14.30 Introduction to Statistical Methods in Economics
Spring 2009

Problem Set #4

14.30 - Intro. to Statistical Methods in Economics

Instructor: Konrad Menzel

Due: Tuesday, March 17, 2009

Question One

Suppose that the PDF of X is as follows:

$$f(x) = \begin{cases} e^{-x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}.$$

1. Determine the PDF for $Y = X^{\frac{1}{2}}$.
2. Determine the PDF for $W = X^{\frac{1}{k}}$ for $k \in \mathbb{N}$.

Question Two

Suppose that the PDF of a random variable X is as follows:

$$f(x) = \begin{cases} \frac{2}{25}x & \text{for } 0 < x < 5 \\ 0 & \text{otherwise} \end{cases}$$

Also, suppose that $Y \equiv X(5-X)$. Determine the PDF and CDF of Y. You can solve this in two ways. First, you can compute $f_Y(y)$ using the formula given in class:

$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{d}{dy} g^{-1}(y) \right|,$$

taking care that $g(x)$ is piece-wise monotonic. Second, you can solve this by finding $F_Y(y) = P[Y \leq y]$ directly, as we did in recitation. You will receive extra-credit if you can do it both ways.

Question Three

(Bain/Engelhardt, p. 226)

(6 points) Let X be a random variable that is uniformly distributed on $[0, 1]$ (i.e. $f(x) = 1$ on that interval and zero elsewhere). Use two techniques from class ("2-step"/CDF technique and the transformation method) to determine the PDF of each of the following:

1. $Y = X^{\frac{1}{4}}$.
2. $W = e^{-X}$.
3. $Z = 1 - e^{-X}$.

Question Four

(Bain/Engelhardt p. 227)

If $X \sim Binomial(n, p)$, then find the pdf of $Y = n - X$.

Question Five

(Bain/Engelhardt p. 227)

Let X and Y have joint PDF $f(x, y) = 4e^{-2(x+y)}$ for $0 < x < \infty$ and $0 < y < \infty$, and zero otherwise.

1. Find the CDF of $W = X + Y$.
2. Find the joint pdf of $U = \frac{X}{Y}$ and $V = X$.
3. Find the marginal pdf of U.