14.471: Fall 2012: Recitation 6: Optimal Taxation and Public Production I: Production Efficiency: Diamond and Mirrlees (1971)

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estion: Do optimal commodity taxes imply production efficiency? How to set optimal commodity taxes? Do on optimal commodity taxes and production efficiency change when one introduces public investment?

Introduction

Bring together theories of taxation, public investment and welfare economics

Key Result: Aggregate production efficiency is desirable if taxes are set at the optimal level

Result will be illustrated in 3 settings

Single consumer, no public consumption and only commodity taxes (graphically and calculus)

Several consumers, no public consumption and only commodity taxes

General setup

One-consumer Economy: Geometric Analysis:

Do not allow lump sum taxes

- this would permit the economy to achieve a Pareto optimum

- goal of one-consumer case is to introduce results for several consumers case where only poll taxes are realistic

Keep government spending constant and hence ignore its impact on utility

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Figure 1: Production frontier
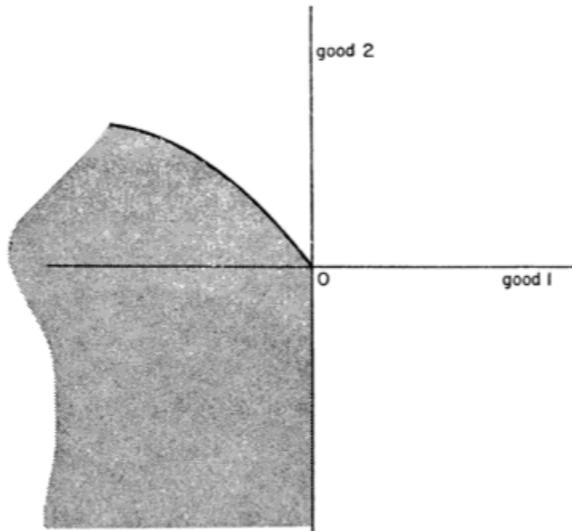
FIGURE 1

- Figure (1)

    - features input horizontally and output vertically

    - shows production frontier with decreasing returns to scale

    - can be shifted to the left (see Figure 2 in paper) with fixed level of government spending

    - would be only constraint in planned economy but decentralization through market is more realistic when number of households grows

Figure 2: Production frontier and offer curve
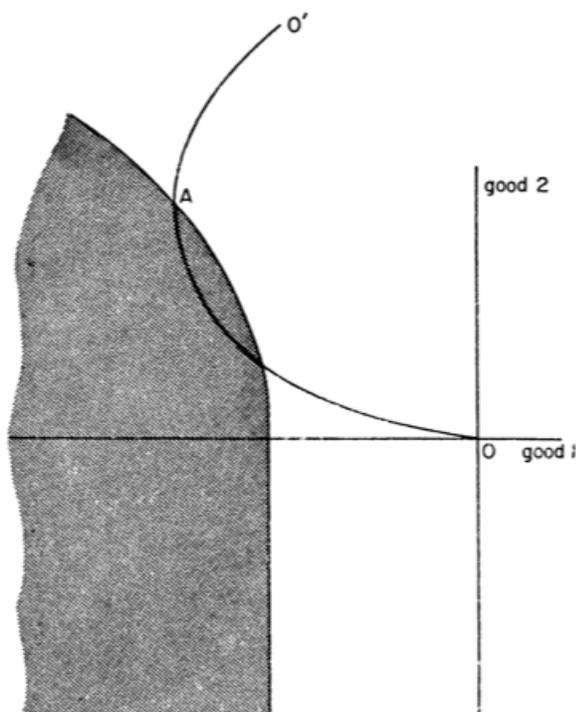
FIGURE 4

- In figure (2) the planner is constrained to:

    - Technological feasibility: production frontier

    - Consumer equilibria: Transactions - which the consumer is willing to undertake at some relative price -
      on the "offer curve" (price-consumption locus)

        * Labor-consumption bundles which maximize consumer's utility given some budget constraint (see
          Figure 3 in the paper)

    - Uniform pricing

- The government choses a price vector q to maximize the the indirect utility of the consumer $v(q)$ subject to
  the constraints that:

    - Government supply of the vector z is feasible, i.e. $G(z) \leq 0$

    - Government supply z equals the consumer demand $x(q)$, i.e. $x(q) = z$

    - In short:

$$\max_q v(q) \quad s.t. \, G(x(q)) \leq 0 \tag{1}$$

    - Note: since we use prices rather than quantities, (1) does not change when you extend to many consumers

- In figure (3) the optimal point is A since we wish to move as far along the offer curve as possible subject to
  the production frontier:

    - Relative price will correspond to the slope of the budget line OA

    - All the points above indifference curve II and in the shaded production set are Pareto-superior to A and
      technologically feasible but **not attainable by market transactions without lump sum transfers**

