## Labor Supply

Where we're going:

I'm going to spend about 4 lectures talking about labor supply. Along the way, l'm going to introduce some econometric issues and tools that we commonly use. Today's lecture and probably some of the next lecture will cover the theory of static and intertemporal labor supply.

You should read Blundell and Macurdy's Handbook chapter, as well as David Card's critique of intertemporal labor supply. The chapter in Cahuc and Zylberbeg might also be helpful. The empirical papers I hope to cover over the next few lectures are:

Eissa and Liebman (QJE 96) (on the earned income tax credit)
Angrist and Evans (AER 96) (on labor supply of women)
Card and Robbins (NBER5701) (self sufficiency project experiment)
And a couple of papers on the intertemporal labor supply of cab drivers and bicycle couriers

Gelbach (AER 02) will be covered in the first problem set.

Questions before I start?

## Decomposition of labor supply in term of substitution, income, and endowment effects

The static (one period) model of labor supply you've seen before, probably in terms of a graph, with leisure on the $x$ axis and consumption on the $y$ axis.

The set-up is straightforward:
$\max U(C, L, X)$
where $C$ is consumption, $L$ is leisure (any time spend not working), and $X$ are individual attributes.

Utility is quasi-concave, so: $U_{c}>0, U_{c c}<0, U_{L}>0, U_{L L}<0$
subject to: s.t. $C+w L=Y+w T, L \leq T \quad(C=Y+w(T-L))$
$w$ is the wage rate, $Y$ is non-labor income, and $T$ is the total time available, and a single sonsumption good is taken as the numeraire (the price of consumption is one).

The 'price' of one unit of not working is the wage (opportunity cost).

Note that it's not clear in the static model over what length of time is being examined. Are we looking over a week, or a year? Also, it's not clear how an individual decides to allocate their time: by hours, or by days? For now, let's think of T in terms of a one week period, with the individual deciding whether not to work, or to work in jobs that pay the same wage, but for different hours (e.g. part-time or full-time).
$M=Y+w T$ (full income)

The lagrangian is:
$L=U(C, L, X)-\lambda(C+w L-M)-\mu(L-T)$
F.O.C.:
$U_{c}(C, L, X)=\lambda$
$U_{L}(C, L, X) \geq \lambda w+\mu$

Alternatively, we can express the solution in terms of the marginal rates of substitution:
$\frac{U_{L}}{U_{c}}=M R S_{L}(C, L, X) \geq w$, which is equal if the individual works at all.

We can use the first order conditions to solve for consumption and leisure choices that maximize utility:

Interior Solution: $L^{*}=L^{*}(w, M, X) \leq T, C^{*}=C^{*}(w, M, X)$, or we instead look at work rather than leisure, since: $H^{*}=T-L^{*}$, where H is hours worked (l'd prefer to use W for weeks worked, but W already stands for wages).

Note, the reservation wage, when an individual is indifferent between working and not working, can be computed by setting: $U_{L}(C, L, X)=\lambda w$ and $\mathrm{T}=\mathrm{L}$.

Many studies in labor focus on the response time worked from changes in the wage. Easier to go through the response to leisure from a change in wage, because leisure is a 'good', whereas hours worked is a 'bad'. For interior solution:

$$
\begin{aligned}
\frac{\partial L^{*}}{\partial w} & =\frac{\partial L^{*}(w, Y+w T, X)}{\partial w} \\
& =\left.\frac{\partial L^{*}}{\partial w}\right|_{M=\text { curr.Inc }}+\frac{\partial L^{*}}{\partial M} \frac{\partial M}{\partial w} \\
& =\left.\frac{\partial L^{*}}{\partial w}\right|_{M=\text { curr.Inc }}+\frac{\partial L^{*}}{\partial M} T
\end{aligned}
$$

Holding individual attributes constant (this could be a big assumption, as we'll see when looking at this response over time).

