

14.54 International Trade

—Lecture 10: Production Functions—

Today's Plan

- 1 Midterm Results
- 2 Properties of Production Functions (2 Factors)
- 3 Isoquants
- 4 Input Choice and Cost Minimization
- 5 Relative Factor Demand

Graphs on slides 7, 10-17, and 19 are courtesy of Marc Melitz. Used with permission.

- We will now introduce another factor of production: capital
 - Can also think about other production factors: land, skilled versus unskilled labor, ...

What issues can be addressed when production requires more than a single factor?

- In the short-run, some factors are more 'flexible' than others: how quickly and at what cost can factors move from employment in one sector to another?
- Example of labor and capital:
 - After a U.S. state is hit with a regional shock, unemployment rate falls back to national average within 6 years (most inter-regional employment reallocations also involve worker reallocations across sectors)
 - In comparison, capital depreciates over 15-20 years and structures over 30-50 years
 - In the short-run, labor is more "flexible" than capital across sectors
- Distributional consequences across factors from changes in goods prices even in the long run

Production Function

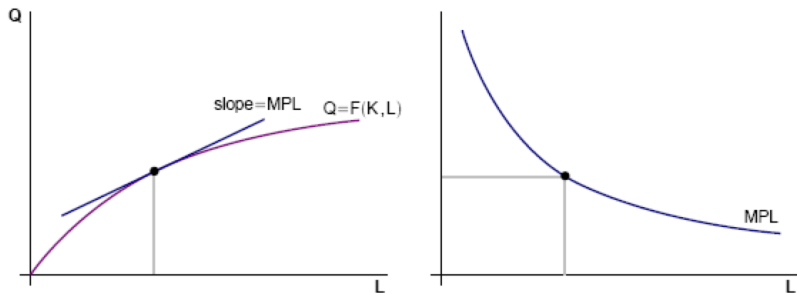
- Under constant returns to scale, a production function with one factor can be summarized by a single number: unit input requirement (an overall productivity index)
- With more than one factor, a production function also characterizes the substitutability between the factors of production (as well as an overall productivity index)
- We will now assume that $Q_C = F_C(K_C, L_C)$ and $Q_F = F_F(K_F, L_F)$
- We will continue to assume constant returns to scale:
 $F(tK, tL) = tF(K, L)$ for any $t > 0$
- And will also assume diminishing marginal returns to a single factor
- ... as well as factor complementarity

Properties of Production Function

- Marginal Products
 - $MPK = \partial F(K, L) / \partial K = F^K(K, L)$ and $MPL = \partial F(K, L) / \partial L = F^L(K, L)$
 - For any production function, must have $MPK \geq 0$ and $MPL \geq 0$
- Diminishing marginal returns to a single factor assumption:
 - $MPK = F^K(K, L)$ is **decreasing** in K and $MPL = F^L(K, L)$ is **decreasing** in L
- Factor Complementarity:
 - $MPK = F^K(K, L)$ is increasing in L and $MPL = F^L(K, L)$ is increasing in K
- Note: these assumptions are **all** compatible with constant returns to all factors

Production Function and Marginal Product

- Hold capital K constant:



- An increase in K will shift up both curves

Implications of Constant Returns to Scale for Marginal Products

- Recall the assumption that $F(tK, tL) = tF(K, L)$ for any $t > 0$
 - Differentiate with respect to K :

$$F^K(tK, tL)t = tF^K(K, L) \Leftrightarrow F^K(tK, tL) = F^K(K, L)$$

- A proportional increase in K and L leaves the MPK unchanged
- Similarly:

$$F^L(tK, tL) = F^L(K, L)$$

- A proportional increase in K and L leaves the MPL unchanged

Production with Competitive Output and Factor Markets

- Factors are paid the value of their marginal products: $r = pMPK$ and $w = pMPL$
- Marginal cost pricing (also average cost pricing with constant returns to scale):

$$p = MC = AC = \frac{rK + wL}{Q}$$

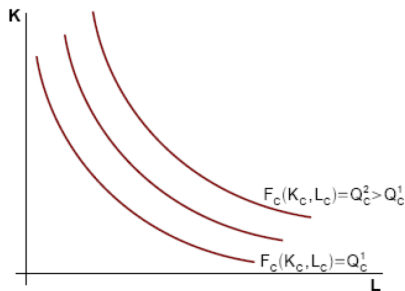
Hence, no economic profits $p = pQ - rK - wL = 0$

