MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

Recitation 17
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1. An amateur criminal is contemplating shoplifting from a store. Police officers walk by the store according to a Poisson process of rate λ per minute. If an officer walks by while the crime is in progress, the criminal will be caught.

   (a) If it takes the criminal t seconds to commit the crime, find the probability that the criminal will be caught.

   (b) Repeat part (a) under the new assumption that the criminal will only be caught if two police officers happen to walk by while the crime is in progress.

2. (Problem 5.12) Beginning at time $t = 0$ we begin using bulbs, one at a time, to illuminate a room. Bulbs are replaced immediately upon failure. Each new bulb is selected independently by an equally likely choice between a Type-A bulb and a Type-B bulb.

   The lifetime, X, of any particular bulb of a particular type is an independent random variable with the following PDF:

   $$\text{For Type-A Bulbs:} \quad f_X(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

   $$\text{For Type-B Bulbs:} \quad f_X(x) = \begin{cases} 3e^{-3x} & x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

   (a) Find the expected time until the first failure.

   (b) Find the probability that there are no bulb failures before time t.

   (c) Given that there are no failures until time t, determine the conditional probability that the first bulb used is a type-A bulb.

   (d) Find the variance of the time until the first bulb failure.

   (e) Find the probability that the 12th bulb failure is also the 4th type-A bulb failure.

   (f) Up to and including the 12th bulb failure, what is the probability, that a total of exactly 4 Type-A bulbs have failed?

   (g) Determine either the PDF or the transform associated with the time until the 12th bulb failure.

   (h) Determine the probability that the total period of illumination provided by the first two Type-B bulbs is longer than that provided by the first Type-A bulb.

3. (Problem 5.16) Consider a Poisson process. Given that a single arrival occurred in a given interval $[0, t]$, show that the PDF of the arrival time is uniform over $[0, t]$.