We can compute $\left.\frac{\partial L^{*}}{\partial w}\right|_{M=\text { curr.Inc }}$ simply from slutsky equation:
[see Varian or other micro text for derivation of Slutsky equation]

$$
\left.\frac{\partial L^{*}}{\partial w}\right|_{M=\mathrm{curr} . \mathrm{Inc}}=\left.\frac{\partial L^{*}(w, M, X)}{\partial w}\right|_{U=U^{*}}-\frac{\partial L^{*}\left(w, C+w L^{*}\right)}{\partial M} L^{*}
$$

(so here we are using the income expression on the left hand side of the budget constraint, whereas for the full expression, we defined income as full income - the right hand of the budget constraint).
$\left.\frac{\partial L^{*}}{\partial w}\right|_{M=\text { curr.Inc }}$ is the Marshalian demand for leisure whereas $\left.\frac{\partial L^{*}(w, M, X)}{\partial w}\right|_{U=U^{*}}$ is the Hicksian demand for leisure.

Recall from your earlier micro classes that the substitution effect, $\left.\frac{\partial H^{*}(w, M)}{\partial w}\right|_{U=U^{*}}$, is negative - if the price of leisure goes up, holding utility constant, people buy less leisure. We also usually assume leisure is a normal good. If wages increase, the total amount of things an individual can 'buy' - consumption of leisure, goes down. So this income effect is negative. Both effects are negative, so this gives us the standard result that the

Marshallian response (elasticity) to the wage change is larger (in absolute value) than the hicksian response.

Substituting the slutsky equation into the full response from a change in the wage:

$$
\begin{aligned}
\frac{\partial L^{*}}{\partial w} & =\left.\frac{\partial L^{*}(w, M)}{\partial w}\right|_{U=U^{*}}-\frac{\partial L^{*}\left(w, C+w L^{*}\right)}{\partial M} L^{*}+\frac{\partial L^{*}\left(w, C+w L^{*}\right)}{\partial M} T \\
& =\left.\frac{\partial L^{*}(w, M)}{\partial w}\right|_{U=U^{*}}+\frac{\partial L^{*}\left(w, C+w L^{*}\right)}{\partial M}\left(T-L^{*}\right)
\end{aligned}
$$

We have the substitution effect, the income effect and the endowment effect. The endowment effect leads individual to take more leisure, since a rise in the wage rate makes them overall richer (the income effect is smaller than the endowment effect).

Note the difference between this result and that from conventional demand theory, which only includes income and substitution effects (which are both negative if the good with a price increase is a normal good). The conventional case concerns goods that are consumed, not sold - no endowments. Here, an individual not only consumes leisure, but also may 'sell' it to obtain a wage, to obtain consumable goods.

The main point of the static model is to show that the response from a wage change is ambiguous - there is both a positive and a negative effect, and it is not clear which one dominates, and under what circumstances.

For prime age men, most evidence indicates that the income effect dominates (men tend to work less from an increase in wage). For women and others with currently lower labor force participation, the response to an increase in wage tends to generate more of a response towards working more hours.

Note, the response is not ambiguous if an individual is currently not working. An increase in the wage will lead to a zero or positive increase in hours worked. Individuals that work a
little are more likely to have the substitution effect dominate the income effect. The less an individual works, the smaller the income and endowment effects are.

This is useful to know in practice, because many empirical studies suggest that movements in labor supply are principally owing to variations in the participation rate, and that the elasticity of the supply of female labor, especially that of married women, is greater than that of men.

The general empirical equation for estimating the overall marshallian labor supply elasticity is:

$$
\log H_{i}=\beta_{0}+\beta_{1} \log w_{t}+\beta_{2} X_{i}+v_{i}
$$

Where $\beta_{1}$ is the estimated elasticity of hours worked (instead of hour of leisure taken: T-H) with respect to wages. Note the endogeneity problem (or the omitted variables problem): clearly there may be many factors that affect both wages and hours worked. We will look at some ways of addressing this shortly.

A few interesting points arise from working with this simple model - you can just as easily realize these points by working through the graphical version of the static model.

1) the elasticity of supply of labor depends individually on preferences for leisure and consumption, as well as individual circumstances (e.g. with children, without).
2) Differences in responses to wage changes may be driven purely by differences in tastes (because the indifference curves take on different shapes).
3) static model assumes wages are parameters (that don't change when other characteristics change), but clearly this is not the case. Perhaps as a first approximation, or for simplification, it's OK.

