14.452: Mock Final Exam

You have two and a half hours. Please answer all questions. Good luck.

1 Short Questions

1. Consider a continuous time neoclassical growth model that admits a representative consumer with the following constant absolute risk aversion preferences at time $t = 0$,

$$U(0) = \int_0^\infty \exp(-\rho t) \frac{\exp[-\alpha c(t)]}{-\alpha} dt,$$

where $c(t)$ is consumption and $\alpha > 0$. Explain why such an economy could have a well defined steady-state equilibrium without technological progress, but would not exhibit a balanced growth path when there is labor-augmenting technological progress at a constant rate (where in the balanced growth path the interest rate and the growth rate of consumption are constant).

2. "In the post-war era, a significant fraction of the differences in income per capita across countries can be explained by differences in their physical and human capital investments." True or false?

3. Consider a model where research firms can improve both a technology A_H complementary to skilled workers and a technology A_L complementary to unskilled workers. The aggregate production function is $Y = \left[\gamma (A_H H)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)(A_L L)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$. Suppose that the costs of improving the two technologies are $\frac{1}{\eta_H} A_H$ and $\frac{1}{\eta_L} A_L$, and that research firms' objective is to choose A_H and A_L to maximize

$$\beta \left[\gamma (A_H H)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)(A_L L)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - \frac{1}{\eta_H} A_H - \frac{1}{\eta_L} A_L,$$

where β is the fraction of output they capture as rents for their technologies. Derive an equation for A_H/A_L as a function of H/L [Hint: take the ratio of the first-order conditions of the maximization problem]. Show that an increase in H/L will always cause "skill-biased" technical change. Interpret this result.

2 Long Questions

1. Consider the canonical OLG model with two-period lived households and log preferences

$$\log(c_1(t)) + \beta \log(c_2(t+1))$$

for each individual. Suppose that there is no population growth. Individuals work only when they are young, and each generation supplies one unit of labor inelastically. Production technology is given by

$$Y = AK^{\alpha}L^{1-\alpha},$$

there is no technological progress and capital depreciates fully after use.

(a) Define a competitive equilibrium in this economy.

(b) Consider a balanced growth path (BGP) equilibrium where the equilibrium wage is equal to w^* at each date and the interest rate is r^* (recall that the interest rate is given by r =marginal product of capital-1). Suppose that $r^* > 0$. Explain how you would apply the First Welfare Theorem to conclude that the competitive equilibrium is Pareto optimal.

(c) Explain why this argument breaks down when $r^* < 0$. Explain why $r^* < 0$ could be part of a competitive equilibrium.

(d) What happens to the interest rate if individuals can simply store their earnings from their youth to old age? Is the equilibrium in this case Pareto optimal?

2. Consider an infinite-horizon economy that admits a representative household with preferences at time 0 given by

$$\int_0^{\infty} \exp(-\rho t) \frac{C(t)^{1-\theta} - 1}{1-\theta} dt.$$

Population is given by $L(t)$ and grows at rate n. Labor is supplied inelastically. The unique final good is produced with the production function

$$Y(t) = \frac{1}{1-\beta} \left[\int_0^{N(t)} x(\nu,t)^{1-\beta} d\nu \right] L(t)^{\beta},$$

where $\beta \in (0,1)$, $x(\nu,t)$ denotes intermediate goods of type ν used in final good production at time t, and $N(t)$ is the number of intermediate good types available at time t. Once a particular type of intermediate good is invented, it can be produced by using ψ units of final good. The innovation possibilities frontier of the economy is

$$\dot{N}(t) = \eta N(t)^{-\phi} Z(t), \tag{1}$$

where $\phi > 0$ and $Z(t)$ is total amount of R&D spending. The resource constraint of the economy is $C(t) + X(t) + Z(t) \leq Y(t)$, where $X(t)$ is spending on intermediate goods. There is free entry into research and a firm that invents a new intermediate good type receives a perpetual patent and becomes the monopolist producer of that good. The economy starts with $N(0) > 0$ intermediate goods at time $t = 0$.

(a) Interpret the innovation possibilities frontier in (1).

(b) Define an equilibrium. Characterize the static equilibrium output, wages and monopolist profits for a given number of intermediate goods, and determine the free entry condition.

(c) Show that, if $n = 0$, then there will be no sustained growth in this economy.

(d) Now suppose there is population growth, i.e, $n > 0$, and show that there is a BGP equilibrium. Does the equilibrium have transitional dynamics? Is the equilibrium Pareto optimal? Why or why not?

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