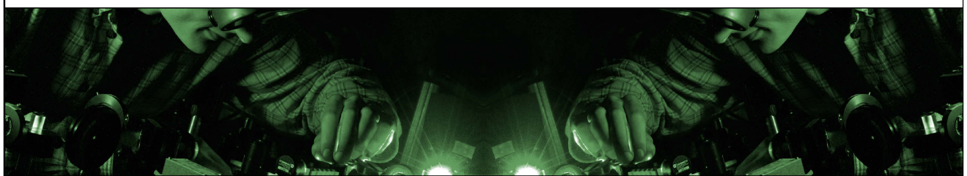

November 3, 2016

6.453 *Quantum Optical Communication* Lecture 16

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6.453 *Quantum Optical Communication* - Lecture 16

- Announcements
  - Turn in problem set 8
  - Pick up problem set 8 solutions, lecture notes, slides, old mid-terms and their solutions
- Quantum Cryptography
  - One-time pad cryptography
  - Bennett-Brassard protocol quantum key distribution
  - Clauser-Horne-Shimony-Holt form of Bell's inequality
  - Ekert protocol quantum key distribution

Perfectly Secure Digital Communication: The One-Time Pad

- Alice has a plaintext message to send to Bob securely
- She sends ciphertext = plaintext $\oplus$ random binary key

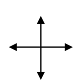
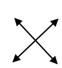
  …1101000…$\oplus$…0100101… = …1001101…
- Ciphertext is a completely random binary string

  impossible to recover plaintext from ciphertext without the key
- Bob decodes ciphertext $\oplus$ *same* binary key = Alice's plaintext

  …1001101…$\oplus$…0100101… = …1101000…
- Security relies on single use of the secret key
- Decoding relies on Alice and Bob having the *same* key

Quantum Key Distribution (QKD): Bennett-Brassard (BB84) Protocol

- Underlying Principle: the state of an unknown qubit cannot be determined… so eavesdropping on an unknown qubit is detectable
- Alice and Bob randomly choose photon-polarization bases

  horizontal/vertical  or  +45/-45 diagonal

  for transmission (Alice) and reception (Bob)
- Alice codes a random bit into her polarization choice
- When Alice and Bob use the same basis…
  - their measurements provide a shared random key
  - eavesdropping (by Eve) can be detected through errors she creates

Quantum Key Distribution (QKD): Bennett-Brassard (BB84) Protocol

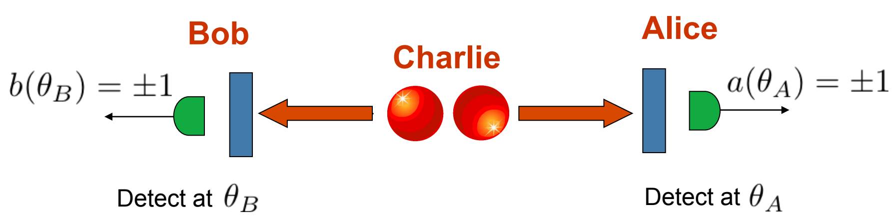
- BB84 Obviously Secure for:
  - Single-photon sources
  - Lossless propagation
  - Ideal photon counters
- BB84 Systems to Date Use:
  - Weak coherent state sources
  - Lossy and noisy propagation media
  - Geiger-mode avalanche photodiode detectors
- BB84 Systems Must Therefore Perform:
  - Sifting
  - Error detection and correction
  - Privacy amplification

Clauser-Horne-Shimony-Holt Inequality: Setup

- Charlie Produces Polarization-Entangled Photon Pair:

$$|\psi^-\rangle = (|H\rangle|V\rangle - |V\rangle|H\rangle)/\sqrt{2}$$

- Alice and Bob Do Polarization Analysis:



+1 if photon is detected; -1 if no photon is detected

- Measurements Repeated and Averaged

CHSH Inequality: Local Hidden Variable Theory

- Perform Repeated Measurements to Determine:

$$C(\theta_A, \theta_B) = \langle a(\theta_A) b(\theta_B) \rangle$$

for $\theta_A = 0, -\pi/4 \quad \text{and} \quad \theta_B = 3\pi/8, \pi/8$

- If Polarizations Determined by Local Hidden Variable μ :

$$\begin{aligned} S &\equiv |C(0, 3\pi/8) + C(-\pi/4, 3\pi/8) + C(-\pi/4, \pi/8) - C(0, \pi/8)| \\ &= |\int \mathrm{d}\mu \, \{\underbrace{[a(0,\mu) + a(-\pi/4,\mu)]}_{\text{must } = \pm 2 \text{ or } 0} b(3\pi/8, \mu) \\ &\quad + \underbrace{[a(-\pi/4,\mu) - a(0,\mu)]}_{\text{must } = 0 \text{ or } \pm 2} b(\pi/8,\mu)\} p(\mu)| \le 2 \end{aligned}$$

CHSH Inequality: Quantum Mechanics

- Polarization Bases for $k = A, B$:

$$\begin{aligned} |\mathbf{i}_k\rangle_k &\equiv \cos(\theta_k)|H\rangle_k + \sin(\theta_k)|V\rangle_k \\ |\mathbf{i}'_k\rangle_k &\equiv \sin(\theta_k)|H\rangle_k - \cos(\theta_k)|V\rangle_k \end{aligned}$$

- Quantum Measurement Theory for $C(\theta_A, \theta_B)$:

$$\begin{aligned} &C(\theta_A, \theta_B) \\ &= |\langle\psi^-|(|\mathbf{i}_A\rangle_A \otimes |\mathbf{i}_B\rangle_B)|^2 + |\langle\psi^-|(|\mathbf{i}'_A\rangle_A \otimes |\mathbf{i}'_B\rangle_B)|^2 \\ &- |\langle\psi^-|(|\mathbf{i}'_A\rangle_A \otimes |\mathbf{i}_B\rangle_B)|^2 - |\langle\psi^-|(|\mathbf{i}_A\rangle_A \otimes |\mathbf{i}'_B\rangle_B)|^2 \\ &= -\cos[2(\theta_A - \theta_B)] \end{aligned}$$

CHSH Inequality: Quantum Mechanics

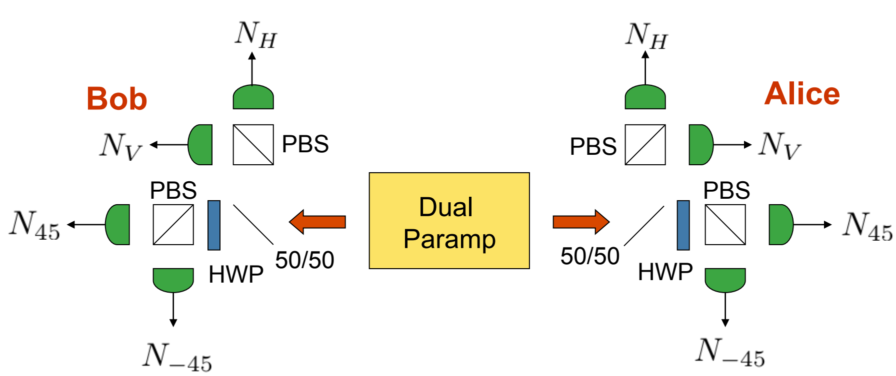
- Quantum Mechanics Can Violate Local Hidden Variables

$$
\begin{aligned}
S &\equiv |C(0,3\pi/8)+C(-\pi/4,3\pi/8)+C(-\pi/4,\pi/8)-C(0,\pi/8)| \\
&= |\cos(3\pi/4)+\cos(5\pi/4)+\cos(3\pi/4)-\cos(\pi/4)| \\
&= \left|-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right| = 2\sqrt{2} > 2
\end{aligned}
$$

- Experiments with Bi-Photon Sources Show $S > 2$

Ekert Protocol Quantum Key Distribution

- Passive Random Selection of Polarization Basis



- Alice + Bob Check $S \approx 2\sqrt{2}$ to Detect Eavesdropping
- Alice + Bob Generate Shared Random Key as in BB84

Coming Attractions: Mid-Term Exam + Lecture 17

- Mid-Term Exam:
  Tuesday, November 8
  - Closed book
  - One 8 1/2 x 11 handwritten formula sheet is permitted

- Lecture 17:
  Quantization of the Electromagnetic Field
  - Maxwell's equations
  - Plane-wave mode expansions
  - Multi-mode number states and coherent states