Incorporating taxes, transfers, welfare, and lump sum costs to working into model:

If taxes and transfers do not affect the shape of indifference curves (a reasonable assumption, but one discussed more by Killingsworth), then introduction of taxes or transfers merely alters the shape of the individual's budget line. Similarly, we might also consider one time costs associated with working or not working (e.g. transportation costs, day care costs, welfare, ...). If additions only affect the budget line, the analysis is the same as before, except the budget line looks slightly different. Specifically,

$$
C=Y+w \operatorname{elf}(H \leq 0)+w(T-L)(1-\tau)-L C 1(H \geq 0)
$$

where welf is welfare payments if not working, $\tau$ is the tax rate of work, and LC is the labor costs with working. Since the budget constraint is now discontinuous, it's a pain to derive the first order conditions. It's a lot easier just to consider how these changes affect the budget constraint graphically (see Killingsworth for a discussion)

Note, the response to a transfer or tax depends both how the revenue raised via those taxes is used. Even if we focus on situations where revenue raised to make transfer payments to other individuals, the effect of a tax on individuals is still ambiguous because of substitution and income/endowment effects. Just as the effect from lowering the wage was ambiguous, so too is the effect from increasing taxes. On the one hand, the price of leisure is reduced, making it more likely to reduce employment. On the other hand, income is reduced, making leisure less attractive (if leisure is a normal good). The response will also depend on people's tastes.

Therefore, the aggregate, overall, response from a change in the budget constraint will depend on many things, and the overall (average) effect on labor supply is a priori unknown. It's an empirical question, one that we will look at shortly below.

## Negative Income Tax Example

Transfer programs tend to generate disincentives to work. Introducing a guaranteed welfare amount has an unambiguous affect on the incentive to work. Suppose we introduce an income floor, where everyone Is guaranteed an income G. The new budget constraint is:

$$
\begin{aligned}
& C=Y+w(T-L)+G \text { if } G \leq Y+w(T-L), \\
& C=G \text { if } G>Y+w(T-L)
\end{aligned}
$$

The marginal tax rate for a non worker, in this case, is 1 . For those affected by introducing the guarantee, the effect in unambiguous in leading these workers to stop working.

An example of a program not as extreme is a negative income tax. The general set-up considers both a guarantee and a subsidy to work. The subsidy is:
$S=G-t(w H)$ if $G>t(w H), S=0$ otherwise,
where $G$ is the guarantee level and $t$ is the tax rate. Non wage income is left out. In practice, Y is hard to assess.
$w H=G / t$ is the breakeven income level when the income supplement ends. If $\mathrm{G}=$ $\$ 20,000$ for family of 4 and $t=.5$, government has to pay families with incomes up to $\$ 40,000$.

We can describe the effects of the NIT graphically, or we can sign the effects more explicitly from the slutsky equation (show first on graph). In effect, what's happened is two simultaneous changes: a shift in $Y$ by $G$, and a fall in wages by -tw (for those with $G>t(w H))$

Total differentiating effect on time worked:
$d h(w, M)=\frac{\partial h}{\partial w} d w+\frac{\partial h}{\partial M} d M$

The first term, $\frac{\partial h}{\partial w}$ can be decomposed as before: $\frac{\partial h}{\partial w}=\left.\frac{\partial h}{\partial w}\right|_{U=U^{*}} d w+\frac{\partial h}{\partial M} h$, so:
$d h(w, M)=\left.\frac{\partial h}{\partial w}\right|_{U=U^{*}} d w+\frac{\partial h}{\partial M}[h d w+d M]$

For individuals working before the program, earning less than the breakpoint (G>t(wH))

| Before the program: | After | change ('d') |
| :--- | :--- | :--- |
| W | (1-t) w | -tw |
| $\mathrm{M}=\mathrm{wT}$ | $\mathrm{wT}+\mathrm{G}$ | G |

So
$d h=\left.\frac{\partial h}{\partial w}\right|_{U=U^{*}}(-t w)+\frac{\partial h}{\partial M}[S]$
both terms are negative if leisure is a normal good. The effect is unambiguous because the income effect from the guarantee even if not working is larger than the income effect generated from working more.

Note that sometimes it's convenient for empirical applications to convert the analysis in terms of elasticities. Take the last example:

$$
\begin{aligned}
& \frac{d h}{h}=d \log h=\left(\frac{w}{h} \frac{\partial h^{c}}{\partial w}\right)(-t)+\left(\frac{M}{h} \frac{\partial h}{\partial M}\right) \frac{S}{M} \\
& \Delta \% h=\eta^{c}(-t)+\eta^{M} \frac{S}{M}
\end{aligned}
$$

If we had variation in $t$ and $S$, we could us regression to try and estimate $\eta^{c}$ and $\eta^{M}$.

