November 1, 2016

6.453 *Quantum Optical Communication*
Lecture 15

Jeffrey H. Shapiro

Optical and Quantum Communications Group
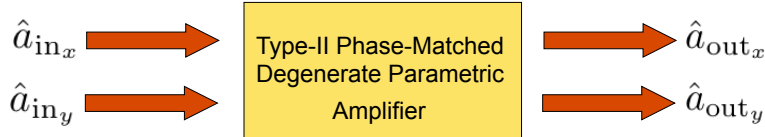
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6.453 *Quantum Optical Communication* - Lecture 15

- Announcements
  - Pick up lecture notes, slides

- Teleportation
  - Fidelity analysis of continuous-variable teleportation

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Quadrature Entanglement: Vacuum-Input Paramp

$\hat{a}_{\text{in}_x}$ → [Type-II Phase-Matched Degenerate Parametric Amplifier] → $\hat{a}_{\text{out}_x}$
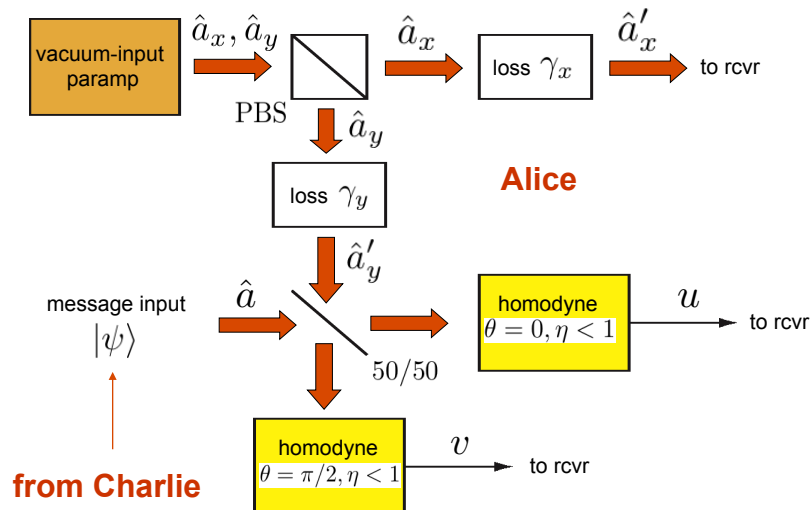
$\hat{a}_{\text{in}_y}$ → → $\hat{a}_{\text{out}_y}$

- Quadrature Variances:

$$\langle \Delta \hat{a}^2_{\text{out}_{x_k}} \rangle = \langle \Delta \hat{a}^2_{\text{out}_{y_k}} \rangle = \frac{2G-1}{4} > \frac{1}{4}, \text{ for } k = 1, 2$$
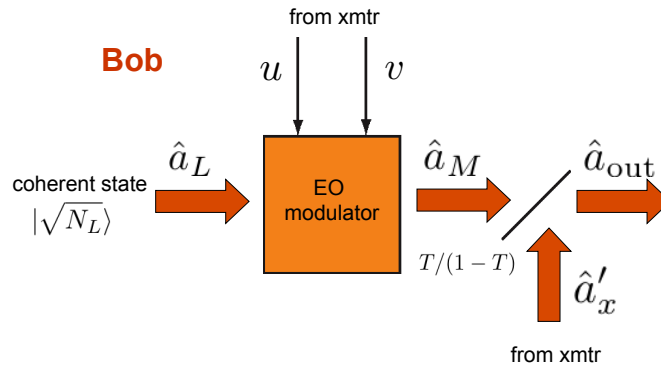
- Quadrature-Difference Variances:

$$\left\langle \left(\frac{\Delta \hat{a}_{\text{out}_{x_1}} - \Delta \hat{a}_{\text{out}_{y_1}}}{\sqrt{2}} \right)^2 \right\rangle = \left\langle \left(\frac{\Delta \hat{a}_{\text{out}_{x_2}} + \Delta \hat{a}_{\text{out}_{y_2}}}{\sqrt{2}} \right)^2 \right\rangle$$

$$= \frac{(\sqrt{G} - \sqrt{G-1})^2}{4} \approx \frac{1}{16G} \ll \frac{1}{4}, \text{ for } G \gg 1$$

Continuous-Variable Teleportation: the Xmtr

[vacuum-input paramp] → $\hat{a}_x, \hat{a}_y$ → [PBS] → $\hat{a}_x$ → [loss γ_x] → $\hat{a}'_x$ → to rcvr

$\hat{a}_y$ ↓

[loss γ_y]

$\hat{a}'_y$ ↓

Alice

message input
$|\psi\rangle$

$\hat{a}$ → [50/50] → [homodyne $\theta = 0, \eta < 1$] → u → to rcvr

from Charlie

[homodyne $\theta = \pi/2, \eta < 1$] → v → to rcvr

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Continuous-Variable Teleportation: the Rcvr

from xmtr

Bob u v

coherent state
$|\sqrt{N_L}\rangle$

$\hat{a}_L$ → EO modulator → $\hat{a}_M$ → $\hat{a}_{\text{out}}$ →

$T/(1-T)$

$\hat{a}'_x$

from xmtr

Continuous-Variable Teleportation: Xmtr Details

- Inputs to the 50/50 Beam Splitter:

$$\hat{a} \text{ and } \hat{a}'_y = \sqrt{\gamma_y}\,\hat{a}_y + \sqrt{1-\gamma_y}\,\hat{a}_{\gamma_y}$$

where $\hat{a}$ is in arbitrary state, $\hat{a}_{\gamma_y}$ is in vacuum state

- Homodyne Detector Measurement Outcomes:

$$u \quad \leftrightarrow \quad \hat{u} \equiv \sqrt{\eta}\,(\hat{a}_1 + \hat{a}'_{y_1}) + \sqrt{2(1-\eta)}\,\hat{a}_{u_1}$$

$$v \quad \leftrightarrow \quad \hat{v} \equiv \sqrt{\eta}\,(\hat{a}_2 - \hat{a}'_{y_2}) + \sqrt{2(1-\eta)}\,\hat{a}_{v_2}$$

where $\hat{a}_u$ and $\hat{a}_v$ are in vacuum states

Continuous-Variable Teleportation: Xmtr Details

- Homodyne-Measurement Signal-to-Noise Ratios

$$\text{SNR}_u \equiv \frac{\eta\langle\hat{a}_1^2\rangle}{\eta\langle\hat{a}_{y_1}'^2\rangle + 2(1-\eta)\langle\hat{a}_{u_1}^2\rangle}$$

$$= \frac{4\eta\langle\hat{a}_1^2\rangle}{\eta[1 + 2\gamma_y(G-1)] + 2(1-\eta)}$$

$$\text{SNR}_v \equiv \frac{\eta\langle\hat{a}_2^2\rangle}{\eta\langle\hat{a}_{y_2}'^2\rangle + 2(1-\eta)\langle\hat{a}_{v_2}^2\rangle}$$

$$= \frac{4\eta\langle\hat{a}_2^2\rangle}{\eta[1 + 2\gamma_y(G-1)] + 2(1-\eta)}$$

Classical Information as Quantum Information

- Alice's Classical Transmission: $u + jv$

- Quantum-Mechanical Equivalent:

$$\hat{u} + j\hat{v} = \sqrt{\eta}\,(\hat{a} + \hat{a}_y'^\dagger) + \sqrt{2(1-\eta)}\,(\hat{a}_{u_1} + j\hat{a}_{v_2})$$

$$\leftrightarrow \sqrt{\eta}\,(\hat{a} + \hat{a}_y'^\dagger) + \sqrt{1-\eta}\,(\hat{a}_{\eta_u} + \hat{a}_{\eta_v}^\dagger)$$

where $\hat{a}_{\eta_u}$ and $\hat{a}_{\eta_v}$ are in vacuum states

Continuous-Variable Teleportation: Rcvr Details

- Input to the EO Modulator: Strong coherent state $|\sqrt{N_L}\rangle$

- Output from the EO Modulator: Coherent state $|K(u+jv)\rangle$

- Asymmetric Beam Splitter Input-Output Relation:

$$
\begin{aligned}
\hat{a}_{\text{out}} &= \sqrt{T}\,\hat{a}_M - \sqrt{1-T}\,\hat{a}_x' \\
&= \sqrt{T}\,K(u+jv) + \sqrt{T}\,\Delta\hat{a}_M \\
&\quad - \sqrt{(1-T)\gamma_x}\,\hat{a}_x - \sqrt{(1-T)(1-\gamma_x)}\,\hat{a}_{\gamma_x}
\end{aligned}
$$

where $\Delta\hat{a}_M$ and $\hat{a}_{\gamma_x}$ are in vacuum states

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Continuous-Variable Teleportation: Rcvr Details

- Parameter Choices:

$$
K\sqrt{\eta T} = 1 \text{ and } \gamma = \gamma_y = (1-T)\gamma_x
$$

- Quantum Input-Output Teleportation Relation:

$$
\begin{aligned}
\hat{a}_{\text{out}} &= \hat{a} + \sqrt{\frac{1-\eta}{\eta}}\,(\hat{a}_{\eta_u} + \hat{a}_{\eta_v}^\dagger) \\
&\quad + \sqrt{1-\gamma}\,(\hat{a}_N + \hat{a}_{\gamma_y}^\dagger) - \sqrt{\gamma}(\hat{a}_x - \hat{a}_y^\dagger)
\end{aligned}
$$

where $\hat{a}_N$ is in the vacuum state

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Continuous-Variable Teleportation: Output State

- Wigner Characteristic Function of Output State:

$$\chi_W^{\rho_{\text{out}}}(\zeta^*, \zeta) = \chi_W^{\rho}(\zeta^*, \zeta)$$

$$\times \quad \exp\left\{-\left[\left(\frac{1-\eta}{\eta}\right) + (1-\gamma) + \gamma s\right]|\zeta|^2\right\}$$

where $s \equiv (\sqrt{G} - \sqrt{G-1})^2$ is the squeezing level

Output State Fidelity

- Fidelity Analysis:

$$F \quad \equiv \quad \langle\psi|\hat{\rho}_{\text{out}}|\psi\rangle$$

$$= \quad \langle\psi| \int \frac{\mathrm{d}^2\zeta}{\pi} \chi_A^{\rho_{\text{out}}}(\zeta^*, \zeta) e^{-\zeta\hat{a}_{\text{out}}^\dagger} e^{\zeta^*\hat{a}_{\text{out}}}|\psi\rangle$$

- Coherent-State Input Special Case:

$$F = \frac{1}{1 + \left(\frac{1-\eta}{\eta}\right) + (1-\gamma) + \gamma s}$$

Coming Attractions: Lecture 16 + Schedule Notes

- Lecture 16:

  Quantum Cryptography
  - Bennett-Brassard protocol
  - Ekert protocol

- Schedule Notes:
  - Term paper proposals are due Tuesday, November 8
  - Mid-term exam Tuesday, November 8

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Fall 2016