- Can think of payment to capital K either as value of MPK or as revenue left over to firm owners (capital owners) after labor has been paid:

$$rK = pQ - wL$$

Production Functions and Isoquants

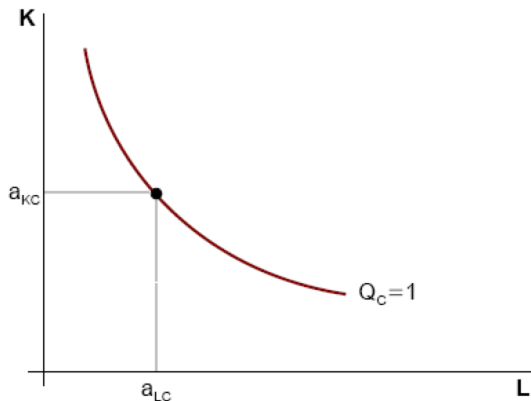
- An isoquant associated with a production function $F_C(K_C, L_C)$ is the set of inputs (K_C, L_C) that can be used to produce a given output level Q_C



- The set of isoquants can be used to represent any given production function
- The curvature of the isoquant captures the substitutability of the factors in production (parallel with indifference curves)

Unit Isoquant

- The unit isoquant is the combination of inputs (K_C, L_C) that can be used to produce one unit of output



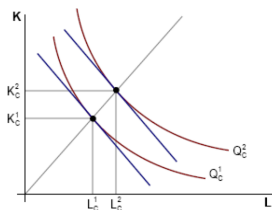
- With multiple factors, firms can choose different combinations of unit input requirements a_{LC} and a_{KC}

Isoquants and Marginal Products

- The slope of any isoquant at (L_C, K_C) is MPL_C / MPK_C
 - Take the total derivative of $F_C(K_C, L_C) = Q_C$:

$$\frac{\partial F_C}{\partial K_C} dK_C + \frac{\partial F_C}{\partial L_C} dL_C = 0 \Leftrightarrow \frac{dK_C}{dL_C} = -\frac{\partial F_C / \partial L_C}{\partial F_C / \partial K_C} = -\frac{MPL_C}{MPK_C}$$

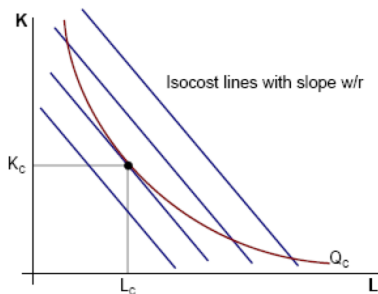
- Thus, with constant returns to scale production, slopes of isoquants do not change along any ray from the origin (why?)



In addition, proportional change in output is equal to proportional change in inputs

Firm Input Choice and Cost Minimization

- Given factor prices w and r , firm minimizes cost of producing Q_C units of output $rK_C + wL_C$ by choosing K_C and L_C such that $MPL_C/MPK_C = w/r$ (see appendix to Ch. 4)

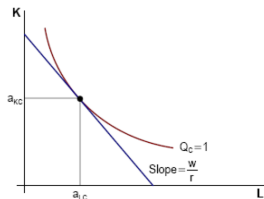


- Note that this is also implied by factor prices equal to the value of their marginal products:

$$w = p_C MPL_C \text{ and } r = p_C MPK_C \Rightarrow \frac{w}{r} = \frac{MPL_C}{MPK_C}$$

Firm Input Choice and Cost Minimization (Cont.)

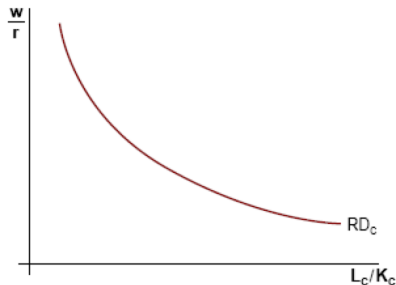
- Under constant returns to scale production, a firm will always produce Q_C units of output with the same L_C/K_C as it uses to produce one unit of output
- At given w and r , a firm choose unit input requirements a_{LC} and a_{KC} such that $MPL_C/MPK_C = w/r$



