * Pi - g[0, 1.7, y, 2], y}, {2 * Pi - g[0, 1.8, y, 2], y}, {2 * Pi - g[0, 1.9, y, 2], y}}, {y, .1, 1.9}]
∞::indet : Indeterminate expression \!\(\*StyleBox[0∞, "MT"]\) encountered.)
∞::indet : Indeterminate expression \!\(\*StyleBox[0∞, "MT"]\) encountered.)
∞::indet : Indeterminate expression \!\(\*StyleBox[0∞, "MT"]\) encountered.)
General::stop : Further output of \!\(\*StyleBox[∞::indet, "MT"]\) will be suppressed during this calculation.)
```

`s8 = Show[g8, q8]`



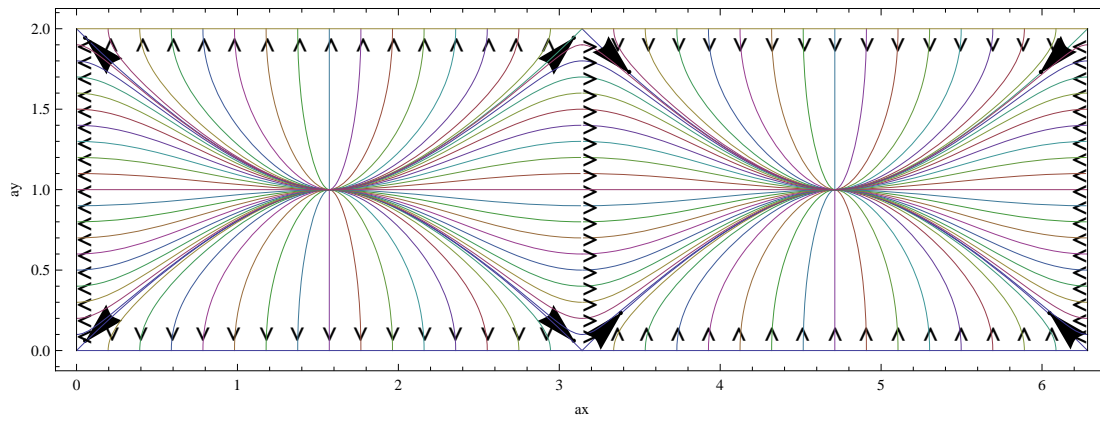
`s7 = Plot[{0, 1, 2}, {x, 0, 2 * Pi}]`



```
Line1 = Graphics[Line[{{0, 0}, {0, 2}}]]
```

```
Line2 = Graphics[Line[{{2 * Pi, 0}, {2 * Pi, 2}}]]
```

```
Show[s1, s2, s3, s4, s5, s6, s7, s8, Line1, Line2, Frame -> True, FrameLabel -> {ax, ay}]
```



Problem 2

a)

Because $u \rightarrow \infty$ for $y < 0$ and $y > d$, $\bar{H} = 0$.

b)

$$\begin{aligned} H_y(y = 0_+) &= 0, & H_y(y = d_-) &= 0 \\ H_x(y = 0_+) &= -K_z(y = 0), & H_x(y = d_-) &= +K_z(y = d) \end{aligned}$$

c)

$$\begin{aligned} \nabla^2 \bar{A} &= -\mu \bar{J} = -\mu J_0 \sin ax \bar{i}_z, & \bar{B} &= \mu \bar{H} = \nabla \times \bar{A} \\ \nabla^2 A_z &= \frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} = -\mu J_0 \sin ax \\ A_{zp} &= A_{zp}(x) \Rightarrow \frac{d^2 A_{zp}}{dx^2} = -\mu J_0 \sin ax \\ A_{zp}(x) &= \frac{\mu J_0}{a^2} \sin ax \\ A_z(x, y) &= A_{zp}(x) + A_{zh}(x, y) = \sin ax \left[\frac{\mu J_0}{a^2} + C \sinh ay + D \cosh ay \right] \\ H_y &= -\frac{1}{\mu} \frac{\partial A_z}{\partial x} = -\frac{a}{\mu} \cos ax \left[\frac{\mu J_0}{a^2} + C \sinh ay + D \cosh ay \right] \\ H_y(y = 0) &= 0 = -\frac{a}{\mu} \cos ax \left(\frac{\mu J_0}{a^2} + D \right) \Rightarrow D = -\frac{\mu J_0}{a^2} \\ H_y(y = d) &= 0 = -\frac{a}{\mu} \cos ax \left(\frac{\mu J_0}{a^2} + C \sinh ad + D \cosh ad \right) \\ C &= -\frac{\frac{\mu J_0}{a^2} + D \cosh ad}{\sinh ad} = -\frac{\mu J_0}{a^2} \cdot \frac{1 - \cosh ad}{\sinh ad} \\ H_y &= -\frac{J_0}{a} \left[1 - \frac{1 - \cosh ad}{\sinh ad} \sinh ay - \cosh ay \right] \cos ax \\ &= -\frac{J_0}{a} \cdot \frac{\cos ax}{\sinh ad} [\sinh ad - \sinh ay + \sinh a(y - d)] \\ H_x &= \frac{1}{\mu} \cdot \frac{\partial A_z}{\partial y} = \frac{a}{\mu} \sin ax [C \cosh ay + D \sinh ay] \\ &= -\frac{J_0}{a} \sin ax \left[\cosh ay \frac{1 - \cosh ad}{\sinh ad} + \sinh ay \right] \\ &= -\frac{J_0}{a} \cdot \frac{\sin ax}{\sinh ad} [\cosh ay - \cosh a(y - d)] \\ K_z(y = 0) &= -H_x(y = 0) = +\frac{J_0}{a} \cdot \frac{\sin ax}{\sinh ad} (1 - \cosh ad) \\ K_z(y = d) &= H_x(y = d) = -\frac{J_0}{a} \cdot \frac{\sin ax}{\sinh ad} (\cosh ad - 1) \end{aligned}$$

d)

Equation of field lines: $A_z(x, y) = A_z(x_0, y_0)$.

$$\begin{aligned}
 A_z(x, y) &= \sin ax \left(\frac{\mu J_0}{a^2} + C \sinh ay + D \cosh ay \right) \\
 &= \frac{\mu J_0}{a^2} \sin ax \left[1 - \frac{\sinh ay}{\sinh ad} (1 - \cosh ad) - \cosh ay \right] \\
 &= \frac{\mu J_0}{a^2} \sin ax \left[\frac{\sinh ad - \sinh ay + \sinh a(y - d)}{\sinh ad} \right] \\
 \tilde{x} &= ax, \quad \tilde{y} = ay, \quad \tilde{A}_z = \frac{A_z a^2}{\mu J_0} \\
 \tilde{x} &= \sin^{-1} \left[\frac{\sin \tilde{x}_0 [\sinh \tilde{d} - \sinh \tilde{y}_0 + \sinh(\tilde{y}_0 - \tilde{d})]}{\sinh \tilde{d} - \sinh \tilde{y} + \sinh(\tilde{y} - \tilde{d})} \right]
 \end{aligned}$$

e)

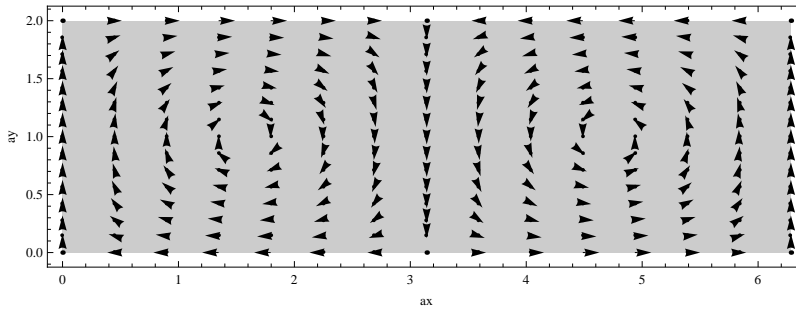


Figure 3: Magnetic field vector arrow plot.

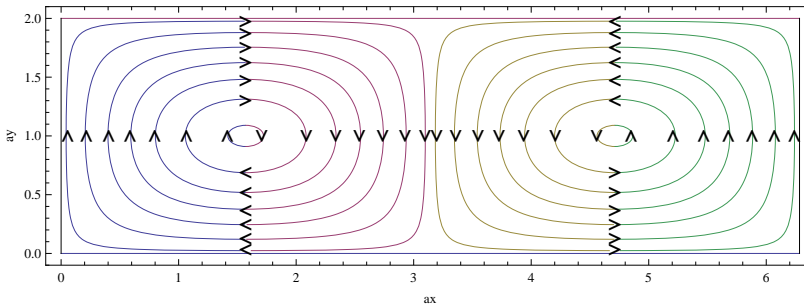


Figure 4: Magnetic field line plot.

