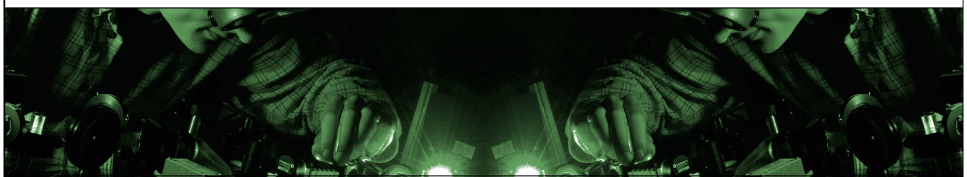
October 4, 2016



6.453 *Quantum Optical Communication*
Lecture 8

Jeffrey H. Shapiro

Optical and Quantum Communications Group

RESEARCH LABORATORY OF ELECTRONICS
Massachusetts Institute of Technology

6.453 *Quantum Optical Communication* — Lecture 8

- Announcements
  - Pick up lecture notes, slides
- Quantum Harmonic Oscillator
  - Positive operator-valued measurement (POVM) of $\hat{a}$
  - Reconciling POVMs and observables
- Single-Mode Photodetection
  - Direct Detection — semiclassical versus quantum
  - Homodyne Detection — semiclassical versus quantum

Measuring the $\hat{a}$ Operator: Definition

- Definition: Measurement of the $\hat{a}$ Operator
  - yields an outcome that is a complex number $\alpha = \alpha_1 + j\alpha_2$
  - joint probability density for getting this outcome is

$$p(\alpha) = \frac{|\langle \alpha|\psi\rangle|^2}{\pi}$$

- Consistency Checks:

$$p(\alpha) \geq 0$$

$$\int \mathrm{d}^2\alpha\, p(\alpha) = \langle\psi|\left(\int \frac{\mathrm{d}^2\alpha}{\pi}|\alpha\rangle\langle\alpha|\right)|\psi\rangle = 1$$

Measuring the $\hat{a}$ Operator: Characteristic Function

- Joint Characteristic Function for the $\hat{a}$ Measurement

$$M_{\alpha_1,\alpha_2}(jv_1, jv_2) \equiv \int \mathrm{d}^2\alpha\, e^{jv_1\alpha_1 + jv_2\alpha_2}\frac{|\langle\alpha|\psi\rangle|^2}{\pi}$$

$$= \chi_A(\zeta^*, \zeta)|_{\zeta = jv/2}$$

- Anti-Normally Ordered Characteristic Function of the State

$$\chi_A(\zeta^*, \zeta) \equiv \langle e^{-\zeta^*\hat{a}}e^{\zeta\hat{a}^\dagger}\rangle = \chi_W(\zeta^*, \zeta)e^{-|\zeta|^2/2}$$

Measuring the $\hat{a}$ Operator: Examples

- Number State $|n\rangle$:

$$p(\alpha) = \frac{|\alpha|^{2n}}{\pi n!} e^{-|\alpha|^2}$$

- Coherent State $|\beta\rangle$:

$$p(\alpha) = \frac{e^{-|\alpha-\beta|^2}}{\pi}$$

- Squeezed State $|\beta; \mu, \nu\rangle, \ \ \mu, \nu \text{ real}$:

$$p(\alpha) = \prod_{i=1}^{2} \frac{e^{-(\alpha_i - \langle \hat{a}_i \rangle)^2 / 2\sigma_i^2}}{\sqrt{2\pi\sigma_i^2}}$$

$$\langle \hat{a}_i \rangle = (\mu + (-1)^i \nu)\beta_i$$

$$\sigma_i^2 \equiv \frac{(\mu + (-1)^i \nu)^2 + 1}{4}$$

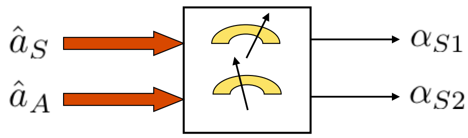
Measuring the $\hat{a}$ Operator: Summary

State	$\langle \alpha \rangle$
$\|n\rangle$	0
$\|\beta\rangle$	β
$\|\beta; \mu, \nu\rangle$	$\mu^* \beta - \nu \beta^*$

State	$\langle \Delta\alpha_1^2 \rangle$	$\langle \Delta\alpha_2^2 \rangle$
$\|n\rangle$	$(n+1)/2$	$(n+1)/2$
$\|\beta\rangle$	$1/2$	$1/2$
$\|\beta; \mu, \nu\rangle$	$(\|\mu - \nu\|^2 + 1)/4$	$(\|\mu + \nu\|^2 + 1)/4$

Reconciling POVMs with Observables for $p(\alpha)$

- Measure Two Commuting Observables on $\mathcal{H} \equiv \mathcal{H}_S \otimes \mathcal{H}_A$