- The minimized unit cost is $wa_{LC} + ra_{KC}$ (constant $AC = MC$)
- Firm produces Q_C units of output using $L_C = Q_C a_{LC}$, $K_C = Q_C a_{KC}$
- Thus only need to solve cost minimization for $Q_C = 1$

Relative Factor Demand

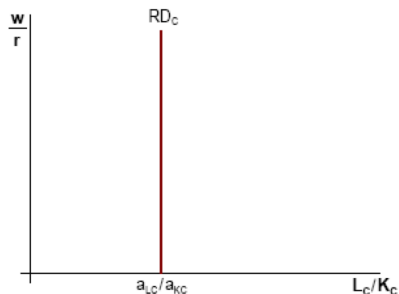
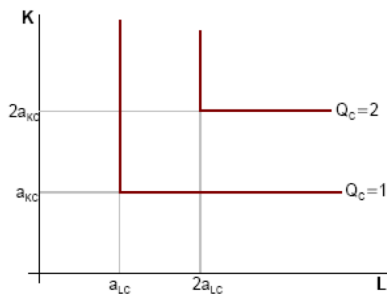
- The relationship between L_C/K_C and w/r can be represented by a relative demand curve for factors (which is independent of the amount of output Q_C produced)



- Note similarity with the relative demand curve for goods (and relationship with factor/good substitutability)
- Also note that textbook uses a relationship between K_C/L_C and w/r (which implies a positively sloped curve)

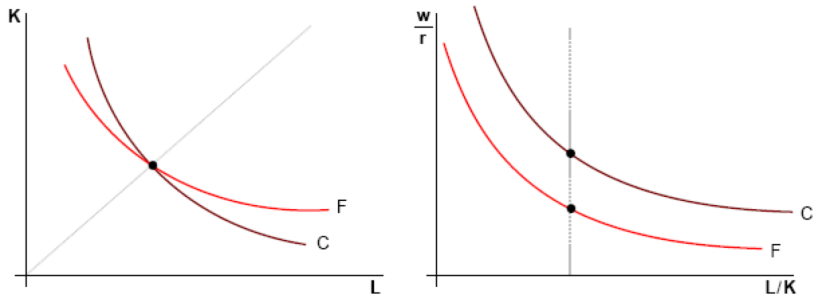
Leontief Technology: Fixed Factor Production

- Under Leontief technology, there is no substitutability between production factors:
 - Producing a unit of output C requires a fixed amount of labor a_{LC} and capital a_{KC}
 - The production function is $Q_C = \min \{L_C/a_{LC}, K_C/a_{KC}\}$



Differences in Technology and Relative Factor Demands

- Consider the following differences between the unit isoquants:



- Definition of factor intensity in production: C is **relatively** labor intensive if, at any given relative factor price w/r

$$\frac{L_C}{K_C} = \frac{a_{LC}}{a_{KC}} > \frac{L_F}{K_F} = \frac{a_{LF}}{a_{KF}}$$

So C firms will always hire relatively more L (and inversely for F firms)

Factor Intensity and Factor Cost Shares

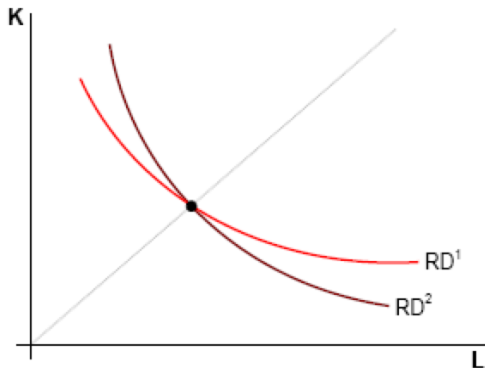
- If C is relatively labor intensive (relative to F) then
 - $a_{LC}/a_{KC} > a_{LF}/a_{KF}$
 - C firms always hire relatively more labor
 - F firms always hire relatively more capital
 - and also

$$\frac{wL_C}{wL_C + rK_C} > \frac{wL_F}{wL_F + rK_F} \text{ and } \frac{rK_C}{wL_C + rK_C} < \frac{rK_F}{wL_F + rK_F}$$

- C firms always devote a higher share of their total cost to labor
- F firms always devote a higher share of their total cost to capital

A Special Case

- What can be said about factor intensity if relative factor demand curves look like this?



- Then factor intensity depends on relative factor price
- This is called a factor intensity reversal
- We will not consider this special case

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Fall 2016

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