f)

In Problem 1e, electric field lines terminate perpendicularly to the electrodes on surface charge. In Problem 2e, the magnetic field lines are parallel to the conducting walls resulting in surface currents.

Mathematica File for Computations Necessary to Generate Magnetic Field Vector Arrow and Magnetic Field Line Plots

$$f[x0_, y0_, y_, d_] = \text{ArcSin}[\text{Sin}[x0] * (\text{Sinh}[d] - \text{Sinh}[y0] + \text{Sinh}[y0 - d]) / (\text{Sinh}[d] - \text{Sinh}[y] + \text{Sinh}[y - d])]$$

$$\text{ArcSin} \left[\frac{\text{Sin}[x0](\text{Sinh}[d] - \text{Sinh}[d - y0] - \text{Sinh}[y0])}{\text{Sinh}[d] - \text{Sinh}[d - y] - \text{Sinh}[y]} \right]$$

$$\text{HxT} = (\text{Sin}[xT] / \text{Sinh}[dT]) * (\text{Cosh}[yT] - \text{Cosh}[yT - dT])$$

$$(-\text{Cosh}[dT - yT] + \text{Cosh}[yT]) \text{Csch}[dT] \text{Sin}[xT]$$

$$\text{HyT} = (\text{Cos}[xT] / \text{Sinh}[dT]) * (\text{Sinh}[dT] - \text{Sinh}[yT] + \text{Sinh}[yT - dT])$$

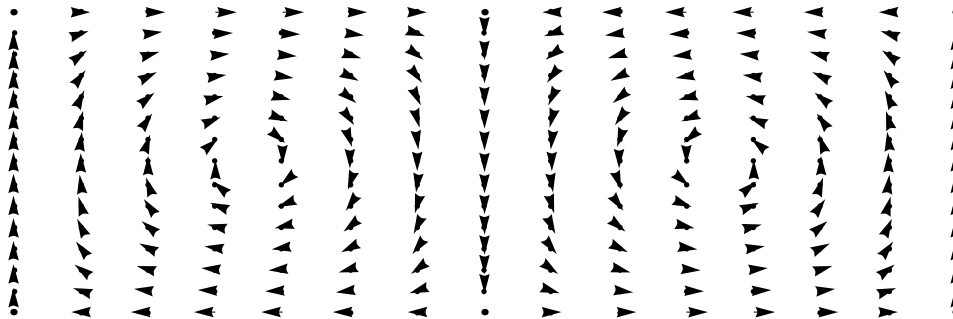
$$\text{Cos}[xT] \text{Csch}[dT] (\text{Sinh}[dT] - \text{Sinh}[dT - yT] - \text{Sinh}[yT])$$

LoadVectorFieldPlots`

dT = 2

2

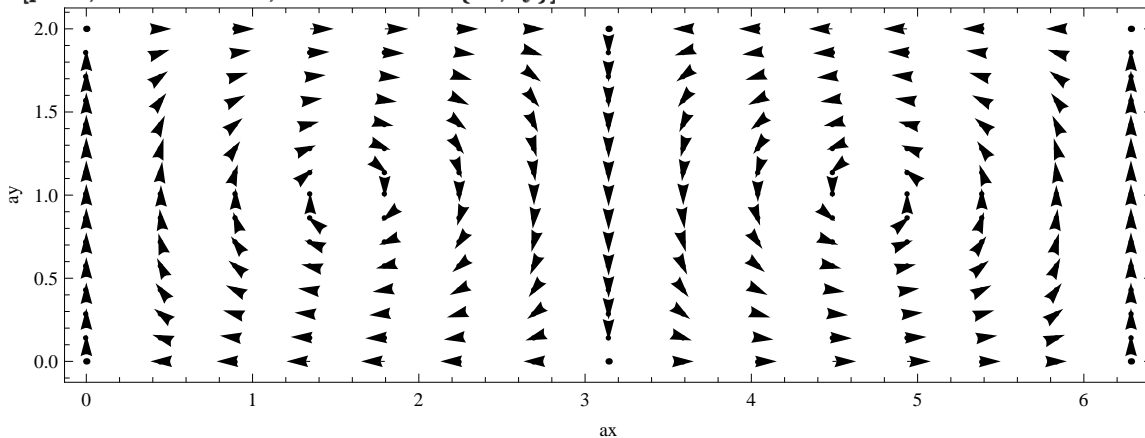
plot1 = PlotVectorField[{HxT, HyT}, {xT, 0, 2 * Pi}, {yT, 0, dT}]



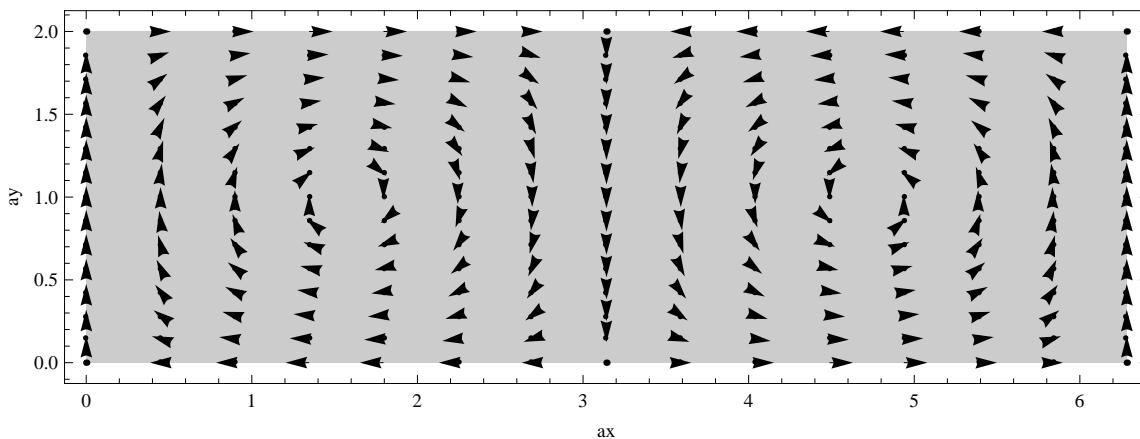
plot2 = {GrayLevel[.8], Rectangle[{0, 0}, {2 * Pi, dT}]}

{GrayLevel[0.8], Rectangle[{0, 0}, {2π, 2}]}

Show[plot1, Frame → True, FrameLabel → {ax, ay}]



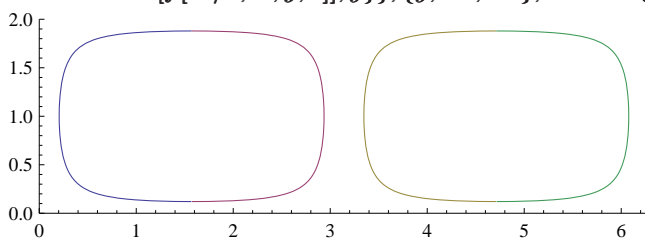
Show[Graphics[plot2], plot1, Frame → True, FrameLabel → {ax, ay}]



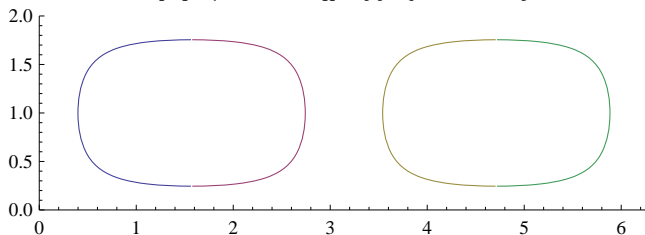
$d = 2$

2

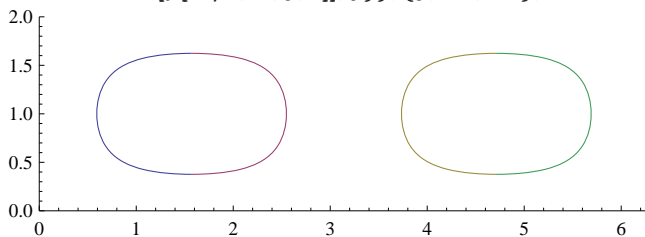
```
p1 = ParametricPlot[{{ArcSin[f[Pi/2, .1, y, 2]], y}, {Pi - ArcSin[f[Pi/2, .1, y, 2]], y}, {Pi + ArcSin[f[Pi/2, .1, y, 2]], y}, {2 * Pi - ArcSin[f[Pi/2, .1, y, 2]], y}}, {y, 0.1, 1.9}, PlotRange -> {{0, 2 * Pi}, {0, 2}}
```



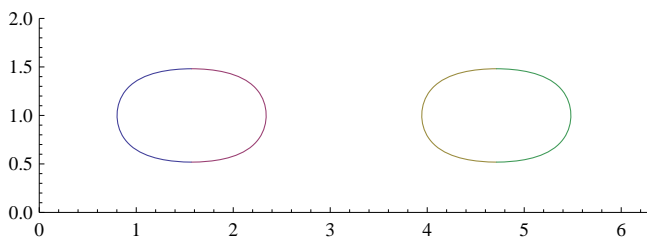