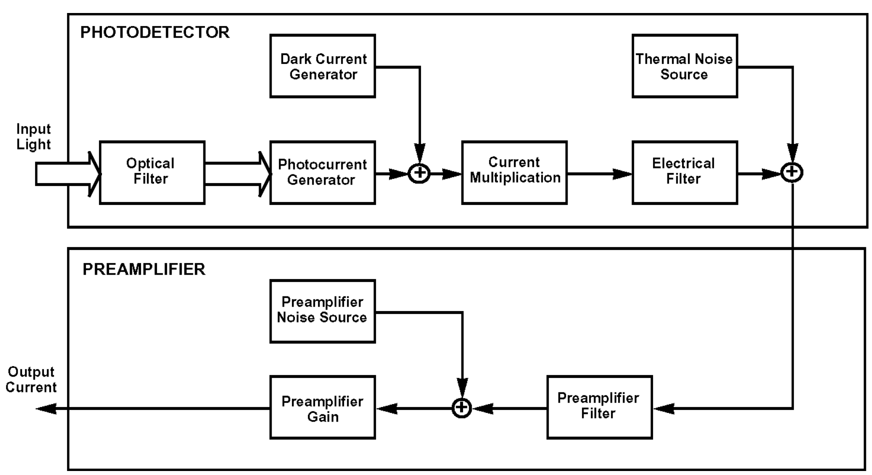


- Signal in State $|\psi\rangle_S$, Ancilla in Vacuum State $|0\rangle_A$

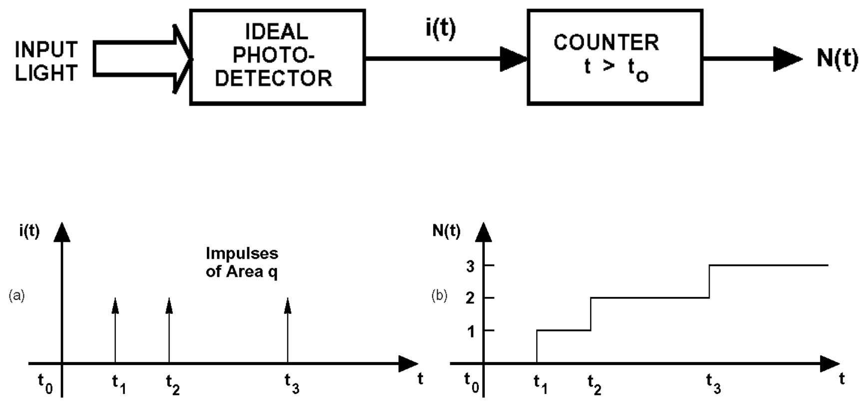
- Commuting Observables are Real and Imaginary Parts of:

$$\hat{a}_S + \hat{a}_A^\dagger \equiv \hat{a}_S \otimes \hat{I}_A + \hat{I}_S \otimes \hat{a}_A^\dagger$$

Real Photodetection Systems



Ideal Photodetection System



Single-Mode Quantized Electromagnetic Field

- Photon-Units Field Operator on Constant-z Plane:

$$\hat{E}_z(x,y,t) = \underbrace{\frac{\hat{a}e^{-j\omega t}}{\sqrt{AT}}}_{\text{excited mode}} + \underbrace{\text{other terms}}_{\text{unexcited modes}}$$

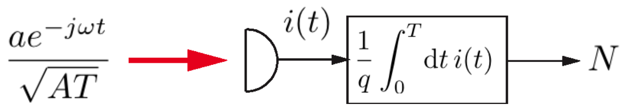
for $(x,y) \in \mathcal{A}, 0 \leq t \leq T$

- Photon Annihilation and Creation Operators: $\hat{a}, \hat{a}^\dagger$

with canonical commutation relation $\left[\hat{a}, \hat{a}^\dagger\right] = 1$

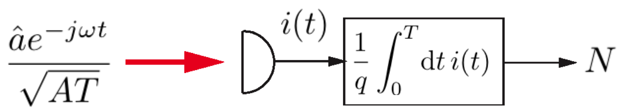
Direct Detection: Semiclassical versus Quantum

- Single-Mode Photon Counter: Semiclassical Description



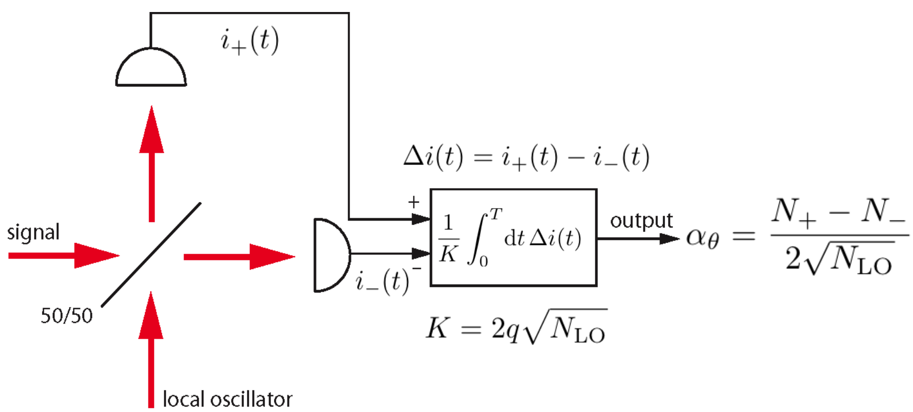
$$\Pr(\, N = n \mid a = \alpha\,) = \frac{|\alpha|^{2n}}{n!}e^{-|\alpha|^2}$$

- Single-Mode Photon Counter: Quantum Description



$$\Pr(\, N = n \mid \text{state} = |\psi\rangle\,) = |\langle n|\psi\rangle|^2$$

Single-Mode Balanced Homodyne Receiver



- Semiclassical Description: $\alpha_\theta \sim N(\mathrm{Re}(ae^{-j\theta}), 1/4)$
- Quantum Description: $\alpha_\theta \longleftrightarrow \hat{a}_\theta \equiv \mathrm{Re}(\hat{a}e^{-j\theta})$

Coming Attractions: Lectures 9 and 10

- Lecture 9:
  Single-Mode Photodetection
  - Heterodyne Detection — semiclassical versus quantum
  - Realizing the $\hat{a}$ measurement
- Lecture 10:
  Single-Mode Photodetection
  - Signatures of non-classical light
  - Squeezed-state waveguide tap