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Figure 3: Production frontier, offer curve, indifference curve and budget line
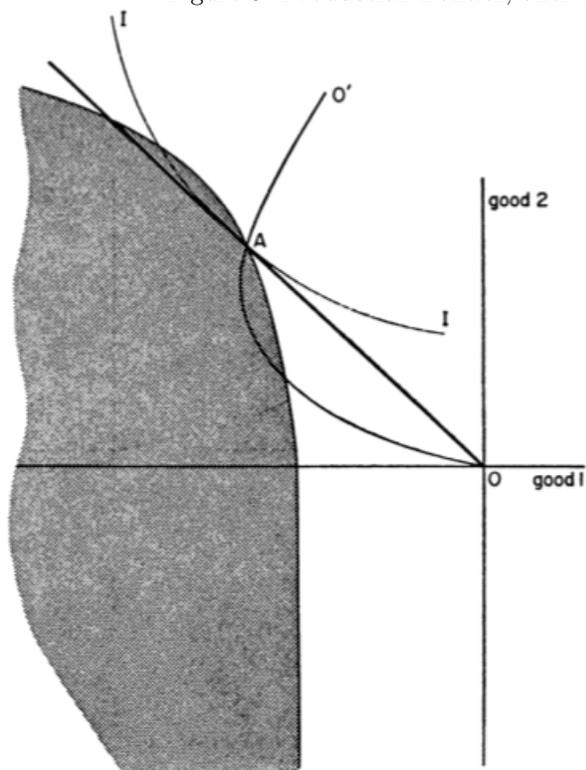
FIGURE 5

- In figure (4) :
  - The pareto-optimum B can be decentralized with market transactions and **with a lump sum transfer**
  - Intersection budget line and horizontal axis represents the payment of a lump sum tax to cover government expenditures in excess of profits from production
  - **Key production efficiency result: Optimal point is on the production possibility frontier** (generalizes to several goods by considering union of such loci)

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Figure 4: Production frontier, offer curve, indifference curve and budget line with lump sum transfers
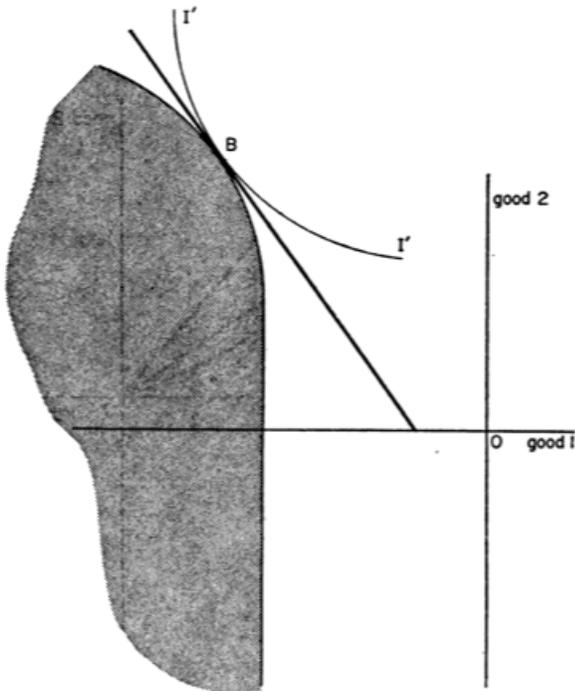
FIGURE 6

3 One-consumer economy: Algebraic analysis:

- Assume
    - now both private production y and still public production
    - Constant returns to scalce (CRS)
    - perfect competition
    - only commodity taxes
    - that government production is efficient $z_1 = g(z_2, ..., z_n)$ to shift our attention to aggregate production efficiency
- Writing $v_k = \frac{\partial v}{\partial q_k}$ and $u_i = \frac{\partial u}{\partial x_i}$, we can show that:

$$v_k = \sum u_i \frac{\partial x_i}{\partial q_k} = -\alpha x_k \tag{2}$$

    - differentiating the budget constraint $\sum x_i q_i = 0$ wrt q_k to yield $x_k + \sum q_i \frac{\partial x_i}{\partial q_k} = 0$ and
    - using that $u_i = \alpha q_i$
- Writing the private production constraint as $y = f(y_2, ..., y_n)$ we get from profit maximization:

$$p_i = -p_1 f_i(y_2, ..., y_n) \tag{3}$$

- From CRS, profits are zero in equilibrium $\sum p_i y_i = 0$

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- Market clearing (private demand is sum of private and public supply: $x(q) = y + z$), private budget constraint and Walras law imply balanced government budget

- Normalize prices: no tax tax on good 1 ($p_1 = q_1 = 1$)

Welfare maximization

- The government picks after and before tax prices and public production to maximize welfare s.t. market clearing, private production y maximizes profit $\sum p_i y_i$ and government efficiency $z_1 = g(z_2, ..., z_n)$

$$\max_{q,p,z} v(q) \quad s.t. \quad x(q) = y + z \tag{4}$$

- Producer prices will be pinned down by (3) and combining the constraints in (4) gives $x_1(q) = y_1 + z_1 = f(x_2 - z_2, ..., x_n - z_n) + g(z_2, ..., z_n)$.