```
p2 = ParametricPlot[{{ArcSin[f[Pi/2, .2, y, 2]], y}, {Pi - ArcSin[f[Pi/2, .2, y, 2]], y}, {Pi + ArcSin[f[Pi/2, .2, y, 2]], y}, {2 * Pi - ArcSin[f[Pi/2, .2, y, 2]], y}}, {y, 0.2, 1.8}, PlotRange -> {{0, 2 * Pi}, {0, 2}}
```



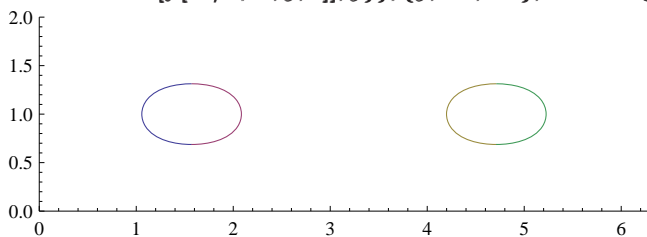
```
p3 = ParametricPlot[{{ArcSin[f[Pi/2, .3, y, 2]], y}, {Pi - ArcSin[f[Pi/2, .3, y, 2]], y}, {Pi + ArcSin[f[Pi/2, .3, y, 2]], y}, {2 * Pi - ArcSin[f[Pi/2, .3, y, 2]], y}}, {y, 0.3, 1.7}, PlotRange -> {{0, 2 * Pi}, {0, 2}}
```



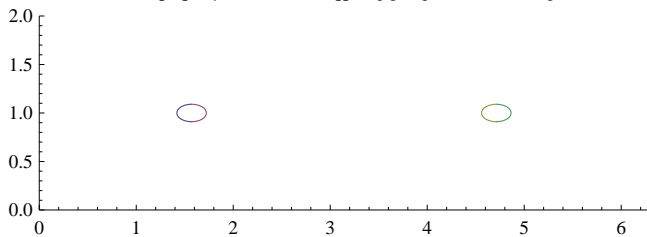
```
p4 = ParametricPlot[{{ArcSin[f[Pi/2, .4, y, 2]], y}, {Pi - ArcSin[f[Pi/2, .4, y, 2]], y}, {Pi + ArcSin[f[Pi/2, .4, y, 2]], y}, {2 * Pi - ArcSin[f[Pi/2, .4, y, 2]], y}}, {y, 0.4, 1.6}, PlotRange -> {{0, 2 * Pi}, {0, 2}}
```



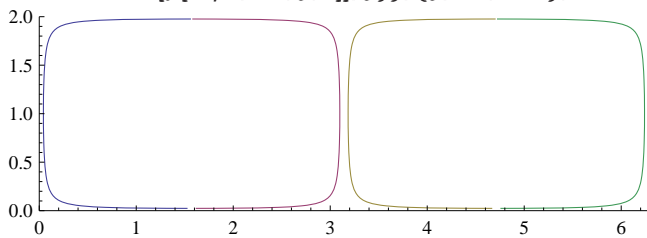
```
p5 = ParametricPlot[{{ArcSin[f[Pi/2, .5, y, 2]], y}, {Pi - ArcSin[f[Pi/2, .5, y, 2]], y}, {Pi + ArcSin[f[Pi/2, .5, y, 2]], y}, {2 * Pi - ArcSin[f[Pi/2, .5, y, 2]], y}}, {y, 0.5, 1.5}, PlotRange -> {{0, 2 * Pi}, {0, 2}}
```



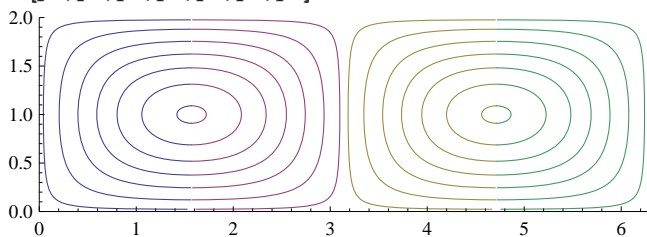
```
p6 = ParametricPlot[{{ArcSin[f[Pi/2, .58, y, 2]], y}, {Pi - ArcSin[f[Pi/2, .58, y, 2]], y}, {Pi + ArcSin[f[Pi/2, .58, y, 2]], y}, {2 * Pi - ArcSin[f[Pi/2, .58, y, 2]], y}}, {y, .58, 1.42}, PlotRange -> {{0, 2 * Pi}, {0, 2}}
```



```
p7 = ParametricPlot[{{ArcSin[f[Pi/2, .02, y, 2]], y}, {Pi - ArcSin[f[Pi/2, .02, y, 2]], y}, {Pi + ArcSin[f[Pi/2, .02, y, 2]], y}, {2 * Pi - ArcSin[f[Pi/2, .02, y, 2]], y}}, {y, .01, 1.99}, PlotRange -> {{0, 2 * Pi}, {0, 2}}
```



```
Show[p1, p2, p3, p4, p5, p6, p7]
```



```
g1 = Graphics[{{Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {Pi/2, 1.99}], Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {Pi/2, 1.89}], Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {Pi/2, 1.77}], Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {Pi/2, 1.64}], Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {Pi/2, 1.49}], Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {Pi/2, 1.32}]}]