## Empirical Example 1:

## The Self-Sufficiency Project Experiment

The self-sufficiency project was started in 1991 designed to explore such a program. In 1991, the SRDC collected a list of all long-term single parent welfare recipients (on welfare for at least 1 year) in Vancouver and rural New Brunswick (in Canada). They randomly selected half of these people on this list and invited them to participate in a work subsidy program. The SSP offered a temporary, but generous earnings supplement to these selected single parents. To take advantage of the supplement offer, the individuals had to begin working full time ( 30 hours or more per week) and stop receiving welfare within a year of being offered the supplement. The supplement was paid on top of earnings. Those who were eligible to receive it could do so for up to three years after finding full-time work, as long as they were still working full time and not recieiving welfare.

The supplement is calculated as half the difference between a participants earnings from employment and $\$ 37,000$. So the supplement is reduced by 50 cents for every dollar of increased earnings:

$$
C+w+[18,500-w / 2 L] 1(w \leq 25,000) 1(H=30 h / w e e k)=Y+w T
$$

<draw on board>

As an example, a single parent with two children in Vancouver received \$17,111 annually in social assistance (welfare) in 1991. If they obtained a job working 35 hours at $\$ 7$ an hour and worked 52 weeks, their annual income would be $\$ 12,740$, much less than welfare!! Unless there was utility from working, static model predicts unambiguously parent would not work. The work subsidy from the program is $\$ 12,130$, so if the parent participated in the program, she would earn instead $\$ 24,870$. In general, most participants faced incomes $\$ 3,000-\$ 7,000$ higher, when including the subsidy, compared to welfare incomes.

One thing we might like to know is the effect of the program on every participant. Let's define $D_{i}$ as an indicator for whether a participant $i$ was selected to receive the subsidy, $D_{i}=1$, or not, $D_{i}=0$. Let $Y_{1 i}$ be woman $i$ 's circumstances (hours worked, welfare status, family health, etc...) if $D_{i}=1$ (say, for example, 3 years into the program), and $Y_{0 i}$ be her circumstances otherwise. Assume both these potential outcomes are well defined for everyone. The problem is, only one is ever observed for each woman. Formally, this can be expressed by writing the observed outcomes, $Y_{i}$, as:

$$
Y_{i}=Y_{0 i}\left(1-D_{i}\right)+Y_{1 i} D_{i}
$$

We cannot know the counterfactual outcome for each woman without being able to observe alternate universes (which we can't). Instead, the Self-sufficiency project tries to estimate the average effect on participants using observed outcomes only. If we simply take the difference in mean outcomes between those selected and those not, we get:

$$
\begin{aligned}
E\left(Y_{i} \mid D_{i}=1\right)-E\left(Y_{i} \mid D_{i}=0\right) & =E\left(Y_{1 i} \mid D=1\right)-E\left(Y_{0 i} \mid D_{i}=0\right) \\
= & E\left(Y_{1 i}-Y_{0 i} \mid D_{i}=1\right) \\
& +\left[E\left(Y_{0 i} \mid D_{i}=1\right)-E\left(Y_{0 i} \mid D_{i}=0\right]\right.
\end{aligned}
$$

and because participants are randomly assigned, which means $D_{i}$ is independent of any initial circumstances before the program, the expected counterfactual outcome for participants selected is the same as the expected outcome for participants not selected. Expected outcomes are the same for both groups, conditional on program assignment. The last term of this expression, $\left[E\left(Y_{0 i} \mid D_{i}=1\right)-E\left(Y_{0 i} \mid D_{i}=0\right]=0\right.$, is zero. Thus, by comparing observed means, random assignment allows us to estimate the average effect from being offered the subsidy.

Essentially, the study looked at the following:

$$
H_{i t}=\beta_{0}+\beta_{1} T_{i}+v_{i t}
$$

where $T_{i=1}$ if offered the subsidy, 0 otherwise. Notice we are examining the labor supply effect over different periods in time, including the period after the subsidy is removed. As we'll see when we turn to the intertemporal model of labor supply, it may matter whether such policy change is temporary or permanent.

