14.452: Economic Growth
Problem Set 3

Due date: November 25, 2009.*

Exercise 1: Consider the canonical OLG model with log preferences

$$\log(c_1(t)) + \beta \log(c_2(t+1))$$

for each individual. Suppose that there is population growth at the rate n. Individuals work only when they are young, and supply one unit of labor inelastically. Production technology is given by

$$Y(t) = A(t) K(t)^{\alpha} L(t)^{1-\alpha},$$

where $A(t+1) = (1+g) A(t)$, with $A(0) > 0$ and $g > 0$.

1. Define a competitive equilibrium and the steady-state equilibrium.
2. Characterize the steady-state equilibrium and show that it is globally stable.
3. What is the effect of an increase in g on the equilibrium?
4. What is the effect of an increase in β on the equilibrium? Provide an intuition for this result.

Exercise 2: Consider the baseline OLG model with population growth at the rate n. Suppose the economy starts with the steady state capital-labor ratio.

1. Suppose the competitive equilibrium is dynamically inefficient, i.e., $r^* < n$, and show that there exists a feasible sequence of unfunded social security payments $\{d(t)\}_{t=0}^{\infty}$ which will lead to a competitive equilibrium that Pareto dominates the competitive equilibrium without social security.
2. Suppose the competitive equilibrium is dynamically efficient, i.e., $r^* > n$, and show that any unfunded social security system will increase the welfare of the current old generation and reduce the welfare of some future generation.

*You can turn in the problem set in Lecture on Tuesday, November 24; or you can turn it in the TA's mailbox on Wednesday, November 25.

Exercise 3: Consider the following continuous-time neoclassical growth model:

$$\int_0^\infty \exp(-\rho t) \frac{c(t)^{1-\theta} - 1}{1-\theta} dt,$$

with aggregate production function

$$Y(t) = AK(t) + BL(t),$$

where $A, B > 0$. Assume population is constant at L.

1. Define a competitive equilibrium for this economy.
2. Set up the current-value Hamiltonian for the representative household. Characterize the solution. Combine this solution with equilibrium factor prices and derive the equilibrium path. Show that equilibrium displays non-trivial transitional dynamics in the sense that capital and output do not grow at constant rates.
3. Determine the evolution of the labor share of national income over time.
4. Analyze the impact of an unanticipated increase in B on the equilibrium path.
5. Prove that the equilibrium is Pareto optimal.

Exercise 4: Consider the following endogenous growth model due to Uzawa and Lucas. The economy admits a representative household and preferences are given by

$$\int_0^\infty \exp(-\rho t) \frac{C(t)^{1-\theta} - 1}{1-\theta} dt,$$

where $C(t)$ is consumption of the final good, which is produced as

$$Y(t) = AK(t)^\alpha H_P^{1-\alpha}(t)$$

where $K(t)$ is capital and $H(t)$ is human capital, and $H_P(t)$ denotes human capital used in production. The accumulation equations are $\dot{K}(t) = I(t) - \delta K(t)$ and $\dot{H}(t) = BH_E(t) - \delta H(t)$, where $H_E(t)$ is human capital devoted to education (further human capital accumulation), and the depreciation of human capital is assumed to be at the same rate, δ, as physical capital for simplicity. The resource constraints of the economy are $I(t) + C(t) \leq Y(t)$ and $H_E(t) + H_P(t) \leq H(t)$.

1. Interpret the second resource constraint.
2. Denote the fraction of human capital allocated to production by $h(t)$ (so that $h(t) \equiv H_P(t)/H(t)$) and calculate the growth rate of final output as a function of $h(t)$ and the growth rates of accumulable factors.

3. Assume that $h(t)$ is constant, and characterize the BGP of the economy (with constant interest rate and constant rate of growth for capital and output). Show that in this BGP, $r^* \equiv B - \delta$ and the growth rate of consumption, capital, human capital and output are given by $g^* \equiv (B - \delta - \rho)/\theta$. Show also that there exists a unique value of $k^* \equiv K/H$ consistent with BGP.

4. Determine the parameter restrictions to make sure that the transversality condition is satisfied.

5. Now analyze the transitional dynamics of the economy starting with K/H different from k^* [Hint: look at dynamics in three variables, $k \equiv K/H$, $\chi \equiv C/K$ and h, and consider the cases $\alpha < \theta$ and $\alpha \geq \theta$ separately].

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