```



```
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi/2, .7}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi/2, .53}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi/2, .38}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi/2, .26}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi/2, .14}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi/2, 0.05}]]]
```

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```
g2 = Graphics[{Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {3 * Pi/2, 1.99}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {3 * Pi/2, 1.89}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {3 * Pi/2, 1.77}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {3 * Pi/2, 1.64}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {3 * Pi/2, 1.49}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {3 * Pi/2, 1.32}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {3 * Pi/2, .7}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {3 * Pi/2, .53}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {3 * Pi/2, .38}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {3 * Pi/2, .26}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {3 * Pi/2, .14}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {3 * Pi/2, 0.05}]]]
```

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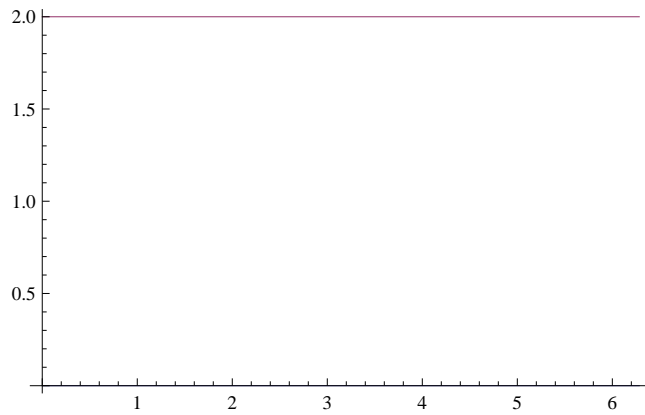
```
g3 = Graphics[{Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {.04, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {.2, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {.39, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {.57, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {.78, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {1.05, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {1.4, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi - 1.45, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi - 1.07, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi - .82, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi - .62, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi - .42, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi - .23, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi - .06, 1.}, {0, 0}, {0, 1}]]]
```

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```
g4 = Graphics[{Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi + .04, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi + .2, 1}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi + .39, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi + .57, 1}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi + .78, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi + 1.05, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" < ", FontSize->14, FontWeight->"Bold"], {Pi + 1.4, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {2 * Pi - 1.45, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {2 * Pi - 1.09, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {2 * Pi - .83, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {2 * Pi - .62, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {2 * Pi - .42, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {2 * Pi - .23, 1.}, {0, 0}, {0, 1}],
Text[StyleForm[" > ", FontSize->14, FontWeight->"Bold"], {2 * Pi - .06, 1.}, {0, 0}, {0, 1}]]
```

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```
p8 = Plot[{0, 2}, {x, 0, 2 * Pi}]
```



```
Line1 = Graphics[Line[{{0, 0}, {0, 2}}]]
```

```
Line2 = Graphics[Line[{{2 * Pi, 0}, {2 * Pi, 2}}]]
```

```
Show[p1, p2, p3, p4, p5, p6, p7, p8, g1, g2, g3, g4, Line1, Line2, PlotRange -> {- .1, 2.1},  
Frame -> True, FrameLabel -> {ax, ay}]
```