Assignment to $T$ is random, so there is no omitted variables bias: $v$ is independent of $T$, and the sample is conditional on being a single parent, and on welfare for at least one year.

The counterfactual is obvious: what would have happened had the treatment not been given, and the average effect for the population is identified.

## Expected effects:

The wage rate for working full time is being subsidized. The anticipated effect over the 3 year period the program is offered is unambiguously zero or positive, since there is no short-run income effect among the population. But here's where the intertemporal model is useful: it reminds us that the program is short term, and the effect of the program is transitory. Lifetime wealth increases as a result of the program, for those that participate. The effect of labor supply after the subsidy runs out is ambiguous, since there is a (small) wealth effect spread out over the lifetime. What is interesting is that the model predicts once the subsidy goes away, individuals that did change their labor supply should reduce it once the subsidy has ended, maybe even to levels lower than the control group.

Results:

Figure 3.1

It is interesting to note that only $30 \%$ of those treated decided to switch to working full time, compared to $15 \%$ of the control group in the first year of the study. Therefore, many in the treatment group did not take up the subsidy, and analyzing the labor supply dynamics of the program are confounded by including the part of the treatment group that was eligible, but did not participate.

Another way to examine the experiment is to focus on the 'Treatment of the treated'. That is, to focus on just the effects of those that did take up the program, among those who were eligible. It's crucial to note that the characteristics of this group are unlikely to be similar to the characteristics of the control group, so any analysis of the treatment of the treated must be treated with this understanding of likely omitted variables bias.

Figure 3.4 shows the effect of the treatment on the treated over time. Obviously the full time employment for this group will initially increase to almost 100\%, by definition of those who took up the program. But now notice how quickly the fall in full employment among them occurs - almost 20 percentage points over the 3 years, compared to a slow, but steady rise for those that did not take up the program. Note, the slower change in employment between the control group and the treated group that did not take up the program indicates the potential that these groups are different.

Figure 6.1. What is interesting is how relatively small the labor supply decrease occurred following immediately after the drop in subsidies. I would have expected to see a larger reaction. On the other hand, by the time the subsidy was removed, the treatment group had no different employment rates than the control group, which is generally what would be expected from the theory, if wealth effects were small. It seems the overall effect of the program was to increase the speed at which welfare parents took up full-time employment.

Analyzing the program. Should this program be adopted? Tables 7.4 and 7.5 look at the benefits and costs of the program. For individuals that would have stayed on welfare, but
instead work with the subsidy, the cost is less to subsidize than for welfare. But the program also subsidizes individuals that would have started full time employment anyway for these individuals (wind-fall recipients), the program cost even more. Targeting only long-term welfare recipients and requiring fulltime employment for take-up helps minimize the number of wind-fall recipients, but the study reveals that overall, this program costs more - the 5 -year per program group member cost for the treatment group was $\$ 40,000$, whereas for the control group it was $\$ 37,000$ (ignoring administrative costs). If we include additional earnings as a benefit (payment for value added), then we might be able to justify program - earnings were, on average, about \$4,000 higher for the treatment group than for the control. But this analysis completely ignores redistributional issues: the earnings are received by the participant, the subsidy is received by the participant, and the costs are born by the tax-payer.

A variation of this program might lower overall costs if it could do a better job targeting those that would have stayed on welfare otherwise. It would be interesting to see the same program set up for only those with at least 2 years on welfare (not clear what results would show, however). Another consideration is that if this program was introduced, it might provide incentive to stay on welfare longer. A separate sample for the SSP was carried out, where a random sample from a group of welfare recipients with less than 1 year on welfare was told they would be eligible for the program if they were still on welfare after a year. Interestingly, Card, Robins, and Lim find minimal effects.

What were the effects of the self-sufficiency project? Introducing the program led to about a 25 percentage point rise in full-time employment rate relative to the control group (from 5 to $30 \%$ ). Note that about $70 \%$ of those on welfare didn't react to the program at all. Five years into the experiment, the control group had caught up: about the same percentage of welfare recipients were employed full time in both groups (about 30\%). So the program seems to have had a very large effect in making some welfare recipients start work early,
but little long-run effect. Strong tendency to stay on welfare even if offered very large financial incentive to work. Interesting to know why.