- Hence we can rewrite (4) as follows:

$$\max_{q,z} v(q) \quad s.t. \quad x_1(q) - f(x_2 - z_2, ..., x_n - z_n) - g(z_2, ..., z_n) = 0. \tag{5}$$

- Differentiating the Lagrangian $L = v(q) - \lambda\left(x_1(q) - f(x_2 - z_2, ..., x_n - z_n) - g(z_2, ..., z_n)\right)$ w.r.t. q_k gives:

$$v_k - \lambda\left(\frac{\partial x_1}{\partial q_k} - \sum_{i=2} f_i \frac{\partial x_i}{\partial q_k}\right) = 0 \tag{6}$$

- Using (3) for producer prices and using that $p_1 = 1$, rewrite (6) as:

$$v_k - \lambda\left(\sum_{i=1} p_i \frac{\partial x_i}{\partial q_k}\right) = 0 \tag{7}$$

- Differentiating L w.r.t. z_k we have:

$$\lambda(f_k - g_k) = 0 \tag{8}$$

- Conclusion: Provided that $\lambda \neq 0$ (social cost to a marginal need for additional resources), then Equation (8) implies **equal marginal rates of transformation in public and private production efficiency and thus aggregate production efficiency.**

Optimal Tax Structure

- The relations (7) determine the optimal tax structure: they relate producer and consumer prices.

  - Intuition: Any further increase in consumer prices results in a change in welfare v_k which is the same ratio to the cost of satisfying the change in demand arising from the price increase.

- Let us reintroduce taxes in (7) and use that (i) x_i depends on $p + t$, (ii) $\frac{\partial x_i}{\partial q_k} = \frac{\partial x_i}{\partial t_k}$ and that (iii) p is held constant in this derivation.

$$v_k = \lambda\left(\sum_{i=1} p_i \frac{\partial x_i}{\partial t_k}\right) = \lambda \frac{\partial}{\partial t_k}\left(\sum_{i=1} p_i x_i\right) \tag{9}$$

- From the consumer budget constraint we get $\sum x_i p_i = \sum x_i q_i - \sum x_i t_i = -\sum x_i t_i$. Hence:

$$v_k = -\lambda \frac{\partial}{\partial t_k}\left(\sum_{i=1} t_i x_i\right) \tag{10}$$

  - Proportionality between:
    * Marginal utility of a change in the price of a commodity

* The change in tax revenue resulting from a change in the corresponding tax rate (calculated at constant producer prices)
- Assuming individualistic welfare, using (2), we get:

$$x_k = \frac{\lambda}{a} \frac{\partial}{\partial t_k} \left(\sum_{i=1} t_i x_i \right) \tag{11}$$

  – We equalize for all commodities the ratio of

  * Quantity of the commodity

  * Marginal tax revenue from an increase in the tax

  – We have the information to test for optimality of the tax structure

4   Production efficiency in the many-consumer economy

- The efficiency proof by contradiction that optimal production will generally be on the production frontier follows the following 3 steps:

  1. Suppose that welfare- production is not on the production frontier

  2. Any small change in prices q will not change production requirements by much -> our new demands are still technologically feasible (Assuming that aggregate demand functions $X(q)$ are continuous)

  3. We can increase welfare by modifying q : Contradiction -> 1 cannot be true –> We have to be on the frontier

- If there is a commodity that:

  – No consumer purchases but some consumer supplies (certain labor skills), we should raise its price

  – No consumer supplies but some consumer purchases (electricity), we should reduce its price

5   Extensions

- Summarize the efficiency result in economy with (i) consumers, (ii) private producers and (iii) public producers

  – Conclusion: all sectors not containing consumers should be viewed as a single sector, and treated so that aggregate production efficiency is achieved

Intermediate good taxation

- If we separate the private sector into more than 1 sector, we can tax transactions between firms
- Example: 2 CRS private production sectors and 1 consumer sector
- We want efficiency for private production possibilities taken together
- Then cannot have any intermediate good taxes since these would prevent efficiency:

  – In the absence of profits, taxation of intermediate goods must be reflected in changes in final good prices

  – Hence, revenue could have been collected by final good taxation causing no greater change in final good prices and avoiding production inefficiency

International trade

- Assume indifferent to welfare rest of the world

- Trade gives extra options to transform goods into others

- Efficiency: equate marginal rates of transformation between producing and importing

  – Final good sales direct to consumer should be subject to a tariff equal to tax on same sale by domestic producers

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